ROBERT G. TOMPKINS*

The Austrian Stock Market: Objective Dispersion Processes and Option Pricing

1. Introduction

This paper examines the nature of the objective dispersion processes for spot prices on sixteen of the most actively traded stocks on the Austrian stock market and compares these to futures prices of the Austrian Traded Index (ATX), which measures the overall Austrian stock market (hereafter referred to as 'markets'). Our first contribution is to provide the descriptive statistics of a number of stocks previously not presented in the literature (recently issued or merged stocks) and covering the market turbulence of 1997 and 1998.

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This paper will compare and contrast the statistical properties of returns, the volatility of returns, the time series dynamics of volatility clustering and test for leverage effects. Of particular interest for us are the implications of these findings for the pricing of contingent claims.

For all markets examined, we reject the hypotheses that: 1) the return series are independently and identically distributed (i.i.d.) and 2) they conform to Geometric Brownian Motion (GBM). However, the degree to which the return series deviate from such processes varies among the assets.

Alternative models are considered to explain these empirical results. These include models that assume non-normality in the underlying log-price process, incorporate a subordinated stochastic volatility process and allow price and volatility processes to be correlated. Due to the rich nature of these models, they do not lend themselves to parameterisation using standard methods (such as maximum likelihood methods) and a simulated method of moments approach was utilised. The choice of target empirical moments was done to address most stylised results previously pointed out in the literature. These attributes include statistical moments from the daily return series: 1) the unconditional skewness and 2) the unconditional kurtosis. Volatility clustering is captured using two measures of the average autocorrelation of absolute returns for short and intermediate lagged periods as attributes 3) and 4). To better understand the nature of the unconditional volatility process for the seventeen markets, two attributes are considered. These include 5) the coefficient of variation for the unconditional volatility at a 20-day time horizon and 6) the rate of decay of the standard deviation of unconditional volatility as the estimation horizon is increased. Finally, to examine the leverage effect [pointed out by CHRISTIE (1982) among others], a new variable 7) is constructed which measures the correlation between the unconditional 20-day volatility with recent relative price levels.

While we observe significant differences in the statistical moments of the return series and time series dynamics among the stocks and ATX futures, we find that a single model appears to explain the dynamics for all seventeen markets. To capture all aspects of the unconditional return and volatility processes, a model, which assumes a skewed jump process with subordinated (and correlated) stochastic volatility, is required. Given that this model adequately explains key aspects of the objective return process, we examine the possible implications of this alternative process for the pricing of European options.

This paper is organised as follows: the first part briefly reviews previously presented empirical evidence, which indicates that stock and stock index futures prices do not follow an independent and identically distributed (i.i.d.) Geometric Brownian Motion (GBM) process. Two possible models, which have been proposed in the literature to explain these results, were considered, as is the rationale for the selection of seven empirical attributes to capture key aspects of non-normality and inter-dependence. A description of the data sources used for this research follows and presents empirical results for the seven attributes across the markets. After this, alternative models were examined using a simulated method of moments approach to assess their ability to explain the key attributes. With the best model determined, option prices that are consistent with these processes were estimated for ATX futures and compared to the prices from the BLACK (1976) formula. Significant biases are found relative to these models which were consistent with previous research examining the biases in option pricing introduced by stochastic volatility. Finally, conclusions and suggestions for further research appear.

2. Price Processes for Stocks under the "Empirical" Measure

It is well established that the unconditional return series for individual stocks, stock indices and stock index futures do not conform to the assumptions of an i.i.d. lognormal dispersion process [see STOLL and WHALEY (1990)]. Returns for stocks and stock index futures usually display excess kurtosis and (for many periods) significantly negative skewness compared to a normal distribution. Returns for stocks also display intertemporal dependence. In the examination of absolute returns for both individual stocks and for stock index futures markets, significant positive autocorrelations have been found. This result has led DING, GRANGER and ENGLE (1993) to conclude, "It is clear that the [S&P 500] stock market return process is not an i.i.d. process" (page 87).

Another line of empirical research has examined the unconditional volatility series directly. A typical approach to better understand the volatility process is to examine the statistical moments of the process. BURGHARDT and LANE (1990) examined the variability of the unconditional volatility process using a volatility cone approach. We extended this by looking at the sampling properties (restricted to the standard deviation) of the unconditional volatility measured at a 20-day time horizon. Using non-overlapping data, we obtained an average estimate of the unconditional volatility and the standard deviation of this average. Under the assumption that an i.i.d. price process is generating these volatilities, the expected coefficient of variation of the 20-day volatility is known. This measures the variability of volatility for a given time horizon.

However, the time varying dynamics of the variability of volatility as the time horizon of estimation is extended, is of additional interest. A simple log-linear form was chosen to capture these dynamics. The rate of decay in the standard deviation of volatility implies that the maturity structure of historical volatility experiences long-term memory (interdependence). This can be seen as complimentary to long-term memory effects for absolute returns identified by DING, GRANGER and ENGLE (1993). An additional and important feature of these price processes is the leverage effect pointed out by CHRISTIE (1982) among others.

While a number of theories have been proposed to explain these results, three alternative hypotheses will be examined here. The Constant Elasticity of Variance (CEV) model of COX and ROSS (1976), a non-GBM price process model [such as the Jump diffusion model proposed by MERTON (1976)] and Stochastic Volatility Models were considered. These three models can be nested within a stochastic volatility model. Given the wide range of stochastic volatility models proposed in the literature [see TAYLOR (1994)], it was not obvious which model to select. As models with correlated processes were considered, the HESTON (1993) model was the obvious choice. This model has the additional benefit of having a closed form solution for the pricing of options [see HESTON (1993), BAKSHI, CAO and CHEN (1997) and BATES (2000)]. Variants of the HESTON (1993) model proposed by BARNDORFF-NIELSEN (1997) and BARNDORFF-NIELSEN and SHEPHARD (1999) are also considered.

Specifically, three models are tested here:

MODEL 1

$$dF = \mu F dt + \sigma F dZ(t) \qquad (1)$$

Where Z(t) is a standard Wiener Process and μ and σ are constants, the return series r_t is normally distributed with $r_t = \mu + \sigma Z_t$ and $Z_t \sim N(0,1)$. This is the assumption of the BLACK (1976) model of i.i.d. Geometric Brownian Motion and will be referred to as GBM.

It is clear that Model 1 is a straw man, given the well-known results that returns for stock markets display both non-normality and inter-dependence. However, this can serve as a benchmark for the relative effectiveness of the alternative models and is used to assess the sampling properties of the evaluation approach. Subsequent models will consider stochastic volatility ($\hat{\sigma}$) which will be evaluated in terms of a stochastic variance process ($\sqrt{V}$).

MODEL 2

$$dF = \mu F dt + \hat{\sigma} F dZ_1 \qquad (2.1)$$

With the variance process defined by:

$$dV = \kappa(\theta - V)dt + \xi\sqrt{V}dZ_2 \qquad (2.2)$$

Where Z_1 and Z_2 are standard Wiener processes with correlation ρ. The term κ indicates the rate of mean reversion of the variance, θ is the long-term variance and ξ indicates the volatility of the variance. The terms V and $\sqrt{V}$ represent the variance and the volatility of the process, respectively. This model will be referred to as SV in this paper.

MODEL 3

$$dF = \mu F dt + \hat{\sigma} F dN \qquad (3.1)$$

With the variance process defined by:

$$dV = \kappa(\theta - V)dt + \xi\sqrt{V}dZ \qquad (3.2)$$

where N represents a non-normal price process for the underlying price series and in this research is the Normal Inverse Gaussian distribution (NIG). This model is related to that of BARNDORFF-NIELSEN (1997) who was the first to propose a stochastic volatility model of this form [subsequently extended by ANDERSSON (1999)].

This approach extends the findings of BATES (1996, 2000) and HO, PERRAUDIN and SØRENSEN (1996), who assumed the volatility process is subordinated in a non-normal price process. In this model, the stochastic variance disturbances are assumed to follow a standard Wiener Processes, Z, with correlation ρ existing between the two processes.[1] When correlated processes are considered, this variant of model 3 is related to the model for incorporating a leverage effect in a stochastic volatility model proposed by BARNDORFF-NIELSEN and SHEPHARD (1999). This model will be referred to as NIGSV in this paper. The CEV model of COX and ROSS (1976) which allows for the inclusion of negative correlations between the two stochastic processes is nested in both Models 2 and 3. This allows the leverage effect to be captured.

3. Choice of Attributes and Fitting Parameter Values

A key problem in the empirical testing of stochastic volatility models is the estimation of optimal input parameters into the model. Due to the complexities of our models, it is not clear whether estimation of parameter inputs by maximum likelihood techniques is feasible. This research uses a hybrid between the Generalised Method of Moments (GMM) approach of MELINO and TURNBULL (1990) and the Simulated Method of Moments (SMM) approach of DUFFIE and SINGLETON (1993). This approach subjectively selects essential attributes, simulates price processes consistent with the alternative models and assesses the sum of squared errors between simulated and empirical attributes. Alternative parameterisation of the models was examined and optimised. Similar to ANDERSEN, CHUNG and SØRENSEN (1999), we investigated the sample properties of this estimator technique (using Model 1). This allowed comparisons to be made between the attributes of each market and the models and conclusions to be drawn regarding the overall fit of each model.

At the heart of this estimation technique is the judicious choice of key attributes. It is crucial that the choice of the attributes jointly considers the relevant elements of empirical interdependence and non-normality and provides a means by which the salient features of the alternative models can be captured. Given that both alternative price process (to GBM) and stochastic volatility models have been proposed to explain excess kurtosis in returns, the unconditional kurtosis for daily returns is a logical attribute to choose. Furthermore, some research has indicated that negative skewness is an important attribute for describing the returns of stock indices and stock index futures. Both THEODOSSIOU (1998) and HARVEY and SIDDIQUE (1998) chose to examine the skewness in addition to the excess kurtosis. This attribute would provide evidence for the existence of asymmetric jumps and/or leverage effects.

To examine both of these moments, price changes were estimated based upon continuously compounded returns in the standard manner (using log-price increments). Extreme care was taken to assure that when futures returns were estimated, only futures prices with the same expiration date were used. With this time series of daily returns, the unconditional skewness and kurtosis statistics were determined.

Clearly, given that this research examined two variants of a stochastic volatility model, attributes had to be selected that would allow the salient features of these models to be captured. This required attributes capturing the volatility of volatility parameter, the rate of mean reversion and the correlation between the processes. For this research, the estimation of the standard deviation of the returns was determined using squared returns and these results were annualised (assuming 252 trading days in a year). A number of attributes were selected in order to capture critical dynamics of the volatility process.

The first attribute examines the volatility of volatility. A time series of unconditional volatilities was estimated on a daily basis for a time horizon of 20 days (using non-overlapping data). From this series, the average and standard deviation were estimated. Given widely different levels of these sample statistics, a coefficient of variation statistic was chosen as the key attribute. As a basis for comparison, the coefficient of variation of volatility measured at the 20th lag would be approximately equal to 0.1622 if the underlying return series were independent and identically distributed ($1/\sqrt{2 \cdot 19}$).

While this measured the variability of volatility at a single time horizon, the time varying dynamics of the standard deviation of volatility could not be captured. This was achieved by the estimation of the coefficient of variation of volatility with time horizons from 20 days to 200 days in 20-day increments.[2] From statistical theory, the expected decay in the standard deviation of a volatility estimate (σ^*) is a square root function of the number of observations used for estimation ($SE = \sigma^*/\sqrt{2N}$). Given that the average level of volatility remains constant (which was found empirically for these seventeen markets), the decay in the coefficient of variation should follow the same functional form. If the first volatility observation is at the 20-day horizon, the decay in the standard deviation of volatility from that point forward can be expressed as $\sigma_N \cdot \sqrt{N/(N+1)}$. In this formula, σ_N is the standard deviation of the volatility at the Nth observation and N + 1 is the number of observations in the next time horizon (initially N = 20 and N + 1 = 40). The functional form of this decay can be expressed as a power function of the form $SE = (\sigma^*/\sqrt{2}) \cdot N^{-0.5}$. A linear regression of (the natural logarithms of) the time horizon of estimation regressed upon the levels of the coefficient of variation was used to capture the empirical rate in decay. This can be expressed as:

$$\ln(\hat{\sigma}_N) = \alpha + \beta(\ln(N)) \qquad (4.1)$$

From this regression equation, the decay attribute, follows the following exponential form:

$$e^{\alpha+\beta(\ln(N))} \qquad (4.2)$$

Observed divergences in the Beta coefficient of equation 4.2 from –0.5 give an indication of the degree of the volatility of volatility persistence observed in the unconditional process. This also provides an attribute to capture the interaction between the rate of mean reversion and volatility of volatility in the stochastic volatility process.

Another salient feature of empirical return volatility series is that subsequent realisations may not be independent. To capture serial correlation in absolute returns, autocorrelation dynamics were examined directly rather than using alternative methods relying on maximum likelihood. This was achieved by examining the autocorrelograms of absolute returns previously employed by TAYLOR (1986) and DING, GRANGER and ENGLE (1993). This approach has the additional benefit of known sampling properties. Thus, a simple confidence interval test can be used to reject the null hypothesis of independence.

For the purposes of this research, composite measures of the autocorrelations are required. Given that the markets differ in the manner that the autocorrelations decay, the averages of the autocorrelations from lag 1 to 20 and from lag 51 to 70 were both selected. The first average represents the short-term autocorrelation. The medium-term average provides an indication of how quickly the autocorrelations die out. Unfortunately, such measures may no longer have known sampling properties allowing for a simple parametric confidence interval test. Therefore, to assess the sample characteristics of these composite measures, nonparametric standard errors were determined via simulation. These two attributes provide additional information relevant to the calibration of a stochastic volatility model, as they capture both short-term and medium-term evidence of volatility clustering.

The final attribute must measure the leverage effect and provide a means for a correlation between the stochastic processes to be captured. There is, however, one problem with the determination of the leverage effect: Even if the volatility is a stationary series, the prices are not. To solve this problem, a new variable was constructed which measures recent price movements and is stationary.[3] This variable is an exponentially weighted return series, which indicates whether recent price movements are relatively high or low. It can be shown that given some exponential weighting scheme: where $W = 1 - \theta$ and $\theta \approx e^{-W\Delta t}$, we can define a new series, ω_i, that can be expressed as:

$$\omega_i = \omega_{i-1} + W(r_{i-1} - \omega_{i-1}) \qquad (5)$$

where ω_i is the exponentially weighted price movement, r_i is the daily return and the initial ω_i is set to zero. W represents the weight used in the weighting scheme. This new variable was created and the series of 20-day unconditional volatility were compared. With an arbitrary weight (W) for all markets imposed at 0.03, the correlation between the two variables was estimated.[4] This correlation coefficient serves as an attribute to measure the leverage effect.

With these seven target attributes, the stochastic volatility models were parameterised via simulation. Following the three models proposed previously, price series of 1500 observations were generated consistent with these models. The return and volatility characteristics of these simulated price series were estimated in exactly the same manner as was done for the sixteen stocks and the ATX futures. The resulting simulated attributes were then compared to the empirical attributes using a sum of squared errors statistic. To reduce scaling impacts due to different levels of the attributes, the squared errors were divided by the standard deviation of the attributes across the seventeen markets.[5] This test statistic can be written as:

$$\min \sum \left(\frac{M_i - X_i}{\sigma_i} \right)^2 \qquad (6)$$

where M_i is the attribute for the market, X_i is the attribute of the price series generated by the model and σ_i is the standard deviation of the attributes across all seventeen markets for the total period of analysis.

Finally, 500 samples of 1500 prices consistent with Model 1 (GBM) were drawn to better understand the sample properties of all the attributes and of the test statistic. This provided a non-parametric estimation of the standard errors of the attributes and allowed confidence intervals to be estimated which allows comparisons between markets and models.

4. Data Sources

Sixteen Austrian stocks were selected for this study. These include Austrian Micro Systems (AMS), Bank Austria (BAS), BWT, Brau Union (BRAU), Boehler (BUD), Erste Sparkasse (ERSTE), Flughafen (FLU), Mayr Meinhof (MM), OeMV (OMV), Radex (RAD), Semperit (SEMP), VA Stahl (VAS), VA Tech (VAT), Verbund (VER), Wienerberger (WIE), and Wolford (WOL). In total, these sixteen stocks make up 77.45% of the value of the Austrian Traded Index as of January 30, 1998. In addition, closing prices of the nearest to expiration futures contracts for the ATX futures were examined.

The daily closing stock and futures prices were obtained directly from the Wiener Börse.[6] Given the limited availability of the some of the stocks (due to recent issuance), the period of analysis varies considerably. The analysis period for the sixteen stocks and the ATX futures and the number of daily observations appears in Table 1.

Given that this portion of the research is empirical in nature, a major effort was made to assure the validity of the data used in the analysis and to verify that the analytic methods employed were correct. This was achieved in a number of ways. Firstly, the futures price series were compared with the options (on the futures) price series for the same days to identify obvious errors in recording either price series. This comparison was achieved by comparing the put-call parity values of the options with the underlying futures prices

Table 1: Markets Included in Research, Time Period of Data, Number of Observations

Underlying Asset	Time Period of Analysis	Number of Observations
Austrian		
Micro Systems	12/07/1993–18/12/1998	1349
Bank Austria	02/01/1991–18/12/1998	1974
BWT	11/05/1992–18/12/1998	1641
Brau Union	01/06/1989–18/12/1998	2368
Boehler	10/04/1995–18/12/1998	914
Erste	02/01/1991–18/12/1998	1978
Flughafen	15/06/1992–18/12/1998	1619
Mayr Melnhof	22/04/1994–18/12/1998	1156
OeMV	01/06/1989–18/12/1998	2369
Radex	01/06/1989–18/12/1998	2369
Semperit	18/09/1995–18/12/1998	806
VA Stahl	04/10/1995–18/12/1998	792
VA Tech	25/05/1994–18/12/1998	1134
Verbund	01/06/1989–18/12/1998	2368
Wienerberger	01/06/1989–18/12/1998	2367
Wolford	14/02/1995–18/12/1998	953
ATX Futures	03/07/1993–18/12/1998	1317

for every single date in our database. A screening procedure was imposed: If futures or options prices diverged by more than the normal bid/offer spread (of one tick), the observations were flagged. Once this was done, each price was compared with the original daily price sheets to confirm if a "keypunch" error had occurred. We discovered that only 1–2% of the data had such errors. Nevertheless, these errors were of a sufficient magnitude that they did influence the results and therefore required correction.

The most arduous part of the data cleaning process was the ongoing examination of the data as results of the analysis were obtained. One reason why seventeen markets were examined was to allow a cross-sectional comparison. Apart from the benefit of assessing general tendencies across markets, it is also possible to use anomalous results as an additional check on data validity. This assured that the data series employed in this research was as accurate as is humanly possible.

5. Attributes for Individual Markets

For all the markets, we examined the return statistics for daily, weekly and monthly returns. These returns were calculated using non-overlapping data. For the weekly and monthly series, the return was determined by the logarithm of the price relatives at time T = i and T = i + 5 for the weekly series and T = i and T = i + 20 for the monthly series. The results of these analyses appear in Appendix A1.

The first column describes the market under investigation and the return frequency examined. The second and third columns present the first (mean) and second (standard deviation) moments of the return distribution. The fourth and fifth columns present the statistics for the unconditional skewness and the significance level for a null hypothesis of normality. Under the null hypothesis of normality, the skewness statistic is normally distributed with standard errors: $se = \sqrt{(6/T)}$, where T is the sample size. When the skewness statistic rejects the null hypothesis of normality at the 99% level, the significance level appears in bold type.

The sixth and seventh columns present the statistics for the unconditional kurtosis and the significance level for a null hypothesis of normality. Under the null hypothesis of normality, the excess kurtosis statistic is normally distributed with standard errors: $se = \sqrt{(24/T)}$, where T is the sample size. This statistic is equal to the kurtosis statistic appearing in the table minus 3.0. When this statistic rejects the null hypothesis of normality at the 99% level, the significance level appears in bold type.

The statistic in the sixth column is the Bera-Jarque (BJ) statistic for detecting departures of the data from normality. Under the null hypothesis of normality, the BJ statistic is distributed as χ^2 with 2 degrees of freedom. The critical value at the one-percent level is 9.21. When the BJ statistic is above this level, the statistic appears in bold type.

For all seventeen markets, the dispersion of daily returns is not well described by a normal distribu-

Table 2: Characteristics of the Unconditional Volatility (Standard Deviation) of Returns for Sixteen Austrian Stocks and the ATX Futures

Markets	20 Day Average Volatility	20 Day SD Volatility	Coefficient of Variation	Line Fit of SD of Volatility vs. Time Horizon	Leverage Correlation (20 Day Volatility vs. Recent Relative Prices)	Average Auto-correlation of Absolute Returns (Lags 1-20)	Average Auto-Correlation of Absolute Returns (Lags 51–70)
Austrian Micro Systems	39.90%	19.18%	**0.481**	–0.4487	**–0.2530**	**0.0575**	**0.0218**
Bank Austria	21.00%	14.62%	**0.696**	**–0.2141**	**–0.3590**	**0.1927**	**0.0818**
BWT	21.25%	9.64%	**0.453**	**–0.2569**	0.1660	**0.0950**	**0.0299**
Brau Union	23.72%	10.26%	**0.432**	**–0.1962**	0.0780	**0.0696**	**0.0311**
Boehler	25.95%	11.83%	**0.456**	**–0.1571**	**–0.4950**	**0.1533**	**0.0999**
Erste	25.96%	14.55%	**0.561**	**–0.2815**	–0.1860	**0.0928**	**0.0268**
Flughafen	23.65%	10.86%	**0.459**	**–0.2930**	–0.1920	**0.0837**	**0.0284**
Mayr Melnhof	28.14%	15.44%	**0.549**	**–0.3329**	**–0.3170**	**0.1190**	0.0070
OeMV	26.69%	13.75%	**0.515**	**–0.1565**	**–0.2900**	**0.1384**	**0.0550**
Radex	30.45%	14.07%	**0.462**	**–0.2090**	**–0.2690**	**0.0894**	**0.0301**
Semperit	34.22%	15.75%	**0.460**	**–0.2054**	0.1370	**0.0591**	**0.0221**
VA Stahl	31.80%	12.43%	**0.391**	**–0.1515**	**–0.6740**	**0.1179**	**0.0629**
VA Tech	24.39%	14.27%	**0.585**	**–0.0973**	**–0.5500**	**0.2056**	**0.0860**
Verbund	22.31%	14.98%	**0.672**	**–0.0834**	**0.3740**	**0.1464**	**0.0732**
Wienerberger	24.84%	13.55%	**0.545**	**–0.1447**	–0.0810	**0.1345**	**0.0607**
Wolford	36.02%	17.36%	**0.482**	**–0.2457**	–0.1890	**0.0903**	**0.0157**
ATX Futures	16.66%	7.51%	**0.451**	**–0.1516**	**–0.3220**	**0.1208**	**0.0495**
Expected GBM Attributes	20.000%	3.244%	0.162	–0.5000	0.0000	0.000	0.0000
Standard Error of Attributes	3.244%	0.032%	0.010	0.0778	0.0999	0.005	0.0061

In the furthest left column, the Individual Austrian Stock or the ATX Futures appears. In the bottom two rows appear the attribute values expected from an Independent and Identically Distributed (i.i.d.) Price process associated with Geometric Brownian Motion (GBM). The standard error of the attributes is determined for a series of 500 draws from an i.i.d. GBM process of 1500 observations. These non-parametrically estimated standard errors allow significance testing between the empirical attributes and the assumption of an i.i.d. GBM process. When a T-statistic rejects this assumption at a 95% level or above, the attributes are **BOLDED**. In Columns Two, Three and Four, analysis of the 20 day volatility estimated on an non-overlapping basis appears. Using all available observations, daily returns (differences in the logarithm of daily closing stock or futures prices) were estimated. With these returns, the standard deviation was estimated for a fixed time horizon of 20 days and then annualized using the sqrt(252).The estimation of the 20 day volatility was done on a non-overlapping basis. In the fifth column appears the relationship between the standard deviation of volatility and the time horizon of estimation. The Time Factor term is determined by a regression of the logarithm of the time horizon (T) on the logarithm of the unbiased standard deviation of volatility. Only the beta coefficients of the regression appears. Observed divergences in the Beta coefficient from –0.5 give an indication of the degree of the volatility of volatility persistence observed in the unconditional process. The sixth column provides information about the leverage relationship between the levels of stock or futures prices and volatility. This is the correlation between the unconditional volatility estimated with daily log price increments and estimated on a 20 day rolling time horizon and an exponentially weighted return series, which indicates whether recent price movements are relatively high or low. In the two furthest right columns appear information about the autocorrelations in absolute returns (and volatility clustering). Given the relatively rapid decay of the autocorrelations, we averaged the autocorrelations for each lag from lag 1 to 20 and separately averaged the autocorrelations for each lag from lag 51 to 70.

tion. The BJ values are all greater than their critical values for daily observations and only VAS fails to reject the hypothesis of a normally distributed series for weekly observations. For many of the stocks, we cannot reject the null hypothesis of normality for monthly returns.

The high levels of the Bera-Jarque statistic are due to the skewness and kurtosis statistics diverging significantly from a normal distribution. However, while the excess kurtosis statistic is almost always significant for all time horizons, the skewness statistic is neither significant nor of the same sign for all markets. While most of the stocks and the ATX futures have significant negative skewness statistics, BWT and Verbund have significant positive skewness statistics. Of the remaining stocks only eight of the sixteen have significant negative skewness statistics for daily returns.

Given that our research also finds inconsistencies in the sign and significance of the skewness statistics but consistency in the excess kurtosis significance, one might be tempted to examine solely the excess kurtosis. However, both THEODOSSIOU (1998) and HARVEY and SIDDIQUE (1998) have chosen to examine the skewness in addition to the excess kurtosis. The latter paper shows that asymmetric variance exists and this is captured by the [conditional] skewness. Both authors examine both effects by means of a non-central Student-T distribution. Likewise, in this research, we will examine both elements. However, our research differs from these studies by capturing both effects with a Normal Inverse Gaussian distribution.

As the remaining attributes all capture characteristics of the volatility process, these will be summarised in a single table, Table 2.

In this table, the first column describes the individual market examined. The next three columns display the average annualised volatility measured at a 20-day time horizon. At the bottom of these columns are the expected attributes from a GBM dispersion process with constant variance. The attribute of interest to this research is the Coefficient of Variation statistic. For all seventeen markets, we can compare this measure of the volatility of volatility to what would be expected under the GBM assumption. By determining a standard error of this attribute by simulation, we can reject the hypothesis that the volatility process conforms to the GBM i.i.d. assumption (when T-statistic reject the null hypothesis at 95% level or above, the attribute appears in bolded text).

In the fifth column appears the beta of the regression of the relationship between the [natural logarithms of the] time horizon of the estimation period against the coefficient of variation of volatility. If markets conform to a GBM i.i.d. process, a decay coefficient of –.50 (seen at the bottom of the column) would be observed. For sixteen of the seventeen markets, the rate of decay is (statistically) significantly less that this decay function (the exception is for Austrian Micro Systems). In Column six, the leverage correlation coefficient appears. This measures the relationship between the 20-day unconditional volatility and the recent relative prices. Consistent with the negative leverage effects CHRISTIE (1982) pointed out for individual stocks, many of these stocks display a significant negative leverage effect (although for only ten markets is this effect significant and for three, the effect is insignificantly positive). These should be interpreted with care given that the simulated standard error of this attribute is fairly high (at 0.0999). While these effects remain significant, it is only at the 95% level for number of the markets.

The final two columns represent the average autocorrelations of absolute returns for lagged periods from 1–20 days and 51–70 days. Underlying the assumption of an i.i.d. price process, these have a prior expectation of zero. For almost all seventeen markets, these averaged autocorrelations are significantly (statistically) positive (the exception is Mayr Melnhof with an insignificant average autocorrelation of absolute returns from 51–70 days).

The Results from both Appendix A1 and Table 2 indicate that both the return series and the volatility process significantly diverge from a prior assumption of a GBM i.i.d. process. With these attributes as target conditions, the proposed alternative models were examined.

6. Fitting Alternative Models

In previous sections the models to be tested were presented. It only remains to discuss technical issues in the simulation process and the parameterisation of the stochastic volatility models before proceeding directly to the results.

The simulated method of moments approach was done in a two-stage process. The first stage was to determine representative distributions for the later simulations. Specifically, this entailed the generation of 500 samples of 1500 draws from an independent normal distribution. According to standard procedures, the random number generation process used a standard Box-Muller method and the anti-thetic approach suggested by BOYLE (1977). With these 500 samples, price series were constructed that conformed to GBM with constant variance (using equation 1). The distributional and time series attributes of each series were assessed and compared to the theoretical moments of an i.i.d. GBM process. Utilising formula 6, the sum of squared errors between each of the 500 samples and the true attributes of an i.i.d. GBM process was determined. The two distributions with the lowest sums of squared errors relative to the priors were selected as representative normal distributions. For the sake of convenience,

Table 3: Attribute Values for Simulated GBM Processes, Standard Deviation of Attributes, and Two Representative Processes

GBM (I.I.D) Process	CoV 20-day Vol.	Uncond. Skewness	Uncond. Kurtosis	Leverage Corr.	Auto-Corr. (1–20 lags)	Auto-Corr. (51–70 lags)	Time Decay Line Fit	SSE
Expected Results	0.1622	0.0000	3.0000	0.0000	0.0000	0.0000	–0.5000	0.0000
Average Result of 500 Simulations	0.1617	0.0000	3.0020	0.0000	–0.0014	0.0001	–0.5085	6.9806
Standard Deviation of 500 Simulations	0.0010	0.0607	0.1345	0.0999	0.0051	0.0061	0.0778	4.5646
Representative GBM Price Process (Z1)	0.1648	0.0004	3.0705	–0.0042	–0.0011	–0.0082	–0.5000	0.3636
Representative GBM Volatility Process (Z2)	0.1586	–0.0160	3.0529	–0.0182	–0.0020	–0.0002	–0.5002	0.3647

The furthest left column indicates various simulations of an Independent and Identically Distributed (I.I.D.) Geometric Brownian Motion (GBM) price process. The row titled "Expected Results" is not based upon simulation but is what is expected from statistical theory. The next two rows titled "Average Results of 500 Simulations" and "Standard Deviation of 500 Simulations" indicate the sampling properties of 500 series of simulated prices of 1500 observations based upon random draws from an I.I.D. GBM process. The next two rows represent the two draws from the I.I.D. GBM process which has the lowest sum of squared errors relative to the expected theoretical results. These will be used in the second stage of the analysis as representative normal distributions and will be referred to as Z1 and Z2. Columns 2 to 8 indicate the actual attribute values. The furthest right column (the ninth column) indicates the sum of squared errors (SSE) statistic for that row. Of importance to a later stage of analysis is the average standard deviation of the SSE. This result will allow comparisons to be made between alternative models.

the normal disturbances for the underlying price process will be referred to as Z_1 and the normal disturbances for the volatility process will be referred to as Z_2. These two series were uncorrelated. Thereafter, whenever simulation used either of these distributional forms, the same sets of random numbers were used (to reduce errors introduced by the selection of random numbers). Table 3 details the results of the simulated GBM price series and provides the sample standard deviation of the 500 simulated series.

In this table, the theoretical attribute values for a GBM process are listed as are the average attribute values and the standard deviations of the attributes across the 500 simulations. Of crucial interest are the sampling properties of the attributes and especially the sum of squared errors (SSE) statistic. The standard deviation of this statistic was found to be equal to 4.5646 and will be used subsequently as a means to establish confidence intervals for the comparison of the alternative models to a GBM assumption. Finally, the char acteristics of the two representative draws of the GBM process appear in the bottom two rows.

To generate the NIG distributions, random numbers were generated using the method suggested by RYDBERG (1997). These simulations required the input of the four moments of the distribution. The first moment (mean) was set to 0.0 and the second moment (variance) to 1.0. The third (skew) and fourth (excess kurtosis) moments were chosen to be less than the observed moments for daily returns in Appendix A1. This was done because of the fact that the stochastic volatility will interact with the NIG distribution and yield simulated moments that are amplified compared to the NIG moments. Given these effects could not be ascertained prior to the simulation, four NIG distributions were simulated and all were examined as potential candidates for the underlying price process. These four possible NIG distributions appear in Table 4 as NIG#1 to NIG#4.

This approach contains an apparent inconsistency: while the random numbers estimated conform to a NIG distribution, prices were simulated assuming that a risk-neutrality condition exists. When the underlying price process follows an alternative process, such as a NIG distribution, a correction to the drift term is required to allow risk neutral evaluation. An appropriate risk-neutral drift adjustment is:

Table 4: Sample Statistical Moments of Simulations of Four Normal Inverse GAUSSIAN (NIG) Distributions

NIG Distribution	Mean	Standard Deviation	Skewness	Kurtosis
NIG #1	0.01821	1.00088	–0.0049	4.7807
NIG #2	0.00078	0.93955	–0.9289	8.7576
NIG #3	–0.0067	1.01818	–0.1936	3.4129
NIG #4	0.0035	0.99176	–0.0794	7.627

We simulated four NIG distributions using the method suggested by RYDBERG (1997). This method requires the four moments of the distribution to be input. The first moment (mean) was set to 0.0 and the second moment (variance) to 1.0. The third (skew) and fourth (excess kurtosis) moments were less than the observed moments for daily returns of the Austrian Stocks and the ATX. This was done, due to the fact that the stochastic volatility will interact with the NIG distribution and amplify the resultant simulated moments.

$$a_t = \frac{\delta}{\Delta t} \cdot \left[\begin{array}{l} \sqrt{\alpha^2 - \left(\beta^2 + \sigma_{t-1} \cdot \sqrt{\Delta t}\right)^2} \\ - \sqrt{\alpha^2 - \beta^2} \end{array} \right] - \frac{\mu \cdot \sigma_{t-1}}{\sqrt{\Delta t}} \quad (7)$$

Where a_t is the risk neutral adjusted drift, the Greek letters, α, β, δ, and μ are the parameters of the NIG distribution and σ_{t-1} is the random volatility (from stochastic volatility process in equation 3.2) for the previous discrete observation. A complete proof of the derivation of this drift adjustment appears in Appendix A2.[7] This drift is then used in equation 3.1 with the drift term (μ) equal to a_{t-1} from equation 7 and the disturbances are drawn from the NIG distribution.

Finally, to simulate correlated processes, we estimated a new set of random numbers (Z′) for the volatility process using the usual method for drawing samples from a standardised bivariate distribution:

$$Z' = D \cdot \rho + \sqrt{(1-\rho)^2} \cdot Z_2 \qquad (8)$$

Where D represents the random disturbances for the underlying price process (in the case of GBM, the Z_1 set of normal draws, in the case of a NIG process, the draws from one of the four samples). The term, Z_2, refers to the representative normal distribution selected for the volatility process and the term, ρ, refers to the correlation coefficient between the two processes.

Once the distributions were drawn, the second stage of the simulated method of moments approach was done in three steps. The first step was to simulate price and volatility series that were consistent with the proposed models and secondly, determine the attribute values for this simulated series. The third step entailed varying the parameter inputs into the models to minimise the sum of squared errors relative to the observed empirical attributes.

To efficiently perform the third step, a starting point was to examine the sensitivities of the overall sum of squared errors to a small incremental change in each of the parameter inputs (holding the other parameters constant). Of the four variables in both of the stochastic volatility models, it was found that only three of the factors were critical. For a given long-term volatility, θ (taken as roughly equal to the average 20-day volatility in Table 2), the crucial factors are the rate of mean reversion, κ, the volatility of the volatility, ξ, and the correlation between the price and volatility process, ρ. It was found that small changes in the level of the long-term volatility had a minor effect on the sum of squared errors. Given that only three parameters needed to be varied, the parameterisation simply compared the sum of the squared errors using the initial seeded parameter values to the sum of squared errors for the same model (and random numbers) but varying the three critical parameters (κ, ξ and ρ). By varying only three parameters both up and down, thus, only eight alternatives to the original results had to be compared. If one of the new combinations of parameters achieved a lower sum of squared errors, this model would replace the previous model.

The search routine continued to search the "cube" of eight adjacent alternative parameter combinations until no new combination yielded better results. The initial search procedure first used fairly high increments in the adjacent corner search (for example 1.0 for κ and 0.1 for ξ and ρ). When optimal parameters were found, the increments for the search was progressively reduced (for example, as low as 0.01 for κ and 0.0001 for ξ and ρ) until no further reduction in the sum of squared errors was achieved. When no further improvement in the sum of squared errors is possible, the final combination of parameter values is deemed the optimal estimation of the alternative stochastic volatility models.

Table 5 displays the empirical attributes (taken from Appendix A 1 and Table 2) for each of the seventeen markets and compares these attributes consistent with Model 1 (GBM Constant Variance). With a null hypothesis that each of the attributes conform to an i.i.d. GBM process, T-statistics for each attribute were estimated using the previously estimated standard errors for each attribute (by simulation in Table 3). The attribute levels for the representative GBM sample (Z_1) appears at the bottom of Table 5 with the estimated standard errors underneath. When the T-test fails to reject the null hypothesis that the empirical attributes are consistent with an i.i.d. GBM process at above a 95% level, the empirical attribute appears in bolded type.

The returns for the GBM series have a skewness statistic of 0.0004 and a kurtosis statistic of 3.0705. Regarding the unconditional skewness, fourteen of the seventeen markets display significant levels of non-zero skewness (exceptions are Brau Union, Erste and Wienerberger). For the un-

Table 5: Empirical Attributes of Sixteen Austrian Stocks / ATX Futures & Sum of Squared Errors for a GBM Model

Market	Empirical Moments							Statistical Analysis	
	CoV	Uncond. Skewness	Uncond. Kurtosis	Leverage	AvgCor (1–20)	AvgCor (51–70)	Line Fit	SSE	T-test of GBM Null Hypothesis
AMS	0.481	–0.476	11.443	–0.253	**0.0575**	**0.0218**	**–0.4487**	37.2308	8.16
BAS	0.696	–0.533	18.467	–0.359	0.1927	0.0818	–0.2141	120.9731	26.50
BWT	0.453	0.543	9.075	**0.166**	0.0950	**0.0299**	–0.2569	43.7328	9.58
BRAU	0.432	**–0.083**	9.791	**0.078**	**0.0696**	**0.0311**	–0.1962	43.1651	9.46
BUD	0.456	–0.289	6.859	–0.495	0.1533	0.0999	–0.1571	73.9923	16.21
ERSTE	0.561	**0.035**	22.988	**–0.186**	0.0928	**0.0268**	–0.2815	73.1005	16.01
FLU	0.459	–0.457	12.654	**–0.192**	**0.0837**	**0.0284**	–0.2930	46.4709	10.18
MM	0.549	–0.625	15.709	–0.317	0.1190	**0.0070**	–0.3329	63.5292	13.92
OMV	0.515	–0.238	8.436	–0.290	0.1384	0.0550	–0.1565	67.9732	14.89
RAD	0.462	–0.132	10.137	–0.269	0.0894	**0.0301**	–0.2090	49.4547	10.83
SEMP	0.460	–0.199	8.077	**0.137**	**0.0591**	**0.0221**	–0.2054	39.3844	8.63
VAS	0.391	–0.402	4.607	–0.674	0.1179	0.0629	–0.1515	54.4483	11.93
VAT	0.585	–0.295	8.840	–0.550	0.2056	0.0860	–0.0973	107.9631	23.65
VER	0.672	0.248	11.592	0.374	0.1464	0.0732	–0.0834	106.5343	23.34
WIE	0.545	**–0.078**	11.096	**–0.081**	0.1345	0.0607	–0.1447	76.5822	16.78
WOL	0.482	–0.172	9.512	**–0.189**	0.0903	**0.0157**	–0.2457	43.1651	9.46
ATX	0.451	–0.710	10.116	–0.322	0.1208	0.0495	–0.1516	61.3352	13.44
Average	0.509	–0.2272	11.1411	–0.2014	0.1156	0.0460	–0.2015		
Std. Dev.	0.083	0.3145	4.4109	0.2708	0.0428	0.0275	0.0722		
GBM	**0.1648**	**0.0004**	**3.0705**	**–0.0043**	**–0.0011**	**–0.0082**	**–0.5000**		
Std. Error	**0.0010**	**0.0607**	**0.1345**	**0.0999**	**0.0051**	**0.0061**	**0.0778**	**4.5646**	

This table displays the empirical attributes (taken from Tables 2 and 3) and compares all attributes to those consistent with Model 1 (GBM). The average attribute value for each market and the standard deviation across the seventeen markets appears below. At the very bottom of the table, are the attributes for a representative GBM price series. Standard errors for attributes consistent with a GBM price process appear in the last row and are determined by simulation of 500 GBM series. These standard errors are used to determine T-statistics to test the null hypothesis that the attributes for the seventeen markets are not different from a GBM process. When the null hypothesis is rejected at a 95% level or above, the attribute appears in **BOLDED** text. In the furthest two right-hand columns appears summary statistics. These represent the sum of squared errors for the seven attributes relative to the I.I.D. GBM assumption and T-statistics appear to test the null hypothesis that these conform to this assumption. The standard error for this test is 4.5646 and appears in the lowest row.

conditional kurtosis all markets were significantly leptokurtic. The divergences of these moments for the seventeen markets is not surprising as this was previously examined in Table 2 with the Bera-Jarque statistic. This alone would be sufficient to reject the hypothesis that these securities returns are normally distributed.

For the volatility characteristics, we reject the null hypothesis that the empirical attributes are drawn from an i.i.d. process. For example, the average autocorrelations for all seventeen markets are positive and in most cases significantly different from zero. For ten of the seventeen markets, a significant relationship is found between the levels of the volatility and the underlying price (a negative leverage effect for nine of the ten markets). For all markets, the volatility of volatility (CoV) is significantly too high and the rate of decay in the volatility of volatility (Line Fit) does not conform to the square root of time hypothesis of an i.i.d. process (AMS is not significant at a 95% level but at the 90% level).

The sum of squared errors statistic provides an overall test of both i.i.d. processes and GBM. In last two columns of this table the sum of squared errors (SSE) statistic and a T-test statistic (testing the null hypothesis of a SSE statistic of 0.0) appear. The standard error used for this test is the simulated result for the SSE statistic provided in Table 3 (and appears also at the bottom of this table: 4.5646). It is clear that the Sum of Squared errors is large for all seventeen markets and the T-statistic indicates rejection of the null hypothesis that these seventeen markets conform to an i.i.d. GBM process. Now that we have knocked the straw man down (Model 1), we will consider alternative stochastic volatility models.

The results for the seventeen markets with the optimised parameter values for Model 2 [HESTON (1993)] appears in Table 6. In this table, the

Table 6: Empirical Attributes of Sixteen Austrian Stocks / ATX Futures & Sum of Squared Errors for the HESTON (1993) Model Assuming the Underlying Price Process follows GBM and the Volatility and Underlying Processes are Correlated

Market	Model Parameters HESTON (1993) Optimized values				Simulated Moments							Statistical Analysis	
	θ	κ	ξ	ρ	CoV	Uncond. Skewness	Uncond. Kurtosis	Leverage	AvgCor (1–20)	AvgCor (51–70)	Line Fit	SSE	T-Test vs. GBM Model
AMS	0.42	10.10	2.70	–0.75	**0.355**	–0.558	**5.1380**	–0.5402	0.0814	–0.0038	–0.3316	**8.6063**	**–6.27**
BAS	0.22	3.00	1.00	–0.60	**0.523**	–0.653	**6.8215**	–0.5224	0.2193	0.0911	–0.1312	**13.0153**	**–23.65**
BWT	0.22	8.90	2.10	–0.35	0.384	**–0.542**	**5.1656**	–0.2163	0.1048	–0.0261	–0.2900	**13.9800**	**–6.52**
BRAU	0.25	8.10	2.10	–0.25	0.425	**–0.544**	**5.6408**	–0.2693	0.1374	–0.0254	–0.2514	**5.6602**	**–8.22**
BUD	0.25	2.10	1.90	–0.30	0.535	**–0.833**	7.6640	–0.4992	0.2357	0.1540	–0.0634	**2.8561**	**–15.58**
ERSTE	0.27	6.50	1.70	–0.05	0.478	–0.160	**5.6471**	0.1261	**0.1893**	0.0075	–0.1806	**22.0393**	**–11.19**
FLU	0.24	6.50	1.70	–0.30	0.509	–0.592	**6.4836**	–0.2524	**0.2090**	0.0225	–0.1633	**8.4431**	**–8.33**
MM	0.25	6.00	2.00	–0.55	**0.430**	–0.619	**5.9545**	–0.4284	0.1407	–0.0091	–0.2500	**9.9744**	**–11.73**
OMV	0.28	6.00	2.00	–0.55	0.439	**–0.727**	**6.3888**	–0.4489	0.1514	0.0556	–0.1614	**4.3023**	**–13.95**
RAD	0.32	5.00	2.20	–0.40	0.466	**–0.775**	**6.7671**	–0.5959	0.1727	0.0696	–0.1422	**6.1179**	**–9.49**
SEMP	0.35	9.90	2.70	–0.05	0.366	–0.221	**4.6686**	0.0107	0.1013	0.0033	–0.1200	**4.5120**	**–7.64**
VAS	0.32	4.90	1.10	–0.55	0.370	–0.427	4.6700	–0.5014	0.1236	0.0430	–0.1874	**1.1851**	**–11.67**
VAT	0.25	2.90	1.10	–0.50	0.487	–0.664	**6.2312**	–0.5860	0.2003	0.0967	–0.1246	**3.1882**	**–22.95**
VER	0.24	2.00	1.70	0.10	**0.517**	**–0.360**	**5.8039**	**–0.1501**	0.2270	0.0692	–0.1343	**13.1253**	**–20.46**
WIE	0.26	4.00	1.80	–0.20	**0.382**	–0.379	**5.1508**	–0.1434	0.1198	–0.0047	–0.2193	**7.0395**	**–15.24**
WOL	0.38	8.00	2.00	–0.40	0.386	**–0.612**	**5.4309**	–0.4767	0.1106	0.0179	–0.2618	**5.4027**	**–8.27**
ATX	0.17	4.00	2.20	–0.60	0.462	–0.719	**6.5136**	–0.5051	0.1678	0.0440	–0.1727	**3.1682**	**–12.74**
	Standard Deviation of Simulated Attributes				0.060	0.188	0.824	0.217	0.049	0.049	0.070	5.303	

This table displays the simulated attributes with optimal parameters for Model 2 [HESTON (1993)]. The simulated attribute value for each market and the standard deviation across the seventeen markets appears below. These standard deviations across the seventeen markets are used to determine T-statistics to test the null hypothesis that the attributes for the seventeen markets are different than those generated with Model 2. When the null hypothesis is not rejected at a 95% level or above, the attribute appears in **BOLDED** text. In the furthest two right-hand columns appear summary statistics. These represent the sum of squared errors for the seven attributes relative to the simulated attributes of Model 2 and T-statistics appear to test the null hypothesis that the sum of squared errors for Model 2 is not different from that of the GBM case.

parameter values, simulated attribute and the SSE and T-statistics are presented. The rates of mean reversion (κ) are similar to those found by HESTON (1993). However, the volatility of volatility (ξ) is somewhat higher and for all markets a negative correlation between the processes (ρ) was found.

Null hypotheses were tested; that the simulated attributes are different from the empirical attributes found in Table 5. Given that Model 2 will have different sampling properties than the i.i.d. GBM case, standard errors for T-tests are determined by the standard deviation of the simulated attributes. These appear at the bottom of Table 6. When the null hypothesis is not rejected at a 95% level, the simulated attributes appear in bolded text. Using this as our basis for comparison, it is clear that Model 2 is able to capture most aspects of the volatility process. For almost all the autocorrelations of absolute returns, the volatility of volatility (CoV), the leverage effect and the time decay of volatility of volatility (Line Fit), simulated attributes of Model 2 reject the null hypothesis (of no improvement). However, for the return attributes, Model 2 does not reject the null hypothesis for the unconditional kurtosis for fifteen of the markets and for the unconditional skewness for seven of the markets. As has been pointed out elsewhere in the literature [see BATES (1996, 2000)] stochastic volatility models assuming the underlying price process follows GBM will not generate sufficient levels of excess kurtosis. Regarding the failure to capture the unconditional skewness, the inclusion of the negative correlation between the price and volatility processes has resulted in a more extreme degree of negative skewness than is observed empirically.

To assess the overall goodness of fit, the sum of squared errors statistic and a T-test versus Model 1 appear in the two furthest right-hand columns. The T-statistics test the null hypothesis of no reduction in the SSE statistic from Model 1 to Model 2. The HESTON (1993) model is (in all cases) a significantly better model than GBM for explaining the dynamics of the price processes. The greatest improvement is found for Bank Austria, VA Tech and Verbund with T-statistics above 20.

Following the lines of BATES (1996, 2000), HO, PERRAUDIN and SØRENSEN (1996), BARNDORFF-NIELSEN (1997) and BARNDORFF-NIELSEN and SHEPHARD (1999), we test an alternative stochastic volatility model with the underlying price innovations driven by an asymmetrical distribution with fatter tails than a normal distribution. As an economic interpretation, this will introduce asymmetric jumps into the state space. In this simulation, the stochastic volatility process remains driven by a GBM process (and we used the same random numbers as in the previous simulation) while the innovation of the underlying price process is assumed to follow a NIG distribution. The simulation used the method suggested by RYDBERG (1997).

As with the previous calibration of the stochastic volatility model, we generated four new price series that assumed the price innovation followed (each of) the NIG distributions (appearing in Table 4) and that the volatility dynamics were consistent with the HESTON (1993) stochastic volatility model. Using an appropriate risk-neutral drift adjustment (equation 7), the estimated price series were imported into the same programmes used to describe the empirical dynamics of the sixteen Austrian stocks and the ATX futures markets. As with the previous step, initial seed parameters were used for Model 3 and optimisation was achieved by the same search procedure over decreasing grid increments (actually corners of a cube). This allowed us to select the optimal parameter combination for each sample of NIG numbers. The starting and long term volatility were set approximately equal to the average level of the unconditional 20-day historical volatility reported in Table 2. While all four NIG distributions were examined, we only report the best fitting parameter values for the Model 3 and the optimal NIG distribution. This appears in Table 7.

Table 7: Empirical Attributes of Sixteen Austrian Stocks / ATX Futures & Sum of Squared Errors for the Model Assuming the Underlying Price Process follows a NIG Process and the Volatility and Underlying Processes are Correlated

Market	Model Parameters BN & S (1999) Optimized values					Simulated Moments							Statistical Analysis		
	NIG #	θ	κ	ξ	ρ	CoV	Uncond. Skewness	Uncond. Kurtosis	Leverage	AvgCor (1–20)	AvgCor (51–70)	Line Fit	SSE	T-Test versus GBM Model	Differences in T-Test Statistics Model 3 vs. Model 2
AMS	T2	0.42	8.32	1.89	0.23	0.429	–0.449	10.8168	–0.0211	0.0508	0.0153	–0.3493	**1.0663**	**–7.92**	**–7.67**
BAS	T1	0.22	2.95	0.91	–0.60	0.584	–0.707	**11.5301**	–0.4500	0.1777	0.0834	–0.1771	**3.9883**	**–25.63**	**–9.23**
BWT	T3	0.22	8.30	1.90	0.24	0.412	**0.042**	6.6737	0.2575	0.0998	0.0147	–0.2509	**3.9030**	**–8.73**	**–10.29**
BRAU	T2	0.25	8.33	1.92	0.24	0.433	–0.210	9.9962	–0.0329	0.0574	0.0181	–0.3056	**2.5360**	**–8.90**	**–3.21**
BUD	T1	0.25	2.13	0.75	–0.29	0.528	–0.288	9.2867	–0.2419	0.1665	0.0848	–0.1908	**2.6563**	**–15.63**	–0.24
ERSTE	T4	0.27	9.00	2.52	–0.55	0.475	**–0.410**	**14.4813**	–0.0918	0.0525	–0.0067	–0.3800	**2.3549**	**–15.50**	**–20.03**
FLU	T4	0.24	10.00	2.80	–0.40	0.493	–0.806	15.6602	–0.2774	0.0496	–0.0008	–0.3600	**1.9342**	**–9.76**	**–6.64**
MM	T1	0.25	11.00	3.20	–0.55	0.461	–0.461	**9.1559**	–0.4193	0.0735	0.0044	–0.4464	**1.1961**	**–13.66**	**–8.93**
OMV	T1	0.28	2.13	0.85	–0.33	0.522	–0.345	9.4628	–0.2364	0.1595	0.0759	–0.1958	**1.2506**	**–14.62**	**–3.12**
RAD	T1	0.32	4.01	2.01	–0.41	0.544	–0.531	10.2352	–0.4061	0.1515	0.0532	–0.2367	**3.7605**	**–10.01**	**–2.45**
SEMP	T3	0.35	9.02	1.99	0.01	0.397	–0.198	5.5836	–0.0299	0.1157	0.0265	–0.2576	**3.6784**	**–7.82**	–0.90
VAS	T3	0.32	3.50	1.12	–0.67	0.382	–0.316	4.8881	–0.3871	0.1421	0.0557	–0.1769	**1.6905**	**–11.56**	0.49
VAT	T1	0.25	2.40	1.05	–0.38	0.603	–0.372	10.6659	–0.3390	0.2058	0.0904	–0.1674	**2.2267**	**–23.16**	–1.01
VER	T4	0.24	1.00	1.70	0.05	0.569	–0.053	9.4817	0.0377	0.1530	0.0452	–0.2189	**3.4478**	**–22.58**	**–9.88**
WIE	T1•	0.26	2.00	1.40	–0.11	0.550	–0.068	9.6654	–0.2122	0.1727	0.0575	–0.2235	**1.7391**	**–16.40**	**–5.41**
WOL	T1	0.38	8.00	2.40	–0.50	0.470	–0.415	9.2964	–0.4193	0.0954	0.0162	–0.3036	**1.9553**	**–9.03**	**–3.53**
ATX	T1	0.17	4.04	1.79	–0.50	0.522	–0.498	9.8216	–0.3769	0.1337	0.0518	–0.2552	**2.0124**	**–13.00**	–1.21
	Standard Deviation of Simulated Attributes					0.067	0.222	2.670	0.203	0.051	0.032	0.081	0.985		

This table displays the simulated attributes with optimal parameters (and NIG distribution) for Model 3. The simulated attribute value for each market and the standard deviation across the seventeen markets appears below. These standard deviations across the seventeen markets are used to determine T-statistics to test the null hypothesis that the attributes for the seventeen markets are different than those generated with Model 3. When the null hypothesis is not rejected at a 95% level or above, the attribute appears in **BOLDED** text. In the furthest three right-hand columns summary statistics appear. These represent the sum of squared errors for the seven attributes relative to the simulated attributes of Model 3 and two sets of T-statistics appear. The first series of T-statistics tests the null hypothesis that the sum of squared errors for Model 3 is not different than that of the GBM case. The second set of T-statistics tests the null hypothesis that the sum of squared errors for Model 3 is not different than the sum of squared errors for Model 2.

As with the comparisons of the two previous models, null hypotheses were tested; that the simulated attributes are different from the empirical attributes found in Table 6. As this model will also have difference sampling properties for the target attributes, standard errors for T-tests are determined by the standard deviation of the simulated attributes for this model (appearing at the bottom of Table 7). When the null hypothesis is rejected at a 95% level, the simulated attributes appear in bolded text.

For Model 3, almost all the attributes reject the null hypothesis that the simulated attributes are different from the empirical attributes. The only exceptions are two markets for the unconditional skewness (BWT and Erste) and three markets for the unconditional kurtosis (Bank Austria, Erste and Mayr Meinhof). Model 3 either generated more negative skewness than was observed empirically or could not generate sufficient excess kurtosis. However, in all instances the return moments were closer than those found for Model 2.

For sixteen of the seventeen markets, there has been a further reduction in the SSE statistics (the only exception was insignificant for VA Stahl). The primary reason for improvement is the ability of Model 3 to address the levels of empirical excess kurtosis, and to simultaneously explain the other six attributes. The stochastic volatility parameters (both κ and ξ) for almost all seventeen markets are reduced, suggesting that NIG price innovation is interacting with this model. In the two furthest right-hand columns comparisons between the alternative models appear. The T-test versus Model 1 (GBM) rejects the null hypothesis of no reduction in the SSE statistic for all seventeen markets. The next column compares Models 3 to Model 2. Given this comparison is for alternative models to GBM, standard errors for the SSE statistic are taken as the standard deviation of this statistic for Model 3 (see bottom line of Table 7). Using this to generate T-statistics, Model 3 rejects the null hypothesis of no reduction in the SSE statistic from Model 2 in twelve of the seventeen cases. In no instance does Model 2 have a significantly lower SSE than Model 3.

From the fact that Model 3 is able to simultaneously explain all non-normal and non-i.i.d. aspects of the sixteen Austrian stocks and the ATX futures, this suggests that both stochastic volatility and non-normal distributions contribute to the dynamics of the Austrian stock market. The stochastic volatility models cover volatility attributes, while the inclusion of fat-tailed distributions addresses the excess kurtosis and correlated processes explains the unconditional skewness and the leverage effect.

7. Implications for Option Pricing

From the preceding section, a more realistic price process for the seventeen markets has thus been uncovered. This will serve as a prior process for the estimation of option values. Given that the parameter estimation of the stochastic volatility models relies upon simulation, it is a simple matter to use a similar simulation technique to estimate option prices. This simulation approach determines European call and put options numerically (a Monte Carlo approach) over a variety of strike prices and times to expiration. This is similar in spirit to the approach used by JOHNSON and SHANNO (1987).

Given that for all seventeen markets, Model 3 best explained the empirical attributes (or was insignificantly different from Model 2); we will only simulate option prices from this model. Furthermore, to reduce the amount of output, only results for options on ATX futures are presented.[8] These will be compared to option prices assuming a GBM price innovation with constant volatility [the BLACK (1976) model. The choice of the appropriate NIG distribution and the parameter values for the ATX Futures are taken from Table 7.

An apparent inconsistency for this approach is that the use of Monte Carlo simulations to price options assumes that a risk-neutrality condition exists. This suggests that the state space is continuous and spanned by existing securities. However, the model we simulated introduces a correlated stochastic volatility process and include (negative) jumps and into the state space. Given there are not securities in existence, which allow us to span a state space where the volatility and returns display such dynamics, these models do not allow us to use risk-neutrality and therefore would not allow us (in the strictest sense) to price the options. This is the apparent theoretical inconsistency. However, the determination of the risk-neutral drift adjustment for the NIG process in equation 7 will allow us to compare the simulated options prices to those estimated by the BLACK (1976) formulae. Both will be estimated under an equivalent risk-neutral measure.

In this simulation, price series of three months in length were determined. Given that the estimation of the unconditional (historical) dispersion processes was completed for trading days, options were also priced using trading time instead of calendar time. The assumed number of trading days in a year is 252. Option prices were estimated at time horizons from one week (five trading days) to three months (in 5-day increments). Such options would correspond to typical terms to maturity of actively traded options on ATX Futures.

To gain a better understanding of the impacts of the alternative models across strike prices, we examined fifteen strike prices. The median strike price was centred at the starting value of the simulation and was equal to 100. As we were assuming the underlying asset was a futures contract, the interest rate associated with the drift was set to zero and the appropriate NIG drift from equation 7 was used. This corresponds to an at-the-money option relative to the forward price.

We were also interested in the prices of options with different strike prices. Our analysis was restricted solely to out-of-the-money strike prices. Thus, when the strike price was equal to or below the starting value of 100, the option evaluated was a European put and when the strike price was above 100, the option evaluated was a European call. A non-trivial problem is the choice of strike prices so that as maturities of options vary, meaningful comparisons can be drawn. In previous papers on the impacts of stochastic volatility on option prices, strike price determination has taken one of two forms. Authors have either chosen to fix a single maturity and vary the strike prices in terms of „moneyness" [see HULL and WHITE (1987,1988) or fixed the degree of moneyness (or strike prices) and examined the impacts across different maturities [see HENKER and KAZEMI (1998).

Unfortunately, both methods do not allow meaningful conclusions to be drawn regarding the impacts of the models on option prices across time and a more consistent measure of moneyness is needed. Both NATENBERG (1994) and TOMPKINS (1997) have proposed a more consistent measure of strike price. The strike price is expressed as the logarithm of the ratio of the strike price X of the option relative to the underlying futures price F, divided by the product of the level of the at-the-money volatility and the time remaining until the expiration of the option. This will be expressed as:

$$\frac{\ln(X_{\tau,T}/F_{\tau,T})}{\sigma\sqrt{\tau/252}} \qquad (9)$$

where X is the strike price of the option at time period τ with expiration T, F is the underlying futures price at time period τ with expiration T and the square root of time factor reflects the percentage in a trading year of the remaining time until the expiration of the option. For all the simulations, the initial futures price $F_{\tau,T}$ was set to 100. The sigma (σ) is the at-the-money volatility. This was assumed to be equal to the long-term volatility used in the model estimations for the ATX futures in Table 7 (17% annualised volatility).

This adjustment notes that the distance of an option strike price to the level of the underlying asset is relative, both with respect to the current price of the underlying, the time to expiration and the level of expected variance. This adjustment converts all strike prices into a metric that can be interpreted as a standard deviation. Thus, in our analysis, we examined strike price ranges ± 3.5 standard deviations away from the at-the-money level in 0.5 standard deviation increments. This change in measure will allow more direct comparison of model impacts on option prices where the time to maturity varies but the relative strike prices remain the same. Such standardisation eases evaluation of the biases in option prices compared to the BLACK (1976) model prices and allows comparisons with implied volatility surfaces for related research for four other stock index futures markets previously examined by TOMPKINS (2000).

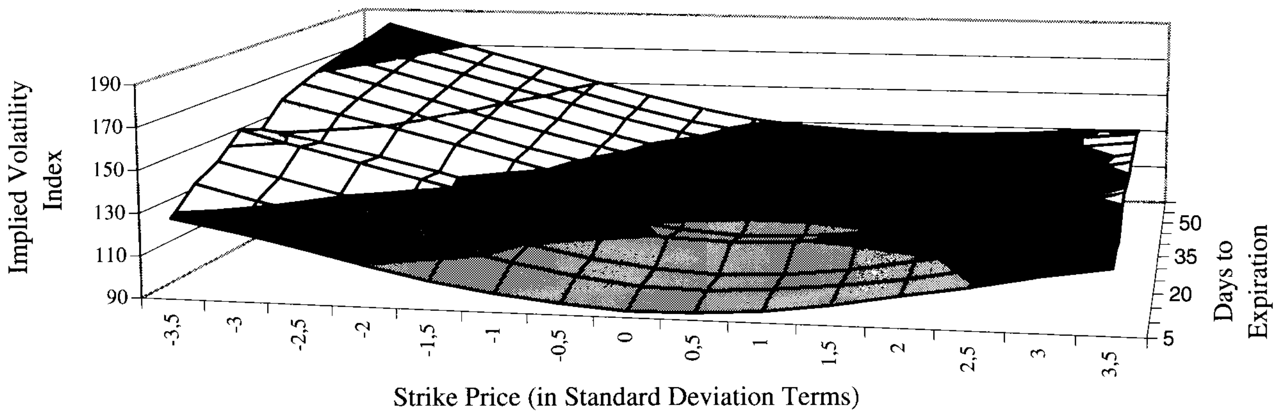
For the Monte Carlo simulation, random numbers consistent with a GBM process were determined using a Box-Muller technique and employed the anti-thetic approach suggested by BOYLE (1977). This series was later used for the determination of the bivariate distribution required when price and volatility processes are correlated. To determine the appropriate NIG distribution (NIG #1), 10000 series of 60 daily increments (business days) were drawn using the method previously used by RYDBERG (1997) for parameter estimation in the previous section. A bivariate distribution was estimated for the volatility series using formula 8 and used the parameters for ATX futures in Table 7.

Volatilities and prices were estimated using an Euler approach (discrete form of formula 3.1 and 3.2). With the prices of each model estimated, the payoffs of the fifteen options (at each point in time) were determined and the result averaged. As interest rates were assumed to be zero, there is no need to discount the result to present value. In parallel, we estimated the prices of all the options for each market using the BLACK (1976) model with the same strike prices as the simulation, the same term to expiration and the volatility equal to the same long-term volatility used in the simulations.

While standardisation of the strike prices eases comparison between BLACK (1976) and Model 3 option prices, the sheer amount of output remains cumbersome. To further simplify comparison, the simulated option prices were expressed as implied volatilities using the BLACK (1976) formula and graphed relative to the standardised strike prices and time to expiration. These implied volatilities were indexed to the level of the volatility used for the estimation of BLACK (1976) prices. The indexed implied volatilities were then presented as a continuous surface. This surface for options on the ATX futures appears in Figure 1. Implied volatility surfaces for the remaining sixteen Austrian stocks are similar to that of the ATX futures. In Figure 1, the simulated implied volatility surface for options on ATX futures displays many of the features of implied volatility surface relative to the Black (1976) model found for actual options on stocks and stock index futures. There is the curvature characteristic of volatility smiles and negative skewness (especially for longer maturities). For short dated options in a range ± 2 standard deviations from the level of the underlying futures, the implied volatilities consistent with the NIG stochastic volatility model are below those of the constant BLACK (1976) implied volatilities (indexed to 100 for all strike prices and times to maturity). For all seventeen markets examined, the BLACK (1976) pricing model overvalues options that are at-the-money and within a significant range around the at-the-money level. The shaded areas below 100 represent this in the graphs. The NIG stochastic volatility out-of-the-money options tend to have higher implied volatilities relative to the BLACK (1976) implied volatilities. This bias is most extreme for options with lower strike prices.

One can observe that there is a significant range of strike prices and maturities where the BLACK (1976) model over-values the options. It is also the case that the overpricing bias tends to be increasingly as the term to expiration is extended. Given that these models include stochastic volatility and a surrogate for a skewed jump process (the NIG) and given that these events are not spanned in the security state, risk premia must exist. How-

Figure 1: Simulated Implied Volatility Surface for Options on ATX Futures Relative to the BLACK (1976) Model



This figure represents a simulated implied volatility surface consistent with the NIGSV model with parameter values determined using the period of analysis from 1993–1998. Option prices were estimated solely to periods from 5 days to 3 months in 5-day increments (trading days). Implied volatilties were estimated using the BLACK (1976) model. The strike prices were expressed in standard deviation terms (away from the average simulated futures price) and the volatilities were indexed to the constant level of 17%. The level of 100 for the indexed implied volatility corresponds to the level expected for options under the BLACK (1976) assumptions.

ever, we observe that these prices are below the prices of the BLACK (1976) model that assumes a complete market (without such risk premia). However, these results are consistent with the biases of stochastic volatility on option prices found in HULL and WHITE (1988).

In formula 20 of their paper, they demonstrate that in the presence of stochastic volatility, option prices will be biased downwards over a fairly wide range around the at-the-money level relative to the BLACK-SCHOLES price (assuming a zero correlation between the volatility and price processes). The range of bias is a function of the level of the volatility and the term to expiration. Furthermore, they observe that option prices (under stochastic volatility) outside this range will be higher than the prices of the BLACK-SCHOLES (1973) [or BLACK (1976) model (s)].

Therefore, our results are consistent with those of HULL and WHITE (1988). One contribution of this research is to examine the biases in the presence of both correlated stochastic volatility and a skewed fat-tailed innovation for the underlying market.

The observed biases can be easily analysed by considering separately the components of Model 3. The curvature and asymmetry for short dated options is due primarily to the inclusion of the NIG distribution. As the NIG distribution remains i.i.d., from the Central Limit Theorem, this distribution will rapidly approach a normal stochastic process. The excess kurtosis and negative skewness effects will dissipate fairly rapidly. In the absence of other factors, this would lead to a skewed and curved implied volatility smile for the shortest time period to expiration. Thereafter (and as the distribution approaches a normal stochastic process), the implied volatility surface would flatten, consistent with a constant volatility assumption. However, the impacts of stochastic volatility increase with the term to expiration. Thus, the two components of the model, non-normal price innovation and stochastic volatility again contribute jointly to the implied volatility surfaces. The NIG distribution causes short-term smile behaviour and stochastic volatility longer-term smile behaviour. The degree of increased asymmetry associated with longer-term options is due to the inclusion of correlated processes. In the case of the ATX futures, this negative correlation of –0.50 produces this effect.

Clearly of interest is the relationship between simulated implied volatility surfaces and actual implied volatility surfaces of options on stock index futures.[9] TOMPKINS (2000) completed similar analysis estimating the dispersion processes for four established stock index futures markets, S&P 500, FTSE 100, DAX and Nikkei Dow. As with this study, a stochastic volatility model correlated to a NIG innovation for the underlying could also explain the dispersion process for these markets. Using a similar procedure as in this paper, implied volatility surfaces consistent with the objective price process for these four markets were estimated and compared to actual smile surfaces determined by options for these four markets for the same period of analysis.

For implied volatility surfaces associated with actual options markets, the degree of curvature tended to be most extreme the shorter the term of the options life. However, for the simulated smiles, the degree of the curvature is much more extreme for longer maturity options. In addition, the actual smiles are much more curved than those associated with smiles consistent with the objective price process. For all four markets, the relative shapes are similar but the amplitude of the effect is dampened. Regarding the skewness bias (lower strike prices having relatively higher implied volatilities), the simulated smiles are able to capture most of the dynamics associated with actual implied volatility surfaces. Therefore, TOMPKINS (2000) concludes that most of the skewness effect observed for options on stock index futures is explained by the inclusion of an asymmetric jump process for underlying returns and negatively correlated price and volatility processes. The inclusion of an appropriate fat-tailed distribution (consistent with the objective return process) is insufficient to generate the empirically observed degrees of curvature.

8. Summary and Conclusions

This paper examines the nature of the objective dispersion processes that can be observed for sixteen Austrian stocks and the ATX futures. For all markets examined, we reject the hypothesis that the return series are i.i.d. and conform to Geometric Brownian Motion. However, the degree to which the return series deviate from a normal distribution diverges across assets. We identified seven attributes that capture the multi-faceted non-normality and interdependence of the empirical dispersion processes.

With these attributes for each of the seventeen markets, we examined alternative stochastic volatility models to understand the nature of these processes. We found that all Austrian equity securities are best understood with a stochastic volatility model correlated to a Normal Inverse Gaussian distribution for the price innovation. These results are consistent with previous research for non-Austrian stock markets [TOMPKINS (2000)].

With these optimal stochastic volatility models, European options were estimated numerically for ATX futures and significant divergences were observed from BLACK (1976) values. The simulated NIG/stochastic volatility process increased the prices of deep OTM options relative to BLACK (1976) prices. The values of options within a wide range of the ATM level were decreased. These results are consistent with biases reported by HULL and WHITE (1988) and TOMPKINS (2000).

It is left for future research to compare how well implied volatility surfaces consistent with the objective process explain actual implied volatility surfaces consistent with the risk-neutral process. For the Austrian market, option markets are too illiquid to allow such comparisons to be drawn. However, in related research for four non-Austrian stock index options markets, systematic differences between these surfaces were found. The nature of this difference remains for future research. Possible lines the research could take include incorporating transactions costs, segmentation of hedging strategies across strike prices, option type and time (by heterogeneous agents) and examination of the risk premia (due to incomplete markets).

Appendix

A 1: Statistics of the Daily, Weekly and Monthly Returns for Sixteen Austrian Stocks (Forwards) and ATX Futures

Underlying Asset	Mean	Std Dev	Uncond. Skew	Significance Level	Uncond. Kurtosis	Significance Level	Bera-Jarque Statistic	Observations
Austrian Micro Systems								
Daily Returns	0.00011	0.02780	–0.47593	**–7.13368**	11.44345	**63.27892**	**4055.111**	1348
Weekly Returns	0.00062	0.06598	0.18912	1.26632	4.38241	**4.62817**	**23.023**	269
Monthly Returns	0.00298	0.12691	–0.09889	–0.33047	2.79751	–0.33833	0.224	67
Bank Austria								
Daily Returns	–0.00044	0.01633	–0.53328	**–9.67044**	18.46745	**140.24152**	**19761.201**	1973
Weekly Returns	–0.00216	0.03903	–0.80726	**–6.54166**	7.31454	**17.48142**	**348.393**	394
Monthly Returns	–0.00814	0.09532	–1.11373	**–4.50109**	12.71452	**19.63039**	**405.612**	98
BWT								
Daily Returns	0.00057	0.01492	0.54251	**8.96921**	9.07532	**50.22098**	**2602.593**	1640
Weekly Returns	0.00283	0.03943	0.69169	**5.11415**	5.88480	**10.66467**	**139.890**	328
Monthly Returns	0.01133	0.08010	0.03515	0.12994	3.34073	0.62981	0.414	82
Brau Union								
Daily Returns	0.00005	0.01644	–0.08335	–1.65554	9.79125	**67.44403**	**4551.438**	2367
Weekly Returns	0.00023	0.04074	0.31731	2.81733	6.23899	**14.37917**	**214.698**	473
Monthly Returns	0.00130	0.09030	–0.11440	–0.50733	3.82476	1.82880	3.602	118
Boehler								
Daily Returns	–0.00020	0.01806	–0.28853	**–3.55922**	6.85905	**23.80185**	**579.196**	913
Weekly Returns	–0.00069	0.04045	–0.15971	–0.87963	5.07062	**5.70206**	**33.287**	182
Monthly Returns	–0.00059	0.08277	–0.61029	–1.67134	3.013382	0.01832	2.794	45
ERSTE								
Daily Returns	–0.00004	0.01885	0.03493	0.63394	22.98753	**181.36228**	**32892.680**	1976
Weekly Returns	–0.00023	0.04117	–0.34957	–2.83629	5.65345	**10.76474**	**123.924**	395
Monthly Returns	0.00082	0.08652	–0.34211	–1.38260	4.23469	2.49498	8.137	98
Flughafen								
Daily Returns	0.00023	0.01641	–0.45731	**–7.50980**	12.65367	**79.26404**	**6339.185**	1618
Weekly Returns	0.00117	0.03638	–0.44210	**–3.24375**	6.06371	**11.23941**	**136.846**	323
Monthly Returns	0.00408	0.06153	–0.34892	–1.27409	3.80720	1.47374	3.795	80
Mayr Melnhof								
Daily Returns	–0.00027	0.02011	–0.62453	**–8.66506**	15.70891	**88.16452**	**7848.065**	1155
Weekly Returns	–0.00133	0.04706	–0.08632	–0.53560	4.15450	**3.58175**	**13.116**	231
Monthly Returns	–0.00609	0.07626	0.02986	0.09202	4.29994	2.00335	4.022	57
OeMV								
Daily Returns	0.00015	0.01903	–0.23758	**–4.71987**	8.43637	**54.00011**	**2938.289**	2368
Weekly Returns	0.00077	0.04770	–0.48117	**–4.27224**	7.72621	**20.98158**	**458.479**	473
Monthly Returns	0.00379	0.09313	0.03588	0.15913	4.09446	2.42681	5.915	118
Radex								
Daily Returns	–0.00025	0.02148	–0.13152	–2.61282	10.13687	**70.89130**	**5032.404**	2368
Weekly Returns	–0.00114	0.05754	–0.20409	–1.81209	6.79890	**16.86485**	**287.707**	473
Monthly Returns	–0.00396	0.12314	–0.59450	–2.63645	5.13679	**4.73802**	**29.400**	118

Underlying Asset	Mean	Std Dev	Uncond. Skew	Significance Level	Uncond. Kurtosis	Significance Level	Bera-Jarque Statistic	Observations
Semperit								
Daily Returns	0.00266	0.02387	–0.19898	–2.30485	8.07673	**29.40195**	**869.787**	805
Weekly Returns	0.01331	0.06001	1.30461	**6.75799**	6.07106	**7.95418**	**108.939**	161
Monthly Returns	0.05500	0.10608	0.09527	0.24598	3.32041	0.41365	0.232	40
VA Stahl								
Daily Returns	–0.00021	0.02141	–0.40230	**–4.61913**	4.60698	**9.22554**	**106.447**	791
Weekly Returns	–0.00095	0.04338	–0.00937	–0.04811	3.41065	1.05365	1.112	158
Monthly Returns	–0.00312	0.09347	–0.87196	–2.22308	4.32393	1.68769	7.790	39
VA Tech								
Daily Returns	0.00002	0.01793	–0.29465	**–4.04901**	8.83971	**40.12370**	**1626.305**	1133
Weekly Returns	–0.00004	0.04336	0.45481	2.79133	5.10724	**6.46639**	**49.606**	226
Monthly Returns	0.00121	0.08721	–0.82912	–2.53299	4.42681	2.17948	**11.166**	56
Verbund								
Daily Returns	0.00079	0.01704	1.31937	**26.20541**	22.73471	**195.98567**	**39097.108**	2367
Weekly Returns	0.00398	0.04518	1.33722	**11.87293**	12.90460	**43.97057**	**2074.377**	473
Monthly Returns	0.01612	0.09363	1.30798	**5.80050**	7.092006	**9.07343**	**115.973**	118
Wienerberger								
Daily Returns	0.00011	0.01801	–0.07755	–1.54001	11.09567	**80.38125**	**6463.517**	2366
Weekly Returns	0.00053	0.04896	0.05820	0.51671	8.74777	**25.51669**	**651.368**	473
Monthly Returns	0.00170	0.08983	0.26202	1.16197	4.58484	**3.51415**	**13.699**	118
Wolford								
Daily Returns	0.00121	0.02516	–0.17226	–2.16978	9.51207	**41.01397**	**1686.853**	952
Weekly Returns	0.00593	0.06808	0.05172	0.29107	9.27646	**17.65980**	**311.953**	190
Monthly Returns	0.02476	0.13292	1.66621	**4.66339**	8.32536	**7.45233**	**77.284**	47
ATX Futures								
Daily Returns	0.00020	0.01179	–0.71028	**–10.51914**	10.11646	**52.69705**	**2887.631**	1316
Weekly Returns	0.00098	0.02584	–0.99494	**–6.58717**	6.60824	**11.94449**	**186.062**	263
Monthly Returns	0.00528	0.05467	–1.05175	**–3.46172**	7.07046	**6.69875**	**56.857**	65

These returns are based upon closing prices for the Sixteen stocks and the Nearest to Expiration ATX Futures closing prices. The first column describes the market under investigation and the frequency of return estimation. The second and third columns present the first (mean) and second (standard deviation) moments of the return distribution. The fourth and sixth columns present the statistics for the unconditional skewness and kurtosis. Under the null hypothesis of normality, the skewness statistic is normally distributed with standard errors: se =sqrt(6/T), where T is the sample size. This appears in the furthest right-hand column of the table. The fifth column indicates a significance statistic testing a normality hypothesis. If the skewness statistic rejects this at a 95% level or above, this is indicated in **BOLDED** text. Under the null hypothesis of normality, the excess kurtosis statistic is normally distributed with standard errors: se =sqrt(24/T). This statistic is equal to the kurtosis statistic appearing in the table minus 3.0. Column 7 indicates a significance statistic testing the null hypothesis of normality. If the kurtosis statistic rejects this at a 95% level or above, this is indicated in **BOLDED** text. The statistic in the sixth column is the Bera-Jarque (BJ) statistic for detecting departures of the data from normality. Under the null hypothesis of normality, the BJ statistic is distributed as Chi squared with 2 degrees of freedom. The critical value at the one-percent level is 9.21. When the BJ statistic is above this level, this statistic appears in **BOLDED** text.

A2: Derivation of the Risk Neutral Drift Adjustment for a Normal Inverse GAUSSIAN Process

The process S, generated by the NIG distribution should be a martingale. In the discrete time setting this means:

$$E\left[S_{t_j} \mid \Im_{t_{j-1}}\right] = S_{t_{j-1}}, \; j = 1,2,3,\ldots,n \qquad \text{(A1)}$$

with $\Im_{t_{j-1}}$ denoting the information at time t_{j-1}.

Since both the drift term, a, and the volatility, σ, are known at time $t_{j-1,}$ and the variance of the series V_j is independent from the history leading up until t_{j-1}, this is equivalent to:

$$e^{a_{t_{j-1}}\Delta t}\, m\left(\sigma_{t_{j-1}}\sqrt{\Delta t}\right) = 1, \qquad \text{(A2)}$$

where $m(x) = E[e^{xV_j}]$ is the moment generating function of the NIG(μ,δ,α,β) distribution.

This function is:

$$m(x) = \exp(\delta(\sqrt{\alpha^2 - \beta^2} - \sqrt{\alpha^2 - (\beta + x)^2}) + \mu x) \qquad \text{(A3)}$$

So the appropriate choice of drift is:

$$a_{t_{j-1}} = \frac{1}{\Delta t} \ln\left(\frac{1}{m\left(\sigma_{t_{j-1}}\sqrt{\Delta t}\right)}\right) \qquad \text{(A4)}$$

Combining equations 1.3 and 1.4 leads to

$$a_t = \frac{\delta}{\Delta t} \cdot \left[\begin{array}{l} \sqrt{\alpha^2 - \left(\beta^2 + \sigma_{t-1} \cdot \sqrt{\Delta t}\right)^2} \\ - \sqrt{\alpha^2 - \beta^2} \end{array}\right] - \frac{\mu \cdot \sigma_{t-1}}{\sqrt{\Delta t}} \qquad \text{(A5)}$$

As a final note, adjusting the drift is only one way to obtain a martingale (for risk-neutral valuation). Given the models we are examining suggest an incomplete market, there are alternative approaches. For example, for the constant volatility NIG process in continuous time, the natural approach would be to estimate the parameters, μ, δ, α and β from the observations of S and adjust β to obtain a risk neutral valuation.

Footnotes

[1] BARNDORFF-NIELSEN (1997) and BARNDORFF-NIELSEN and SHEPHARD (1999) actually assumed that the underlying price process follows GBM and the volatility process follows an Inverse Gaussian distribution. However, this is equivalent to the underlying price process following a NIG distribution and the volatility process follows GBM. See HUBALEK and TOMPKINS (2000).

[2] The estimation of the volatilities was done using overlapping data. This introduces a bias in the estimation of the standard deviation of volatility. This bias was corrected using the HODGES and TOMPKINS (2000) approach.

[3] The author would like to thank Stewart Hodges (University of Warwick) for suggesting the use of this variable.

[4] To simplify the selection of the attribute, a fixed weight of 0.03 was applied to all markets and for all periods of analysis, to allow the leverage correlation factor to not be subject to differing weights. This was found to be close to the optimal weights for each market using a maximum likelihood estimation procedure.

[5] In addition, the natural logarithm of the unconditional kurtosis was examined rather than the absolute levels. This was done due to wide variations in this statistic across the seventeen markets. However, all results are presented as absolute levels.

[6] Additional price data was obtained from Amsterdam Option Traders, Vienna.

[7] In Appendix A2, it is noted there that this is only one manner to adjust this drift to achieve risk-neutrality. Given that this model implies markets are incomplete, there may not exist a unique martingale measure. However, this is certainly a feasible adjustment and will allow relative comparisons to be drawn.

[8] Options prices for all seventeen markets were estimated using Model 3 and for the sixteen stocks, the risk-free Austrian interest rate and the actual dividends paid for each stock. Results are similar across all markets and thus, analysis was restricted solely to options on ATX futures as the representative market.

[9] Our original intention was to examine actual option prices for all the Austrian stocks and the ATX options markets. However, the lack of liquidity in these markets (especially the stock market in Austria) did not provide enough option prices to determine implied volatility surfaces. The liquidity on the ATX option was also problematic.

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