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Time-varying Stock Market Correlations and Correlation Breakdown

1. Introduction

In an economic environment where an international perspective is increasingly relevant, knowledge of the international equity market structure becomes important for several reasons. On the one hand, economists are interested in the worldwide equity comovement structure as it influences capital flows, investment and consumption decisions. On the other hand, investors are interested in comovement relationships for diversification motives. This is especially true because international diversification is often regarded as the best way to improve portfolio performance. However, the extent to which portfolio managers can practically implement risk reduction depends not only on the intertemporal stability of (expected) stock market returns, but also on (expected) correlations, which measure the amount of similarity in the movement of financial markets.

Yet, only recent experience has highlighted the fact that risk parameters are unstable and that international equity correlations can rise quickly and dramatically. Likewise, the global stock market crash in October 1987 increased research interest into how financial disturbances transmit from one market to another and fostered a more cautious attitude towards international diversification and risk management. Firstly, due to the progressive removal of impediments to international investment, international financial markets have become increasingly more integrated. Integrated markets, however, are likely more correlated and could thus erode the advantages of international risk diversification in the long run. Secondly, recent studies suggest that the case for international risk diversification may have been somewhat overstated, since the risk protection brought by diversifying assets across markets is likely to be reduced when it is needed most, namely in periods of high volatility or, worse, extreme negative price movements. Indeed, so-called correlation breakdowns, if they occur, call into question the usefulness of diversifying and hedging operations based on correlations estimated from historical data, since they may be inaccurate precisely when they are most desired.

This paper investigates the following questions. What is the temporal behaviour of international equity market returns? Are they multivariate normally distributed as commonly assumed? If not, does linear correlation correctly measure the dependence structure of international stock markets? Does correlation really and significantly fluctuate

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over time? If yes, is there any particular pattern emerging? Is there a positive time-trend for correlations? Is correlation positively related to volatility? What actually is an extreme event on financial markets? Does correlation breakdown really occur? If yes, does it occur for all countries in the same way? What are the consequences for risk and asset management?

The paper is organised as follows. After a brief review of the literature in Section 2, Section 3 examines the temporal behaviour of selected stock market returns and whether these are normally distributed. It also briefly introduces both the concepts of linear and rank correlation and discusses the shortcomings associated with the use of them. Finally, some preliminary evidence is given that international equity market correlations fluctuate over time. Section 4 investigates the instability of correlation in more depth. Further, it turns to the question of a time-trend inherent to international correlation. Section 5 gives two (new) definitions of extreme events and examines the asymmetry of the correlation structures. Section 6 introduces a procedure for blending variance-covariance matrices from different risk regimes. This enables investors to express views about the likelihood of each risk regime and to differentiate their risk aversion to them. Section 7 provides a brief conclusion of the main empirical findings and their implications.

2. Review of the literature

In the literature, the investigation of the stock market comovement structure is based on many different approaches. To start with, EUN/SHIM (1989) and OERTMANN (1995) investigate the international transmission mechanism of stock market movements by estimating a vector autoregressive (VAR) system. Their results suggest that the dynamic response pattern is generally consistent with the notion of informational efficient international stock markets. Very interestingly, both studies find Switzerland to be the most interactive market. Similarly, KOCH/KOCH (1991) estimate a dynamic simultaneous equations model to describe the contemporaneous and lead-lag relationship across eight national equity markets. Their results reveal growing market interdependence over time, however mainly within the same geographical region. KING/WADHWANI (1990) examine the transmission of volatility between international stock markets. Their empirical evidence indicates that an increase in volatility leads in turn to an increase in the size of contagion effects. In contrast, however, a later study by KING/SENTANA/WADHWANI (1994) concludes that this was only a transitory increase caused by the October 1987 crash. KAROLYI/STULZ (1996) explore the fundamental factors that affect cross-country correlations based on overnight and intraday returns for a portfolio of Japanese stocks using their NYSE-traded American Depository Receipts (ADRs) and a matched-sample portfolio of US stocks. They find that US macroeconomic announcements, shocks to the Yen/Dollar foreign exchange rate and Treasury bill returns, and industry effects have no measurable influence on the correlation between US and Japanese returns, while large shocks to broad-based market indices positively impact both the magnitude and persistence of correlation. Also concentrating on the US and Japanese stock markets, LIN/ENGLE/ITO (1994) show that information revealed during the trading hours of one market has a global impact on the returns of the other market. In order to extract global factors, they describe a signal-extraction model with GARCH processes and an aggregate-shock model, in which investors use return surprises from the other market to set opening prices. The often observed heteroskedasticity in return series suggests to use (multivariate) (G)ARCH models in order to investigate time-varying moments. LONGIN/SOLNIK (1995) find that a bivariate GARCH(1,1) model with constant conditional correlation helps to capture some of the evolution in the conditional covariance structure. Explicit modelling of conditional correlation indicates an increase of international correlation

over the past thirty years. Based on a test inspired from threshold GARCH models (see NG/ENGLE (1993)), they also find that correlation rises in periods of high volatility.
Several studies test conditional (international) capital asset pricing models. BOLLERSLEV/ ENGLE/WOOLDRIDGE (1988) estimate a capital asset pricing model (CAPM) with time-varying covariances. Using a multivariate generalisation of the ARCH-M model introduced by ENGLE/LILIEN/ROBINS (1987), they find that conditional covariances are variable over time and a significant determinant of time-varying risk premiums. On the other hand, DE SANTIS/GERARD (1997) test a conditional CAPM for the world's eight largest equity markets using a parsimonious GARCH parameterisation. They conclude that, although some severe market declines were contagious, the expected gains from international diversification for a US investor is positive and has not significantly declined over the last two decades.

3. Analysis of the return series

In financial markets theory, one of the most important assumption is that continuously compounded stock market returns are multivariate normally distributed. This asumption is also prerequisite of the use of linear correlation as a measure for interdependence between international equity returns. However, although the assumption of (multivariate) normality is broadly applied in almost every area of finance, there is also, starting with MANDELBROT (1963) and FAMA (1965), overwhelming empirical evidence that empirical distributions of stock returns are significantly different from Gaussian distributions. But if returns are non-normal, linear correlation can not capture the whole dependence structure between them. Despite the fact that the concept of linear correlation is a central idea in finance, it is thus necessary to briefly resume the shortcomings associated with the use of it. In addition, the non-parametric rank correlation is introduced. To complete this section, the dependence structure of international stock markets is demonstrated both using linear and rank correlation.

3.1 Description of the data

The sample was restricted to Switzerland (CHF) and the Group of Seven countries (United States (USD), United Kingdom (GBP), Canada (CAD), Germany (DEM), Italy (ITL), France (FRF) and Japan (JPY)). These are the eight largest markets in terms of size and their combined stock market capitalisation is more than 90 percent of that of the world. For each market, monthly nominal returns are calculated in local currency.[1] The sample period is from January 1973 to December 1999.

3.2 Summary statistics

Consider the summary statistics for the monthly returns in Table 1. Over the total period from January 1973 to December 1999, annualised mean returns range from 6.26% for Japan to 12.41% for Italy across the markets. Except for Italy, the medians are higher than the means, indicating that the distributions tend to be skewed to the left for these countries. Maximum and minimum values differ greatly from the mean in all countries. Here, it is United Kingdom that shows the most distinct extreme monthly observations (maximum: +40.05% and minimum: –31.82% – compared to an annualised mean of 10.89%). Annualised standard deviation (volatility) of the returns is lowest in the United States with 15.08%, whereas the returns in Italy are the most volatile ones (25.53%).
The coefficients of skewness and kurtosis allow to evaluate the distributional characteristics of the returns. If returns are exactly normally distributed, the skewness is equal to zero and the kurtosis is equal to three. As already indicated above,

Table 1: Summary statistics for monthly (log-)returns of the selected equity markets

	CHF	USD	GBP	CAD	DEM	ITL	FRF	JPY
Mean (ann.%)	7.97	9.65	10.89	7.06	7.98	12.41	11.71	6.27
Median (ann.%)	12.73	12.66	16.69	8.60	8.87	1.21	19.31	7.32
Maximum (%)	18.25	15.81	40.05	14.96	4.34	27.71	20.42	17.45
Minimum (%)	–27.46	–24.10	–31.82	–23.56	–26.82	–22.38	–24.53	–24.43
Volatility (%)	16.53	15.08	20.46	16.70	17.00	25.53	21.85	18.28
Skewness	–1.07	–0.67	0.23	–0.89	–0.73	0.27	–0.48	–0.31
Kurtosis	8.47	6.89	11.63	6.96	6.36	3.97	4.72	5.32
Jarque-Bera	465.58	228.34	1008.16	255.19	180.97	16.59	52.40	77.44
Probability	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*

Sample: 1973: 01 – 1999: 12 (324 monthly observations). * Significant at the 5% level.

Table 1 shows that except for Italy and United Kingdom, all stock market returns are negatively skewed. Here it is Switzerland that seems to have the most negatively skewed and therefore asymmetric market. The high values of the kurtosis support the fact that financial time-series tend to be heavy-tailed. This means that there are too many observations around the mean and, in contrast, too many outliers (extreme events). To examine the combined effect of skewness and kurtosis, the Jarque-Bera test of normality is conducted. Not surprisingly, the null hypothesis of normally distributed returns is clearly rejected on the 5 percent level of significance for all markets.

3.3 Dependence measures and preliminary empirical findings on time-varying correlations

So far, the time-series were analysed each separately. To analyse the comovement (or dependence structure) of random variables such as equity market returns, there are different (statistical) concepts. The following sub-sections define two of them and discuss their properties: the widely applied parametric linear correlation coefficient and the non-parametric rank correlation coefficient.

Parametric linear correlation

As already briefly mentioned above, the use of linear correlation as a measure of dependence between two random variables is essentially based on the assumption that these are multivariate normally distributed (or, more generally, that returns follow elliptical or spherical distributions – for a definition see EMBRECHTS/MCNEIL/STRAUMANN (1999a)). If the assumption of normality is violated, EMBRECHTS/MCNEIL/STRAUMANN (1999b) identify the following shortcomings associated with the linear correlation as measure for dependence: (1) If the structure of dependence is non-linear, linear correlation can not capture it. (2) As the possible values of linear correlation depend on the marginal distributions of the returns, not all values between minus and plus one are necessarily attainable. (3) Perfect positively (negatively) dependent returns do not necessarily have a linear correlation of plus (minus) one. (4) A linear correlation coefficient of zero does not indicate independence. (5) Linear corre-

lation is not invariant under non-linear strictly increasing transformations of the returns. (6) Correlation is only defined when the variances of the returns are finite. It is not an appropriate dependence measure for very heavy-tailed returns where the variances appear infinite.
Despite these shortcomings, linear correlation is a central idea in finance. It lies for example at the heart of the CAPM and the arbitrage pricing theory (APT). Linear correlation, however, is not the only measure of stochastic dependence between random variables. Another possible measure of dependence is the non-parametric rank correlation.[2]

Non-parametric rank correlation

By turning to the non-parametric rank correlation, certain of the above stated theoretical deficiencies of standard linear correlation can be repaired. Using the SPEARMAN definition of rank correlation, the exact structure of dependence is not of interest and outliers only have limited impact. However, rank correlation can not be manipulated in the same easy way as linear correlation. As the variance is not defined, it is not possible to use it in the context of the mean-variance analysis applied in Section 6. Nonetheless, since it was already showed that returns are non-normal and that there are significant outliers, it is worth to compare the behaviour of linear correlation and rank correlation.

Preliminary empirical findings on international equity market correlation

The following section now turns to the empirical dependence structure of international stock markets. Table 2 reports linear correlations calculated over the whole sample period, mirroring the long-term interdependence of the international equity markets. The highest correlation coefficient is 72.99%, documented for Canada and the United States, and the lowest is 28.66%, reflecting the relationship between returns in Italy and Japan. Switzerland exhibits the highest equally weighted average correlation with the seven other stock markets: the average coefficient is as high as 56.73%. High average values are also measured for the United States (52.56%), France (50.82%) and United Kingdom (50.31%). The lowest mean correlation with other markets is measured for Japan (32.93%), followed by Italy (39.44%).

Table 2: Correlation matrix over the whole period 1973: 01 – 1999: 12

	CHF	USD	GBP	CAD	DEM	ITL	FRF	JPY	Avg
CHF	1	64.44	61.13	58.14	**69.50**	45.91	**58.70**	39.26	**56.73**
USD		1	64.06	**72.99**	47.59	32.73	53.04	33.05	**52.56**
GBP			1	56.59	45.45	39.43	52.23	33.29	**50.31**
CAD				1	42.47	36.45	51.81	31.28	**49.96**
DEM					1	42.94	56.73	31.69	**48.05**
ITL						1	49.96	**28.66**	**39.44**
FRF							1	33.26	**50.82**
JPY								1	**32.93**
								Total Avg	**47.60**

Values are multiplied with 100 [%].

Moreover, the correlation matrix reveals certain ‚regional correlation clusters‘. That is, typically high correlation coefficients are documented for countries which are located in the same region. Examples are USA and Canada (72.99%). Switzerland and Germany (69.50%) and Switzerland and France (58.70%). United Kingdom, on the other hand, seems to be more correlated with the United States and Canada than with other European stock markets (except Switzerland).

It is often claimed that international equity market correlations are unstable over time. Recent studies such as ERB/HARVEY/VISKANTA (1994), LONGIN/SOLNIK (1995), SOLNIK/BOUCRELLE/LE FUR (1996) and OERTMANN (1997), to name just a few, document that correlation matrices are unstable over time. Conversely, other studies such as KAPLANIS (1988) or FORBES/RIGOBON (1999) conclude that the relationship between international equity markets is less unstable as often claimed. To get a first idea of this issue, the above sample period is divided into three sub-periods (January 1973 to December 1986, 1987 and January 1988 to December 1999). Table 3 reports equally weighted average correlations and volatilities for all periods.

Comparing these values, it becomes evident that average correlation has increased notably from 40.55% in the period from January 1973 to December 1986 to 55.16% in the period from January 1988 to December 1999. Even more, in 1987 (including the October 1987 stock market crash), average correlation between the international stock market returns was a high as 68.44%. Thus, it seems that stock market interdependence has largely grown in times of the market crash. Compared with the average correlation computed over the whole sample period (47.60%), it is clearly evident that correlations fluctuate considerably over time.

So far, the analysis was based on linear correlation. It is now interesting to compare these results with the results obtained using rank correlation.

Table 3: Equally weighted average correlations and volatilities over different time periods

Linear correlation	**CHF**	**USD**	**GBP**	**CAD**	**DEM**	**ITL**	**FRF**	**JPY**	**Avg**
1973: 01 – 1999 : 12	56.73	52.56	50.31	49.96	48.05	39.44	50.82	32.93	**47.60**
1973 :01 – 1986 : 12	49.14	45.81	44.04	42.37	35.57	32.63	41.79	33.02	**40.55**
1988: 01 – 1999 : 12	62.90	58.20	58.48	55.38	61.06	49.65	60.20	35.42	**55.16**
1987: 01 – 1987 : 12	76.56	74.65	73.99	76.96	63.90	55.91	78.64	48.52	**68.64**
Rank correlation	**CHF**	**USD**	**GBP**	**CAD**	**DEM**	**ITL**	**FRF**	**JPY**	**Avg**
1973: 01 – 1999: 12	47.13	46.40	42.96	41.69	41.68	37.42	44.59	29.53	**41.43**
1973: 01 – 1986: 12	42.57	40.90	36.74	36.02	31.50	30.95	35.74	29.04	**35.43**
1988: 01 – 1999: 12	53.81	53.55	56.08	49.35	55.80	47.34	56.34	32.09	**50.54**
1987: 01 – 1987: 12	39.96	47.75	27.97	56.34	29.77	30.57	54.05	36.66	**40.38**
Volatility	**CHF**	**USD**	**GBP**	**CAD**	**DEM**	**ITL**	**FRF**	**JPY**	**Avg**
1973: 01 – 1999: 12	16.51	15.06	20.43	16.67	16.97	25.49	21.82	18.25	**18.90**
1973: 01 – 1986: 12	13.98	15.26	22.78	17.33	14.97	27.19	23.18	14.04	**18.59**
1988: 01 – 1999: 12	16.94	12.42	14.61	14.40	16.89	23.23	19.26	21.67	**17.43**
1987: 01 – 1987: 12	31.46	30.64	37.89	28.43	31.39	21.81	26.86	23.27	**28.97**

Values are multiplied with 100 [%].

Generally and in line with the findings of KAPLANIS (1988), average rank correlation is lower than linear correlation in all periods. At the same time, similar to linear correlation, rank correlation is also significantly higher in the period after the crash than in the pre-crash period. In 1987, however, rank correlation is with 40.38% hardly any different from the long-term rank correlation of 41.43%. This seems to confirm that outliers such as the crash in October 1987 only have limited impact on rank correlation.

To summarise, there is preliminary evidence that the interdependence among international stock markets fluctuates widely over time and has increased over the last three decades. It seems that linear correlation increases in periods of high market volatility, while rank correlation seems to be less affected by extreme return fluctuations.

4. Time-measured observations

The previous section gives preliminary evidence that correlations fluctuate widely over time. Additionally, it is often stated that correlations increase in periods of high market turbulence. Another often addressed question is whether the growth in international capital flows and market integration raised the general level of correlation in the past 30 years. Section 3 now investigates the comovement structure of international equity markets in more depth. It is shown that, similar to the expected rates of returns, correlations and volatilities vary widely according to the particular data window considered, that there is a positive relationship between correlation and volatility and that average correlation has increased considerably since the early seventies.

It is not easy to define an appropriate historical analysis. Hence, in the first part of this section, volatilities, covariances and correlations are estimated on the basis of rolling windows. As this has several disadvantages, the argumentation in the second part is based on daily data.

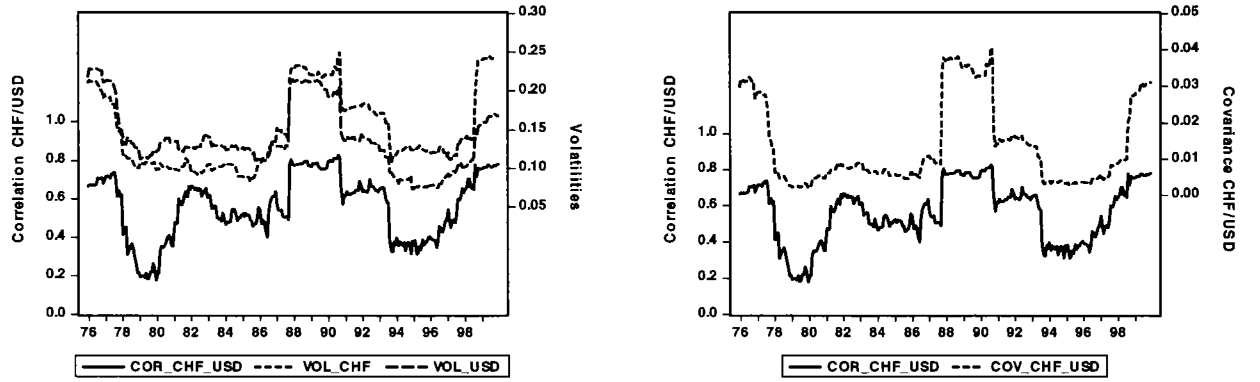
4.1 Time-varying correlations: Rolling windows

It has been recognised for a long time that the sizes and signs of correlations often depend on the sample period and the investment horizon chosen. So, to start with, the computation of volatilities, covariances and correlations is based on rolling windows that include T preceding monthly returns data. Figure 1 shows correlations, covariances and volatilities for Switzerland and the United States calculated for rolling windows with the length of T = 36 months.

Any first inspection of the lines illustrated in these graphs suggests that volatility, covariance and correlation change through time markedly. In addition, it is obvious that the correlation structure is extremely sensitive to extreme events such as the October 1987 stock market crash. As soon as October 1987 falls into the calculation period, correlation increases dramatically and persists at these high levels for the whole sampling period. Looking at Figure 1 also gives evidence that correlation tends to be higher in periods of high market volatility. It seems that US and Swiss volatilities and the correlation between them follow a very similar pattern over time. It is indeed an often noted stylised fact that there is not only a significant positive relationship between volatility and correlation, but that this relationship is even more pronounced if volatility is high. In the literature, there is also overwhelming evidence that correlation and volatility are positively related. LONGIN/SOLNIK (1995) find that their correlation forecast increases in periods of high (conditional) volatility. SOLNIK/BOUCRELLE/LE FUR (1996) and DROBETZ/ZIMMERMANN (2000) statistically verify the presumption that markets tend to move more in parallel in high volatility states using a regression approach. They also find the hypothesis clearly confirmed. To sum up, it seems that covariances increase faster than variances.[3]

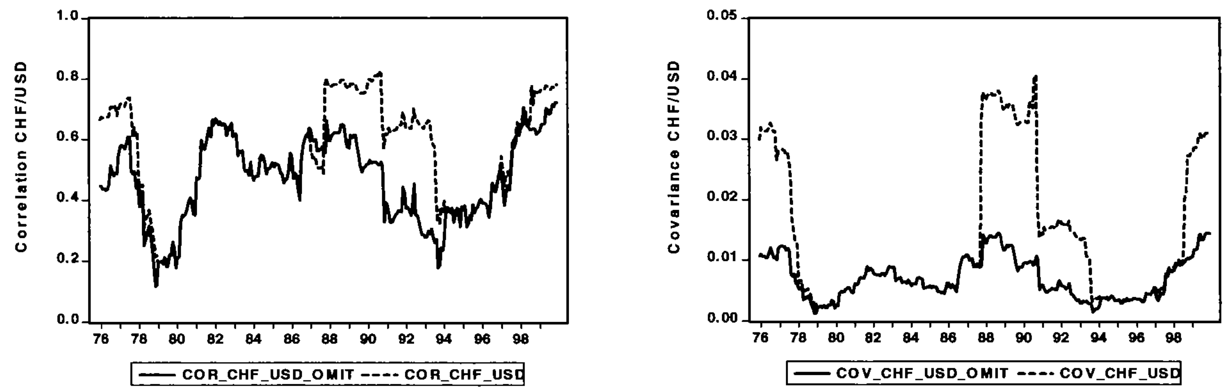
Figure 2 shows the effect of omitting outliers on correlations and covariances between Switzerland

Figure 1: Linear correlations and volatilities (LHS) and covariances (RHS) for Switzerland and the United States



Length of rolling windows: T = 36 months. Calculations based on monthly returns.

Figure 2: Linear correlations (LHS) and covariances (RHS) for Switzerland and the United States: Extreme negative and positive monthly returns omitted



Length of rolling windows: T = 36 months. Calculations based on monthly returns.
Omitted outliers: Switzerland: 10.87 (–27.5%), 8.98 (–20.3%), 9.98 (–16.6%), 8.90 (–14.9%), 9.74 (–12.7%); 10.98 (18.3%), 1.75 (15.8%), 5.90 (12.3%), 2.91 (9.7%), 1.97 (9.2%); United States: 10.87 (–24.1%), 11.73 (–13.9%), 9.74 (–13.8%), 8.98 (–11.8%), 7.74 (–9.9%); 10.74 (15.8%), 1.87 (13.3%), 1.75 (12.6%), 10.98 (12.3%), 1.76 (10.7%).

and the United States. Inspection of the graphs indicates that correlations and covariances are generally lower in absence of these extreme events. While this is especially distinct for the period containing the October 1987 stock market crash, it seems that the recent increase in correlation is surprisingly hardly affected by the large market movements in August, September and

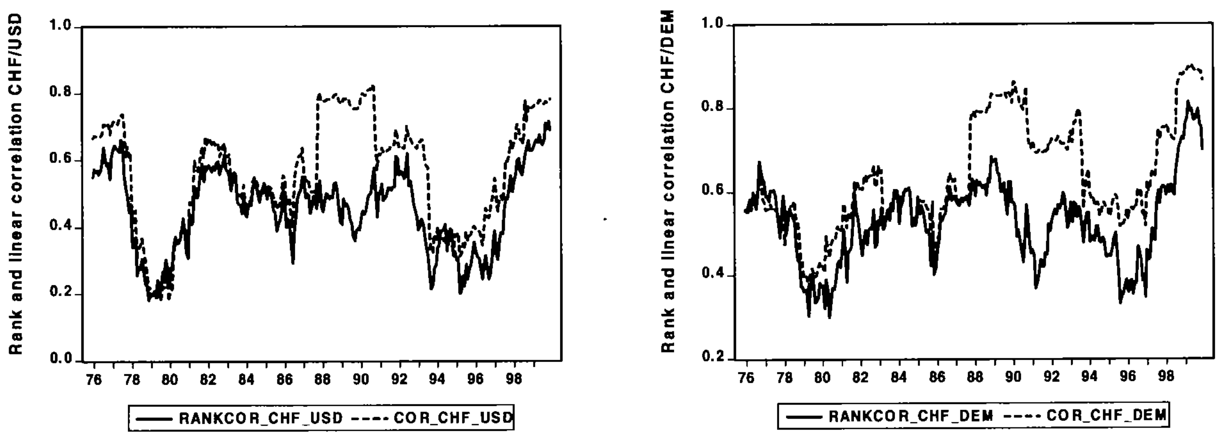
October 1998. Note that the effect of omitting extreme negative and positive returns is more distinct for the covariances as for the correlations. In contrast to correlations, covariances are significantly lower in the periods containing the oil shocks in the early seventies, the October 1987 crash and the recent Asia and Russia crises.

To continue the analysis based on rank correlation, Figure 3 compares rank and linear correlation between Switzerland and the United States and Germany, respectively. Overall, the picture is very similar to the one with omitted outliers. While rank correlation is quite stable over the October 1987 stock market crash, it has increased considerably and in line with linear correlation just recently.

Stability of correlation matrices

An evaluation of changes in market correlation by simply looking at average correlation coefficients does certainly not allow conclusions that are statistically reliable with respect to the whole correlation structure. To get a more reliable picture of the stability of the correlation structure, the above descriptive analysis is now complemented by a statistical test for the stability of the correlation matrices over time. Here, a first opportunity is to construct confidence intervals. Confidence intervals, however, are restricted to pairwise studies. A better measure of the stability of the correlation structure over time would involve all correlation coefficients simultaneously. Therefore, the following analysis is based on the BOX-M test statistic, originally developed by BOX (1949). The BOX-M test statistic is actually a joint test on the equality of two or more variance-covariance matrices, but not the correlation matrices. TANG (1995), however, provides a theoretical extension of the test so that it can be used directly for testing the equality of correlation matrices, too. To test the hypothesis that correlation matrices are equal across time periods, the whole sample pe-

Figure 3: Rank correlations and linear correlations for Switzerland, the United States (LHS) and Germany (RHS)



Length of rolling windows: T = 36 months. Calculations based on monthly returns.

Table 4: Tests of equal correlation matrices (BOX-M test statistics)

One-year sub-periods

73–74	74–75	75–76	76–77	77–78	78–79	79–80	80–81	81–82	82–83	83–84	84–85	85–86
2.42	**2.81**	**2.93**	**2.44**	**2.11**	**2.03**	**2.66**	**2.80**	**2.06**	**1.59**	**2.24**	**2.67**	**2.64**
86–87	87–88	88–89	89–90	90–91	91–92	92–93	93–94	94–95	95–96	96–97	97–98	98–99
3.49	**3.42**	**3.00**	**3.31**	**3.27**	**2.65**	**2.14**	**2.48**	**2.65**	**2.50**	**2.97**	**3.93**	**4.19**
Average 2.75					**Maximum 4.19**					**Minimum 1.59**		

Three-year sub-periods

73/75–76/78	76/78–79/81	79/81–82/84	82/84–85/87	85/87–88/90	88/90–91/93	91/93–94/96	94/96–97/99
7.99	**7.15**	**7.74**	**10.70**	**13.30**	**10.76**	**8.66**	**13.33**
Average 9.96			**Maximum 13.33**			**Minimum 7.15**	

F-value: (approximately) 1.57 (5% significance level).

riod is divided into 27 one-year and 9 three-year periods. The results are reported in Table 4.

The BOX-M test statistics indicate that correlation matrices are statistically significant unstable over time (F-values between 1.59 and 4.19 for one-year sub-periods and 7.15 and 13.33 for three year sub-periods, respectively).[4] In addition, the results show that the shorter the time period considered, the greater the degree of stability in the correlation matrices.

In the literature, a number of different test statistics are applied. KAPLANIS (1988) studies the stability of correlation and variance-covariance matrices of monthly returns of ten markets over a fifteen-year period (1967–1982). Using BOX and JENRICH (1970) tests, she can not reject the null hypothesis that the correlation matrix is constant over two adjacent sub-periods, even though at the 15% level. In contrast, variance-covariance matrices are found to be much less stable.[5] Other studies such as SHAKED (1985) and HEPP (1990) test the equality of correlations restricted to pairwise studies. In contrast to the findings presented here, they both find a positive relation between the stability of the sample correlations and the investment horizon. In other words, their results indicate a higher degree of stability over longer periods than shorter ones.[6] Thus, it seems that testing correlation coefficients pairwise and, on the other hand, testing entire correlation matrices yields completely different and opposite results. This raises the question of which approach is more appropriate. Testing for the entire variance-covariance and correlation matrices avoids the problem of ambiguity created when some pairs of markets show stable relationships while other markets have unstable comovement patterns. However, using a single, entire matrix may reflect double counting, which will not happen in pairwise studies. Nonetheless, as the analysis presented here concentrates only on the contemporary relationships and ignores any lead-lag structure, the advantages of using matrix-wide comparison seem to outweigh the disadvantages.

Overall, it thus seems that structural movements appear to be slight (but statistically significant) on a year-to-year basis, but more pronounced in the long run.

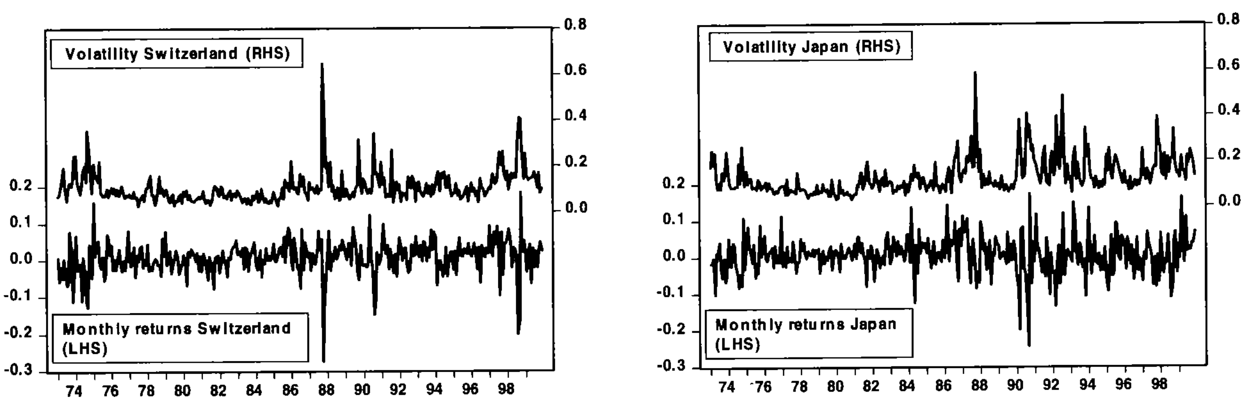
4.2 Time-varying correlations: Daily data

So far, the computation of volatilities, covariances and correlations was based on rolling windows that included T previous monthly returns data. However, one major disadvantage of the use of moving averages is that the resulting time-series are heavily autocorrelated and thus difficult to handle in further econometric analysis. In order to overcome this difficulty associated with the use of overlapping samples, SCHWERT (1989) and FRENCH/SCHWERT/STAMBAUGH (1987) estimate the monthly standard deviation of stock returns using daily returns. They argue that using non-overlapping samples of daily creates an estimation error that is uncorrelated through time. In addition, since volatilities are not constant over time, they emphasise that a more precise estimate of the standard deviation for a particular month is obtained by using only returns within that month. Their argumentation, however, is solely based on volatilities and not on covariances or correlations. Yet, investigating international dependence structures founded on daily data causes one major problem. Namely, national stock markets are opening in diverse time zones with different opening and closing times, thereby making return observations non-synchronous. Thus, daily returns reflect information revealed over different time intervals.

To control for the fact that markets in different countries are not open during the same time, FORBES/RIGOBON (1999) propose to use rolling-average, two-day returns. Thus, the remaining part of Section 4 is based on rolling-average, two-day return series that are chopped into monthly blocks within homoskedasticity is assumed.

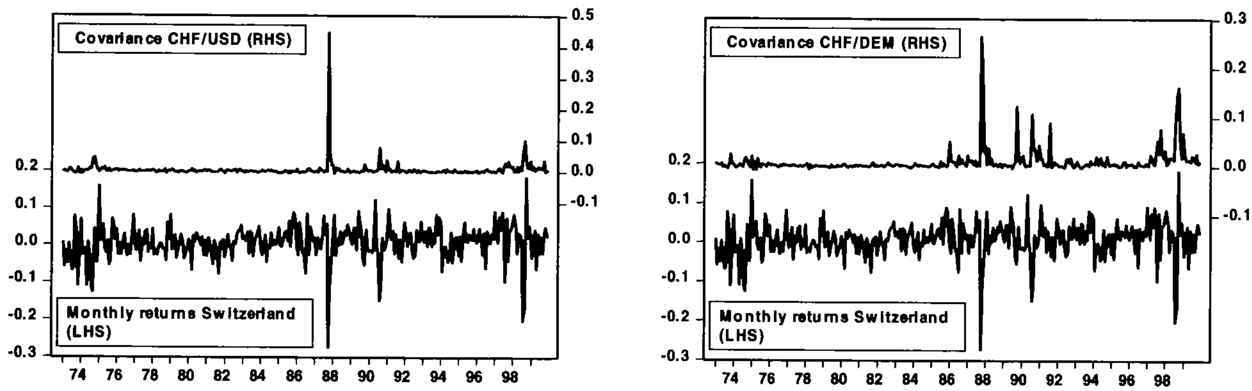
Figure 4 shows the monthly return series and the corresponding time-series of volatilities for Switzerland and Japan, calculated based on daily data and sampled for monthly periods. In Switzerland, especially the large price movements during the early seventies, the October 1987 stock market crash and the Asian and Russia crises in 1998 caused volatility to rocket. In Japan, the nineties were generally very volatile. In general, the volatility clustering patterns are only apparent.

Figure 4: Returns and volatilities for Switzerland (LHS) and Japan (RHS)



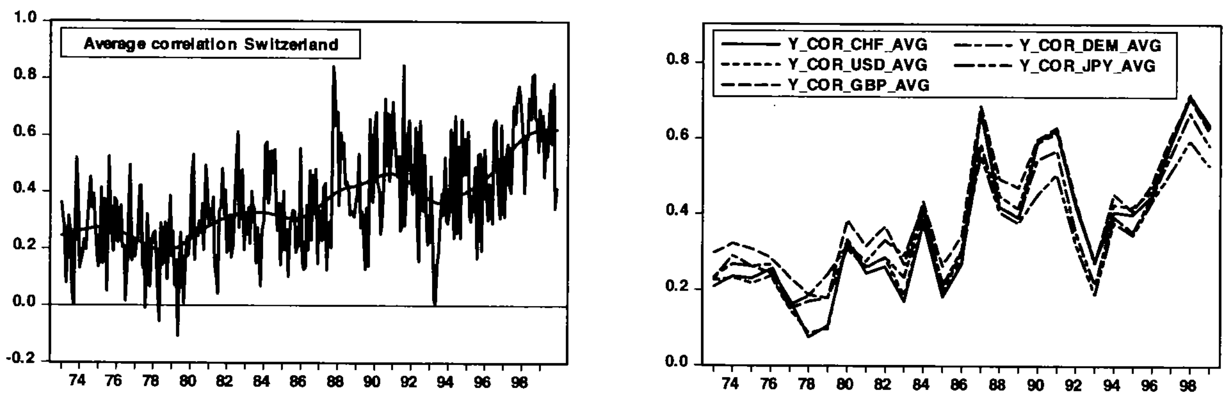
Calculations based on rolling-average, two-day return series (monthly sampled).

Figure 5: Covariances between Switzerland and United States (LHS), and Germany (RHS)



Calculations based on rolling-average, two-day return series (monthly sampled).

Figure 6: Average correlation for Switzerland – monthly sampled (LHS).
Average correlation for Switzerland, United States, United Kingdom, Germany and Japan – yearly sampled



LHS: Calculations based on rolling-average, two-day return series (monthly sampled). (Smoothed series: HODRICK-PRESCOTT (1997) Filter). RHS: Calculations based on rolling-average, two-day return series (yearly sampled).

Similar, Figure 5 shows the return series for the Swiss market and the covariances between Switzerland and the United States and Germany, respectively. Here the peak in October 1987 is very distinct and thus indicates that the October 1987 stock market crash was indeed a exceptional occurrence, at least within the sample period starting in 1973. Figure 5 also seems to give some preliminary evidence that the covariances increased over the last decade.

Figure 6 plots (equally weighted) average correlation for Switzerland and, still based on daily data, but sampled for yearly periods, average correlation for Switzerland, United States, United Kingdom, Germany and Japan. It is evident that the estimates of average correlations are very volatile, reaching levels as high as 0.8 and as low as –0.1. Moreover, the smoothed series shows that there is a distinct upward shift in average correlation and that average correlation is strongly synchronised across the countries. Again, Figures 5 and 6 strongly indicate that variances, covariances and correlations are unstable over time.

4.3 Is there a time-trend?

It is often stated that the progressive removal of impediments to international investment, as well as the growing political, economic and financial integration positively affected international market linkages. In order to investigate whether correlation (and volatility) exhibited such a positive time-trend, simple least-squares lines were fitted over the total sample period (January 1973 to December 1999) for monthly sampled, daily data. The results are reported in Table 5.

Table 5 indicates that, in average, there is no time-trend in volatilities. In contrast, however, average correlations have significantly increased for all countries. Here, the highest increase is measured for Japan (plus 40.86%) and the lowest for Germany (plus 26.11%). Average correlation for Switzerland increased a 35.35% since January 1973. On the whole, average correlation increased almost 40% over the last three decades. These results are fully in line with the findings presented in LONGIN/SOLNIK (1995). While they do not find a secular increase in expected market volatility, they find an average increase in conditional correlation of 36% over the period of 1960 to 1990. On the other hand, they also claim that such a constant linear trend is not consistent with the definition of a correlation coefficient. While this is certainly true, there exists no theory of the exact form of other forms for time-trends. SOLNIK/BOUCRELLE/LE FUR (1996) also fit simple least-squares lines over their total sample period. They conclude that there is no trend easy to identify in their samples. However, as they work with rolling windows and thus with heavily autocorrelated time-series, their results should be interpreted with caution.

Table 5: Time-trends: (Equally weighted average) correlation and volatility

Correlation	CHF	USD	GBP	CAD	DEM	ITL	FRF	JPY	Avg
β	1.091*	1.181*	1.112*	1.023*	0.806*	0.866*	0.996*	1.261*	1.229*
Total	**35.35%**	**38.26%**	**36.03%**	**33.15%**	**26.11%**	**28.06%**	**32.27%**	**40.86%**	**39.82%**

Volatility	CHF	USD	GBP	CAD	DEM	ITL	FRF	JPY	Avg
β	0.148*	0.066	–0.277*	–0.035	0.190*	–0.073	–0.092	0.306*	0.012
Total	**4.80%**	**2.14%**	**–8.97%**	**–1.13%**	**6.16%**	**–2.37%**	**–2.98%**	**9.91%**	**0.39%**

Sample: 1973: 01 – 1999: 12. β multiplied with 1000. * Significant at the 5% level. Heteroskedasticity and autocorrelation consistent standard errors and covariances (according to NEWEY-WEST (1987)).

5. Event-measured observations

The previous section demonstrated that international correlation increases in periods of high market turbulence, irrespective of the direction of the markets. In addition, however, it is often claimed that the correlation structure of large returns is asymmetric, that is, international correlation of large negative stock returns differs from that of usual or positive returns. To put it more precisely, correlation is said to be highest in periods of extremely volatile down or bear markets.
The purpose of Section 5 is therefore to investigate whether correlation between international stock markets really exhibits systematic characteristics with respect to the direction of their movements.[7] Since return distributions are not multivariate normal, the usual standard deviation and correlation of returns do not provide sufficient information. Thus, additional information is now gained by focusing directly on the properties of extreme returns, that is, by focusing directly on the tails of the return distributions. In practical application, however, this raised the question of how exactly an (extreme) event on financial markets is defined. In any case, the literature provides no clear answer.

5.1 Correlation breakdown

It is widely argued that when one equity market dives, so, generally, do others. In other words, correlations tend to 100% as volatility rockets and the markets crash. This pattern of extreme synchronised rises and falls in financial markets is then, according to BOYER/GIBSON/LORETAN (1999), termed 'correlation breakdown'. If returns are drawn from a symmetric distribution such as the multivariate normal distribution, however, correlations in up and down markets should be indistinguishable from each other. For this reason, the usual measure of linear correlation represents average comovement in both up and down markets. It was already shown above, however, that returns are not symmetric but skewed to the left. Thus, only separate correlation estimates in different return environments permit detection of whether correlation increases or decreases in down markets.
ERB/HARVEY/VISKANTA (1994) use a semi-correlation analysis to examine whether correlation coefficients are different when the data are segmented by ex-post returns.[8] In doing so, they calculate correlations for months with below-average return (negative semi-correlation) and for months with above-average performance (positive semi-correlation). In other words, a month is classified as up/up if for a specific pair of countries both market returns are above average, while a down/down market is defined as a month where both returns are less than average. Clearly, under the assumption of symmetric return distributions, there is no statistical reason why returns above the mean should have a different correlation from the returns below the mean. However, their empirical findings indicate that down correlations are substantially higher than up correlations.
LONGIN/SOLNIK (1998), on the other hand, use the results of 'extreme value theory' to model the multivariate distribution of large returns for monthly data from 1959 to 1996 for the five largest stock markets (United States, United Kingdom, France, Germany and Japan). They find in a bivariate framework that the correlation of large negative returns (that is, more precisely, that the returns of both countries are below a given threshold) is much greater than normality would suggest. In contrast, they find correlation between the US and the other four markets not inconsistent with the assumption of multivariate normality for large positive returns. They thus conclude that the benefits of international risk reduction in extremely volatile periods have been overstated.
A third example is given in CHOW/JACQUIER/KRITZMAN/LOWRY (1999). They introduce the vector distance from multivariate av-

erage as a measure for identifying multivariate outliers. Hence, a multivariate outlier represents a set of contemporaneous returns that is collectively 'unusual' in the sense that either the performance of an individual asset or the interaction of a combination of assets is 'unusual'.

What actually is an (extreme) event on financial markets?

Both ERB/HARVEY/VISKANTA (1994) and LONGIN/SOLNIK (1998) implicitly assume an extreme event to be associated with returns being at the same time below a predetermined threshold for both countries. Yet, is this actually a relevant extreme event for international investors? One might argue that if the returns in both countries are already below a certain level, the aim of diversification is already missed. It is exactly the imperfect correlation that should help to avoid those uncomfortable situations with both markets diving. But if such a situation is already present, no investor is any more interested in any correlation.
Using their definition of an extreme event, a second difficulty arises, too. How can those events be interpreted where one market is up and the other market is down and where, consequently, correlation is negative?
The approach of estimating multivariate outliers as suggested by CHOW/JACQUIER/KRITZMAN/LOWRY (1999) is also difficult to justify. Their definition of stress as periods that are unusual (that is, possibly containing extreme negative and positive returns) raises the question whether investors are actually equally risk-averse to negative and positive (multivariate) outliers? The answer here is most definitely no. Altogether, it seems important to properly define what and why a certain situation on financial markets is actually (exogenously) classified as a stress event. Thus, the following analysis is based on the following two (different) definitions of an (extreme) event:

1. An (extreme) event for an investor in country j is given when the monthly return of *stock market j* exceeds an (exogenously) given threshold.
2. An (extreme) event for an investor in country j is given when the monthly return of a *world market portfolio* exceeds an (exogenously) given threshold.

While the first definition avoids the difficulties associated with the above discussed approaches suggested by ERB/HARVEY/VISKANTA (1994), LONGIN/SOLNIK (1998) and CHOW/JACQUIER/KRITZMAN/LOWRY (1999), respectively, the second definition seems to be particularly suitable for investors already internationally diversified.

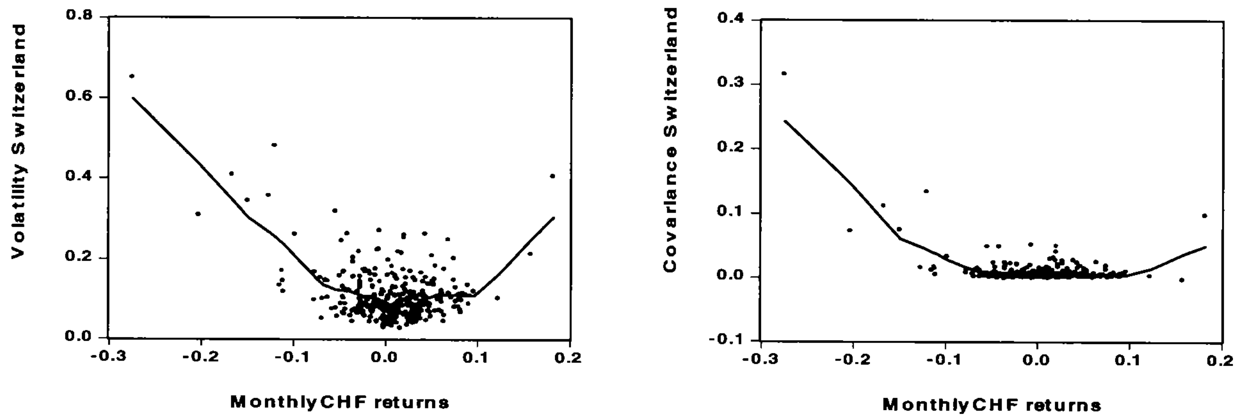
5.2 Empirical evidence

Based on the above given definitions, the remaining part of this section now presents the empirical findings for the selected countries.

Correlation breakdown: Based on daily data

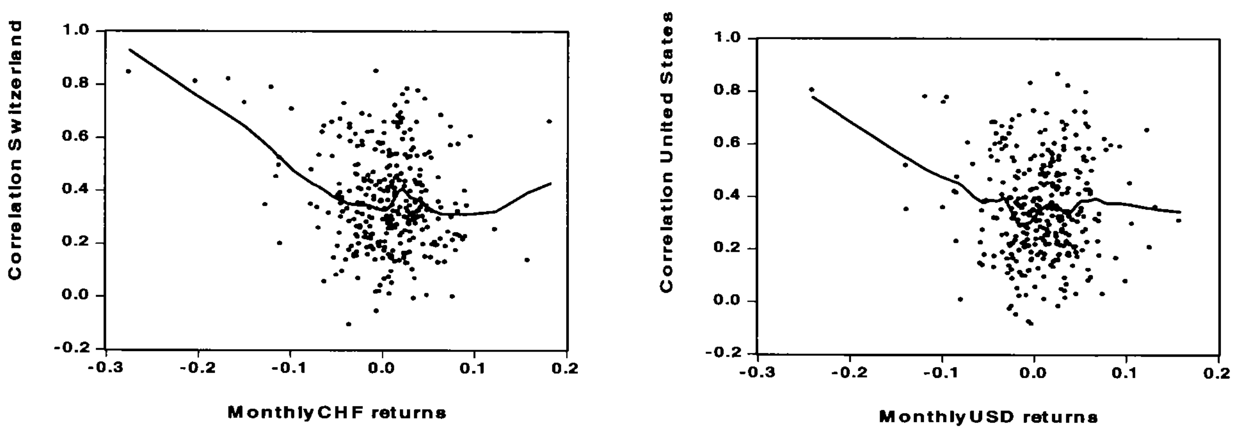
A first opportunity is to consider monthly volatilities, covariances and correlations computed based on monthly sampled, daily data as introduced in the previous section. Figure 7 plots Swiss volatility and average covariance versus Swiss returns. The asymmetry of the volatilities and the covariances due to the impact of extreme negative returns is only apparent. Clearly, the most negative return belongs to the October 1987 stock market crash. This was the month when both volatility and covariance peaked.

Figure 7: Volatility (LHS) and (equally weighted) average covariance (RHS) for Switzerland versus Swiss returns



The graphs plot fitted locally weighted polynomial regressions (monthly sampled).

Figure 8: Average correlation for Switzerland (LHS) and the United States (RHS) versus the respective returns



The graphs plot fitted locally weighted polynomial regressions (monthly sampled).

Figure 8 now extends the analysis to (equally weighted) average correlation for Switzerland and the United States. Similar, but to a less degree, the asymmetry of correlation is also apparent.
To summarise, the above presented analysis is quite sensitive to some individual (extreme) observation pairs. These are, of course, inherent to financial markets and should therefore not be dismissed as special cases. Accordingly, the following analysis now focuses directly on the tails of the monthly return distributions.

Correlation breakdown: Based on monthly data

The first part of the following section is based on definition 1, namely that an (extreme) event for an investor in country j is given when the monthly return of stock market j exceeds an (exogenously) given threshold. More formally, for country j the conditional correlation is calculated as follows:

$$\rho_{i,j} \middle| \underline{\theta} < r_j < \bar{\theta} = \frac{E\left[\left(r_j - \mu_j \middle| \underline{\theta} < r_j < \bar{\theta}\right)\left(r_i - \mu_i\right)\right]}{\sqrt{E\left[\left(r_j - \mu_j\right)^2 \middle| \underline{\theta} < r_j < \bar{\theta}\right]}\sqrt{E\left[\left(r_i - \mu_i\right)^2\right]}} \qquad (1)$$

with $\underline{\theta} = -\infty$ for

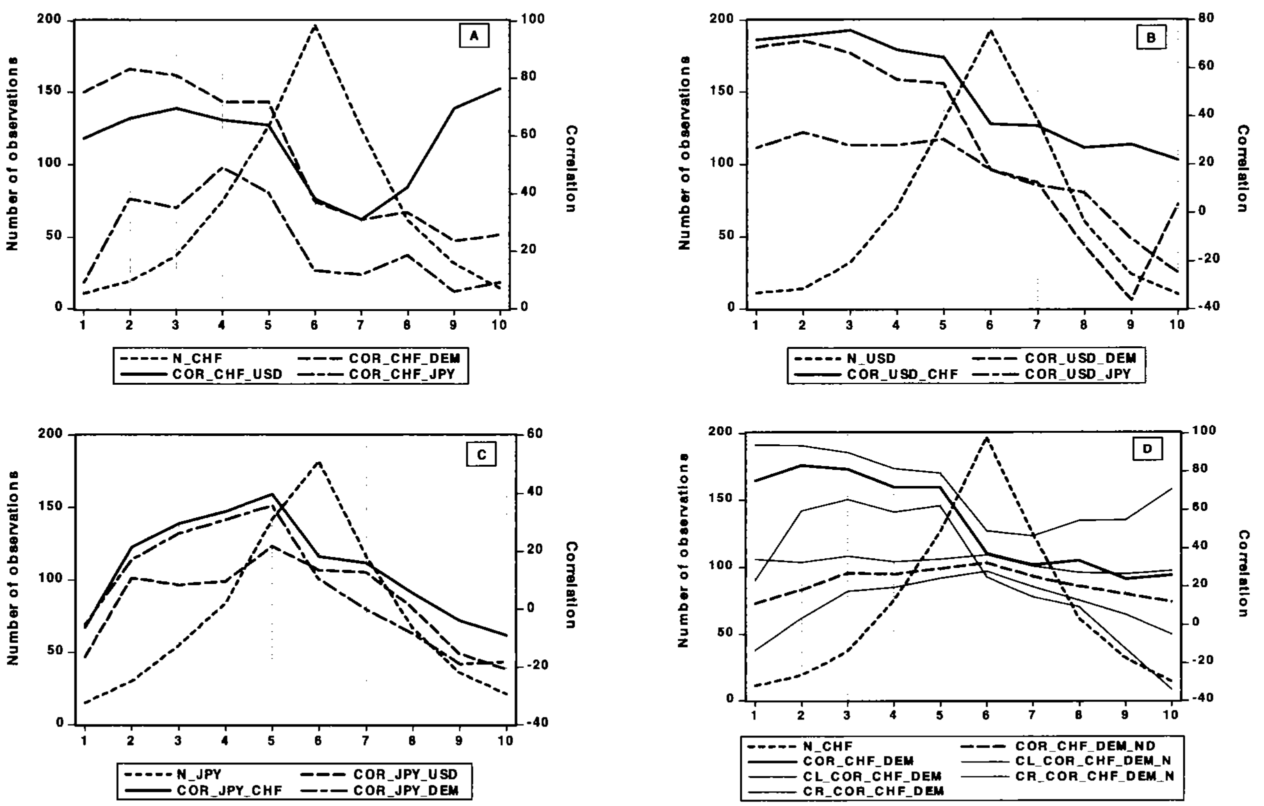
$\bar{\theta} = -8\%(1), -6\%(2), -4\%(3), -2\%(4), 0\%(5)$

and $\bar{\theta} = +\infty$ for

$\underline{\theta} = 0\%(6), +2\%(7), +4\%(8), +6\%(9), +8\%(10)$.[8]

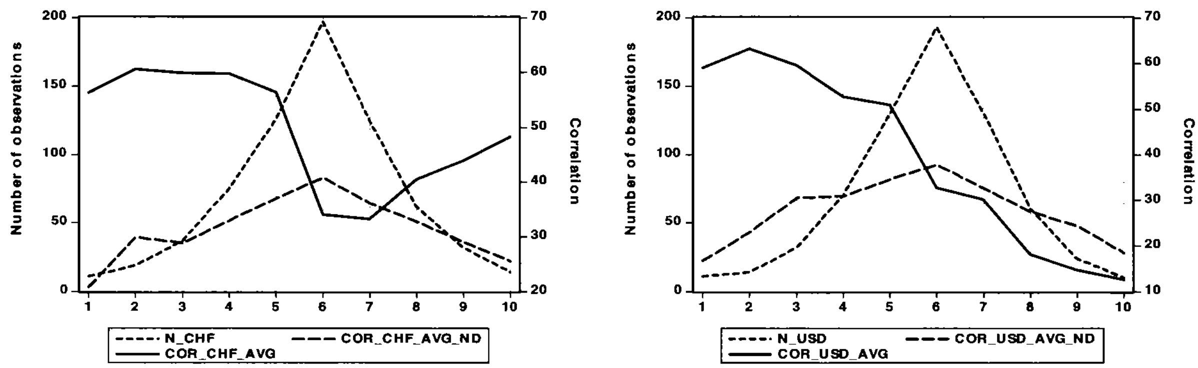
Figure 9 now shows the correlation between Switzerland, the United States and Japan with each of these three markets once being the home market. In the case of Switzerland being the home market (A), correlation between Switzerland and Germany increases with negative returns and de-

Figure 9: Conditional correlation coefficients Swiss (A), US (B) and Japanese (C) markets as home markets, and confidence intervals (D) for correlations between Switzerland (home market) and Germany for empirical and simulated multivariate normally distributed returns



The numbers at the bottom of the graphs correspond to the ones indicated in (1). N: Number of observations. ND: Normal distribution.

Figure 10: Conditional (equally weighted) average correlation coefficients: Swiss (LHS) and US (RHS) markets as home markets: Empirical and simulated multivariate normally distributed returns



The numbers at the bottom of the graphs correspond to the ones indicated in (1). N: Number of observations. ND: Normal distribution.

creases with positive returns. Correlation between Switzerland and the United States, however, is both higher when returns are very low or very high. The picture of the correlation between Switzerland and Japan, however, is very distinct to the one just described for the US market. Here, correlation approximately peaks in the middle of the return distribution and is lowest at both ends of the distribution.

The picture for US investors with the US market as home market (B) is worse, however. Here, the lower the returns, the higher the correlations. In distinct contrast, Japanese investors are in a safe position. For the Japanese market as home market (C), correlations are lowest the more extreme the returns are, irrespective of their direction.

Turning to the question of statistical significance, Figure 9 also plots confidence intervals for the correlations between Switzerland and Germany, for both based on the empirical and simulated multivariate normal distributions (in each case the Swiss market as home market). Inspection of the graph (D) indicates that correlations for negative returns are significantly different from the respective correlations calculated for simulated multivariate normally distributed returns. Due to the small number of observations, however, the statistical significance disappears for correlations calculated in the case of the Swiss market being below the –8% threshold.

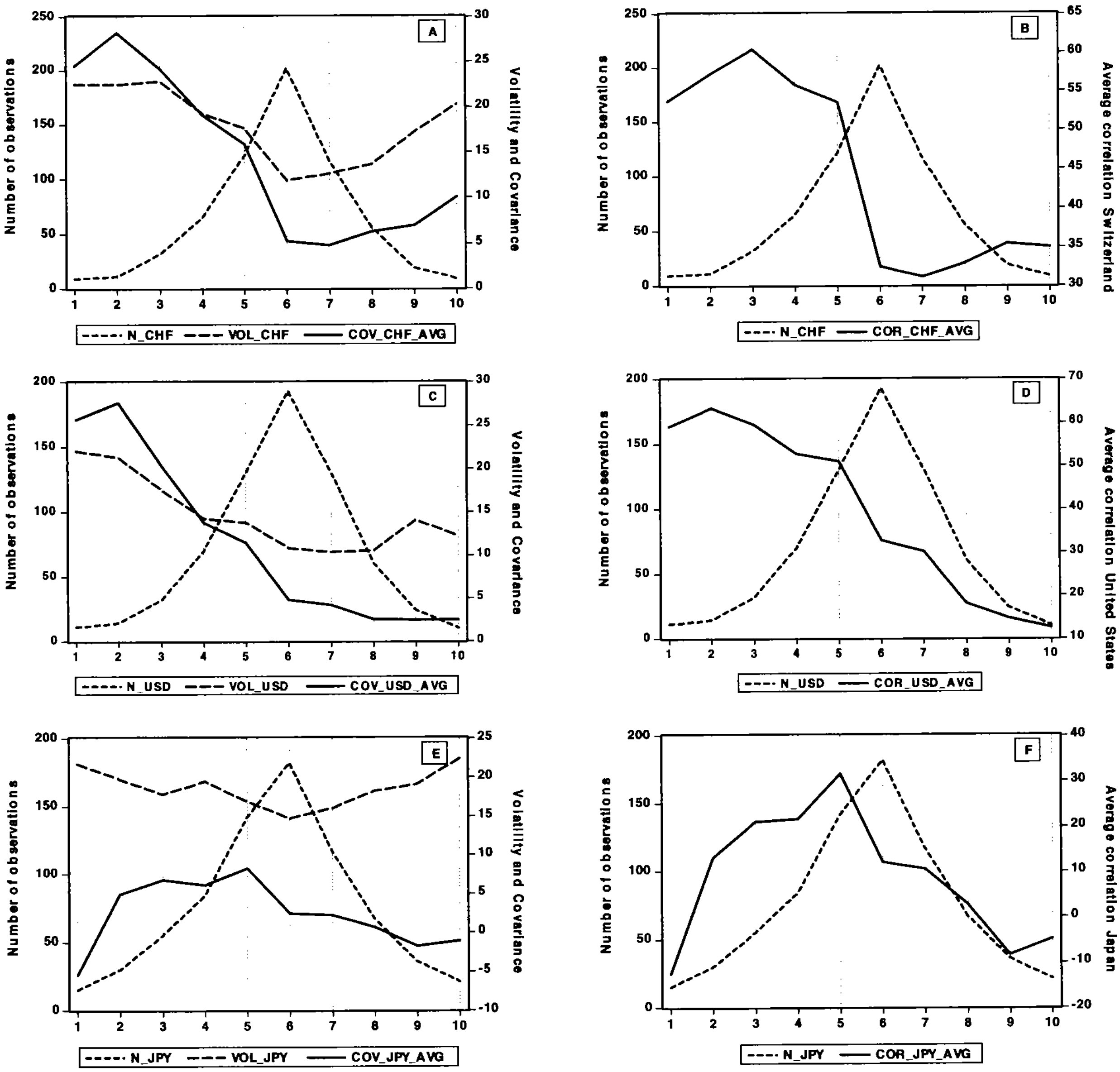
Figure 10 shows (equally weighted) average empirical correlations for Switzerland and the United States, but also graphs the respective average correlations for simulated multivariate normally distributed returns. As indicated in LONGIN/SOLNIK (1998), if all correlation coefficients between any two components of a multivariate normal process are strictly lower than one (in absolute value), then the components of the maximum tend to independence. In particular, the asymptotic correlation of extreme returns is then equal to zero. Inspection of Figure 10 clearly confirms this issue. Here, in contrast to the normal distribution, average correlation for the Swiss market as home market is again higher for extreme returns, irrespective of their direction. This is again completely different for the US market. Here, average correlation tends to decrease with the absolute size of the threshold for positive returns, as expected in the case of multivariate normality, but tends to increase for negative returns. Hence, the probability of having large losses simultaneously is much larger than would be suggested under the assumption of multivariate normality.

The final part of Section 5 is now based on definition 2 of an extreme event, namely that an extreme event for an investor in country j is given when the monthly return of a world market portfolio (here approximated by the MSCI-World Index in local currencies) exceeds an exogenously given threshold. More formally, for country j the conditional correlation is calculated as follows:

$$\rho_{i,j}\big|\underline{\theta} < r_{World} < \bar{\theta} = \frac{E\left[\left(r_j-\mu_j \big| \underline{\theta} < r_{World} < \bar{\theta}\right)\left(r_i-\mu_i\right)\right]}{\sqrt{E\left[\left(r_j-\mu_j\right)^2 \big| \underline{\theta} < r_{World} < \bar{\theta}\right]}\sqrt{E\left[\left(r_i-\mu_i\right)^2\right]}} \qquad (2)$$

The results for the Swiss, the US and the Japanese market are depicted in Figure 11. While both

Figure 11: Volatilities, covariances and (equally weighted) average correlations for Switzerland (A,B), United States (C,D) and Japan (E,F): World market down



The numbers at the bottom of the graphs correspond to the ones indicated in (1). N: Number of observations.

volatilities and average covariances are given on the left-hand side (A,C,E), average correlations are shown on the right-hand side (B,D,F) of Figure 11. For the US and the Japanese markets, there is hardly any difference to the previous analysis. In the case of the Swiss market, however, the picture slightly worsens. Using the second definition, average correlation now decreases similar to the US market when returns increase.

Implications for Asset and Risk Management

The main purpose of Section 5 was to investigate the asymmetric correlation structure of large returns. Using two different definitions of an extreme event, the empirical results clearly indicate that the correlation structure of large returns is asymmetric. Generally, the most asymmetric market is the US market. Here, correlation is highest for large negative returns and lowest for large positive returns. So the probability of having large losses simultaneously is much larger than would be suggested under the assumption of multivariate normality. While the Swiss market also shows significant higher correlation for large negative returns, correlation is also quite high for large positive returns. However, a completely different picture evolves for the Japanese market. For both extreme negative and positive market movements, average correlation tends to be zero or even negative.

Overall, while these results are bad news for Swiss and US risk managers primarily concerned with the left tail of the return distribution, they are good news for Japanese risk managers.

The argument that correlation is positively related to the direction of the market is potentially very important for at least three reasons. First, it has important implications for both asset allocation decisions and risk management. If correlations are higher during bear markets than during bull markets, and bear market moves are greater than bull market moves, this would suggest that the benefits of international diversification are less impressive than conventional wisdom predicts. It is in periods of extreme negative returns that the benefits of international risk diversification are most desired and that the question of international correlation is most relevant to risk-averse agents. Second, what appears in normal circumstances to be a good hedge might become with an increase in correlation a very good hedge, or, might lead to a doubling-up of the respective position. Consequently, it may help explain the 'equity home bias' puzzle, arguably one of the most important puzzles in international finance.[9]

6. Optimal portfolios from event-varying variance-covariance matrices

According to MARKOWITZ (1952), one of the inputs required by investors seeking to hold efficient (internationally diversified) portfolios is an ex-ante estimate of the variance-covariance matrix of the stock market returns. However, the use of ex-post covariance measures as proxies of the ex-ante measures is only justified if the international dependence structure is stationary and if returns are multivariate normal. Yet, the previous sections show that both assumptions fail in reality. Even worse, the instability is particularly distinct in periods of extremely volatile bear markets. Consequently, the common use of constant variances and covariances in global asset allocation within a mean-variance optimisation framework might lead to sub-optimal portfolios. In the same way, if the portfolios can not withstand exceptional periods of market turbulence, they may even not survive and thus can not generate long-term performance. Therefore, portfolios need to be constructed on the basis of carefully estimated variance-covariance matrices. Here, a number of estimation methods have been developed in the literature. The following section, however, is restricted to a method recently proposed by CHOW/JACQUIER/KRITZMAN/LOWRY (1999). They propose to compute two separate variance-covariance matrices, one for normal observations to represent a quiet risk regime, and the other from outlier observations to represent a stressful regime. Their approach is then to blend both variance-covariance matrices into a single one, hereby accounting for different risk aversions and, possibly, assigning the outlier sample a greater probability than its empirical frequency. Based on the above given definition 2 of extreme events (i.e. risk regimes), the following section extends their procedure to more than two risk regimes.

To identify optimal portfolios based on investors' attitude toward k different risk regimes, the full-sample variance-covariance matrix can be replaced by

$$\sum_{i=1}^{k} p_i \Sigma_i \,, \tag{3}$$

where p_i is the probability of falling within risk regime i and sums to unity. Substituting these variance-covariance matrices into the standard equation for expected utility E[U] of a portfolio with a weight vector *w* yields

$$E[U] = \vec{w}'\vec{\mu} - \lambda\left[p_1 \vec{w}'\Sigma_1 \vec{w} + \ldots + p_k \vec{w}'\Sigma_k \vec{w}\right] \tag{4}$$

where λ equals risk aversion to full-sample risk. Equation (4) expresses views about the respective probabilities of the different risk regimes, but it assumes that investors are equally risk-averse to all risk regimes. To differentiate the risk aversion to different risk regimes, the latter are assigned values that reflect the respective relative risk aversion. These values are then rescaled so that they sum to 2 according to

$$\lambda_i^* = \frac{2\lambda_i}{\sum_{i=1}^{k} \lambda_i} \,. \tag{5}$$

Finally, the probability-weighted variance-covariance matrices are multiplied by their respective rescaled risk aversion parameters, resulting in

$$\begin{aligned} E[U] &= \vec{w}'\vec{\mu} - \lambda\left[\lambda_1^* p_1 \vec{w}'\Sigma_1 \vec{w} + \ldots + \lambda_k^* p_k \vec{w}'\Sigma_k \vec{w}\right] \\ &= \vec{w}'\vec{\mu} - \lambda\left[\vec{w}'\Sigma^* \vec{w}\right] \end{aligned} \tag{6}$$

with

$$\Sigma^* = \sum_{i=1}^{k} \lambda_i^* p_i \Sigma_i \,. \tag{7}$$

Table 6 shows the characteristics of two standard mean-variance optimal portfolios based on full-

Table 6: Optimal portfolios from blended and full-sample variance-covariance matrices

Risk regimes i	1	2	3	4	5	6	7	8	9	10
Risk aversion λ_i^*	**0.568**	**0.620**	**0.665**	**0.146**	**0**	**0**	**0**	**0**	**0**	**0**
Probability p_i	1.40%	1.71%	4.98%	10.12%	19.00%	31.46%	18.22%	8.72%	2.96%	1.40%

	Mean (ann.%)	**Max (%)**	**Min (%)**	Volatility (%)	**Skewness**	**Kurtosis**	Jarque-Bera	**Prob.**
Full-sample Σ	6.00	**10.05**	**–19.05**	13.24	**–1.07**	**6.86**	262.07	0.000*
Blended Σ*	6.00	**15.65**	**–14.22**	15.48	**–0.21**	**3.99**	15.54	0.000*
Portfolio weights	**CHF**	**USD**	**GBP**	**CAD**	**DEM**	**ITL**	**FRF**	**JPY**
Full-sample Σ	12.4%	15.0%	–12.5%	38.8%	33.4%	–4.6%	–18.4%	36.0%
Blended Σ*	–3.9%	47.1%	–18.3%	36.7%	–2.9%	–17.0%	–2.6%	61.0%

The numbers defining the different risk regimes correspond to the ones indicated in (1).

sample risk parameters and blended risk parameters under the following assumptions: Firstly, the portfolios' expected return is 6% p.a. and the mean returns for the markets correspond to the ones given in Table 1 (in both cases), secondly, the probabilities of falling within risk regime i correspond to the empirical ones, and, finally, the risk aversion parameters are not chosen arbitrarily, but optimised such that the minimum monthly return observation is maximised (the least negative) and all λ_i are positive (in order to ensure risk aversion).

The results indicate that the resulting portfolio based on the full-sample variance-covariance matrix is skewed to the left and heavy-tailed. Also, the minimum monthly portfolio return is with –19% quite low. In contrast, the second portfolio based on the blended risk parameters is only slightly skewed to the left and hardly heavy-tailed. The minimum monthly portfolio return is with –14% still low, but definitely higher than in the first portfolio. Looking at the respective rescaled risk aversion parameters λ_i^*, it can be seen that only the first four risk regimes are relevant for the calculation of the portfolio. Of course, these correspond to the left tail of the return distribution. Not surprisingly, the composition of the optimal portfolio shifts such that the second portfolio is (more) heavily exposed to the Japanese stock market.

As a result, it thus seems that constructing a portfolio that is almost normally distributed (and hence without negative outliers) corresponds to assuming high risk aversion towards stress periods such as defined in the events 1 to 4. It can thus be concluded that a variance-covariance matrix estimated or blended from stress events better characterises a portfolio's riskiness during market turbulence than a full-sample variance-covariance matrix. In other words, mean-variance optimisation based on a blended variance-covariance matrix produces portfolio strategies with realised returns that have less downside risk exposure than those determined using a full-sample variance-covariance matrix, without sacrificing expected return. Overall, the results support the use of downside-risk approaches to investment decisions which focus on return dispersions below a specified target or benchmark return as proposed by HARLOW (1991). These measures are attractive because they are likely consistent with investors' perception of risk. Specifically, optimisation based on downside measures produce portfolio strategies with realised returns that have less downside risk exposure than those determined using variance.

7. Conclusion

It is concluded that international equity market returns are not multivariate normally distributed since there are too many extreme (negative) observations. While this is a well-known phenomenon, it is nonetheless one reason causing correlation to be unstable over time. Indeed, using the BOX-M test statistic, it is underlined that correlation matrices exhibit large jumps through time. There is also overwhelming empirical evidence that average correlation (and hence systematic risks) almost increased 40 percent since the early seventies. In addition, it is revealed that correlation and volatility are positively related. By turning directly to the tails of the return distributions, it is pointed out that the correlation structure of large returns is asymmetric. Using two different definitions of extreme events, the empirical findings suggest that the most asymmetric market is the US market. Correlation for the US equity market is highest for large negative returns and lowest for large positive returns. While the Swiss market also exhibits higher correlation for large negative returns, correlation seems to be also high for large positive returns. A completely different picture evolves for the Japanese stock market, however. For both extreme negative and positive market movements, average correlation tends to be zero or even negative, which is in fact in line with the behaviour of simulated multivariate normally distributed returns.

Clearly, most of the results are bad news for risk managers and internationally diversified investors. The continuously changing correlation pattern makes it very difficult to select an ex-ante optimal investment strategy. In addition, the observed general increase in correlation eroded the advantage of international risk diversification more and more. Last but not least, since there is a positive link between correlation and market volatility, investors and risk managers do not get the full benefits of international risk diversification in exactly those situations when most desired, namely in high volatility regimes associated with negative returns.

Finally, it is suggested that variance-covariance matrices estimated from different risk regimes provide a better representation of a portfolio's riskiness during periods of market turbulence than a variance-covariance matrix estimated from the full-sample of observations. This supports the use of downside-risk approaches to investment decisions.

Footnotes

[1] The stock market price indices are collected from DATASTREAM. KAPLANIS (1988) shows that the results do not depend on whether returns are expressed in terms of excess returns, in terms of local or a unit currency or in terms of nominal or real returns. In contrast, SOLNIK/BOUCRELLE/LE FUR (1996) show that correlations tend to be higher for local-currency returns than for dollar returns, as foreign currencies bring an element of diversification to domestic portfolios.

[2] For desired properties of dependence measures see EMBRECHTS/MCNEIL/STRAUMANN (1999a). There, they point to an alternative approach to understanding and modelling (more complex) dependence structures, namely copulas. Copulas represent a way of trying to extract the dependence structure form the joint distribution and to extricate dependence and marginal behaviour. This concept, however, is not discussed any further in this study.

[3] KAROLYI/STULZ (1996), but particularly FORBES/RIGOBON (1999) and BOYER/GIBSON/LORETAN (1999), however, claim that linear correlation is conditional on volatility and for this reason biased if computed across different volatility regimes. In other words, they argue that linear correlation is conditional on market movements over the time period under consideration, so that during a period of turmoil when stock market volatility increases, standard estimates of linear correlation will be biased upwards. Moreover they quantify the bias and propose an adjustment to the linear correlation coefficient. Yet, it is necessary to critically examine the notion of 'conditioning'. Particularly in the analysis of FORBES/RIGOBON (1999), conditional refers to a certain sub-sample, that is, a period that is chronologically contiguous. Conditioning one return series to a sub-sample that is chronologically contiguous, however, implicitly conditions the other return series to the same chronologically contiguous sub-sample at the same time. If both return series are conditioned at the same time, their proposed adjustment is no longer valid. To put it another way, note that within any chronologically contiguous time period there is only one (true) correlation between two markets. Thus, their adjustment should (but does not) yield the same adjusted correlation irrespective of whether one market or the other market is conditioned. In addition, their approach is in contradiction to LONGIN/SOLNIK (1998) and EMBRECHTS/MCNEIL/STRAUMANN (1999b). There they show that if all correlation coefficients between any two components of a multivariate normal process are strictly lower than one (in absolute value), then, regardless of how high the correlation is, extreme events appear to occur independently. In particular, the asymptotic correlation of extreme returns is then equal to zero.

[4] Not surprisingly, the most unstable one-year periods are the ones containing the October 1987 stock market crash (F-values of 3.49 and 3.42) and the recent Asia crisis (F-values as high as 3.93 and 4.19). This indicates that extreme negative and positive return observations have a negative impact on the stability of correlation matrices.

[5] Tests not reported here showed that, applying the BOX-M test statistic to variance-covariance matrices, variance-covariance matrices are less stable than correlation matrices. The corresponding F-values are consequently higher than those given in Table 4 for correlation matrices. This is also in line with the results presented in LONGIN/SOLNIK (1995) and TANG (1995). They also find variance-covariance matrices to be less stable than correlation matrices.

[6] As their estimates of the correlation structure become more reliable with increasing time horizons, they conclude that the use of correlation estimates seems only appropriate if it serves the purpose of determining long-term rather than short-term asset allocation strategies. Their conclusion thus supports the practical use of mean-variance models as devices for determining ex-ante optimal international portfolios in the long run. This conclusion is clearly opposite to the findings presented here. These in fact suggest that the selection of an ex-ante optimal investment strategy is even more difficult to identify for a long-term investor than for a short-term investor.

[7] Note that the purpose of this section is neither to provide an explanation nor to give an overview of the existing literature of any particular episode of stock market behaviour such as the October 1987 stock market crash, the recent Asia crisis or the oil crises in the early seventies, to name just the most impressive.

[8] The numbers given in the brackets relate to the description of the axes of the following graphs.

[9] Already LESSARD (1974) argues that in a world of no transaction costs, homogeneous expectations, equivalent internal and external purchasing power of currencies, and independence between exchange rate variations and stock prices, it would be reasonable that all investors hold the world market portfolio of risky securities. However, there is overwhelming empirical evidence that investors hold portfolios which are heavily weighted towards domestic assets instead of the world market portfolio.

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