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Risk Value Analysis of Covered Short Call and Protective Put Portfolio Strategies

1. Introduction

The use of financial derivatives has nowadays become standard in modern portfolio management. An important economic motivation for using derivatives is that they facilitate hedging, that is, they enable managers of an underlying asset portfolio to transfer some parts of the risk of price changes to others who are willing to bear such risk. Options (on stocks, indexes or index futures) are specific derivative instruments that give their owner the right to buy (call-option) or to sell (put-option) a specified number of shares at a specified price (exercise price) of a given underlying asset at or before a specified date (expiration date). For this privilege the owner of the option (long position) pays a fixed premium to the writer (short position) of the option. The non-linear pay-off characteristic of options is unique among securities, which enables investors to create patterns of portfolio pay-offs that are unachievable by a simple combination of conventional investment vehicles (e.g. stocks or fixed-income securities). The only way to achieve a comparable pay-off characteristic without using options requires the use of dynamic portfolio rebalancing.[1] However, from a practical point of view, dynamic portfolio rebalancing is often not feasible or not desireable, investors preferring instead to implement a buy-and-hold strategy over a specific period of time.

While there are many types of option strategies, the protective put and the covered short call are the most widely used. Moreover, these are the only option strategies which regulated financial institutions such as insurance companies, pension or mutual funds can persue. The characteristic feature of the protective put strategy is that the investor holds a number of stocks and purchases put options on the same underlying asset, while with a covered short call he sells short (writes) call options against stocks he already owns. A feature common to both option strategies is that they enable investors to "insure" against losses when the price of the underlying asset declines. However, because the loss limits are achieved differently, each type of strategy results in substantially different pay-off characteristics at the end of the investment period.

In order to make appropriate investment decisions under risk the portfolio manager must be able to compare the hedging effectiveness of both option strategies. Following the classical mean-variance portfolio approach, the hedging effectiveness is examined according to the change in variance and

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expected return as put or call options are introduced into a portfolio containing the underlying asset. The basic limitation of such an approach is the lack of a satisfactory choice-theoretic foundation: The mean-variance framework requires either quadratic utility functions or normal asset return distributions.[2] Neither assumption is necessarily correct. A quadratic utility function is inappropriate, because it implies decreasing marginal utility of wealth, which is inconsistent with the basic requirements of stochastic dominance analysis. Additionaly, BOOKSTABER and CLARKE (1983a, b, 1984, 1985) have presented empirical and theoretical arguments that in the case of options the assumption of a normal distribution is not valid either. Portfolios including options lead to complex, distinctly asymmetrical return distributions with significant moments beyond mean and variance. Hence, these arguments point to the need for an alternative framework to examine the hedging effectiveness in the case of portfolios including options. For this study, we have decided to analyze hedging effectiveness using downside risk measures.

In the literature two kinds of parametric[3] formulations to quantify the downside risk are widely used: skewness and shortfall risk measures. To calculate the skewness of a distribution the third central moment is used, thereby measuring the level of asymmetry of a distribution around the mean. In general, investors prefer positive and dislike negative skewness. Given equality of the means and variances, one distribution is said to be riskier than another if it is more skewed to the left. For example, MERTON/SCHOLES/GLADSTEIN (1978, 1982), SEARS/TRENNEPOHL (1983) and TRENNEPOHL/BOOTH/TEHRANIAN (1988) provide empirical evidence about the volatility and the skewness of stock portfolios including options.[4]

Shortfall risk measures formulate the downside risk as a probability-weighted function of negative deviations from either the expected value or some other predetermined arbitrary target. These risk measures, sometimes referred to as lower partial moments, have attracted considerable interest in the more recent literature on option strategies. Depending on the methodological framework, there are three streams of research: The first uses historical time series[5], the second stochastic simulations[6] and the third an analytical framework to evaluate the shortfall risk and return characteristics of portfolios including options. Concerning the latter approach, which is the focus of this study, ADAM/MAURER/MÖLLER (1996) and ADAM/MAURER (1997) developed under the BLACK/SCHOLES assumptions (i.e. the value of the underlying asset is lognormally distributed) for a wide range of option strategies explicit analytical formulas for the first three central moments as well as for the most widely used downside risk measures: the probability of loss, the expected value of loss and the target semivariance.

The major purpose of this article is to provide evidence concerning the performance of protective put and covered short call strategies under specific assumptions about the investor's risk-tolerance. In particular, given the methodological analytical framework of ADAM and MAURER (1997), two questions will be examined: First, what are the mean-risk characteristics of both option strategies relative to the underlying asset? Second, what is the relative hedging effectiveness of the protective put compared to the covered short call? The paper proceeds as follows: Sections two and three describe the strategies, section four reviews the risk measures we apply later in the paper, and section five presents a case study in which the hedging effectiveness of the option strategies are compared.

2. The Protective Put Strategy

We assume that the investor is holding a number of stocks with current market value s_0 which are nondividend paying during the investor's one-period time horizon. The uncertain market value of the portfolio at the end of the investment period is denoted by the random variable S_1. According to the protective put strategy, the stocks are hedged against price changes by buying put options with exercise price x at the beginning of the holding period. These options are characterised by a time to maturity equal to the end of the investment period and current price $p_0(x)$. The number of puts purchased divided by the number of stocks is called the hedge ratio α_1. It is assumend that $0 \leq \alpha_1 \leq 1$. Furthermore, the put options are of the European type (i.e. they cannot be exercised before their maturity), held to maturity, and then sold for their intrinsic value, i.e. $\max(x - S_1, 0)$. The investor finances the put premiums by taking a discounted loan at the (continously calculated) interest rate r. Defining $p = p_0(x) \cdot \exp(r)$, the repayment of the loan at the end of the investment period equals $\alpha_1 \cdot p$. At the date of maturity we get for the portfolio return (in DM):

$$R_P = S_1 - s_0 + \alpha_1[\max(x - S_1, 0) - p]. \quad (1)$$

An investor who follows the protective put strategy hopes that the underlying stock price rises: the higher the stock price rises, the larger his gains are. Hence, the protective put buyer is in general "bullish" on the underlying stock. However, the protective put is more conservative than a straight stock position, because the investor limits his downside risk by purchasing the put option. The put option has features similar to a term insurance policy with a deductible. The length of the insurance period corresponds to the time to maturity and the deductible is the difference between the initial stock price and the exercise price specified in the put option contract. The deductible is positiv (zero), if the exercise price is below (equal to) the initial stock price, i.e. the put is out-of-the-money (at-the-money). Unlike traditional insurance policies, the investor can also purchase insurance with a negative deductible, when the exercise price exceeds the initial stock price, i.e. the put is in-the-money. Continuing the analogy to insurance, the hedge ratio α_1 corresponds to the coverage level. For $\alpha_1 = 1$ $(0 < \alpha_1 < 1)$ there is a full (partly) coverage if the stock price falls below the exercise price.

The maximum possible loss (MPL) of the protective put strategy at the end of the investment period in the "catastrophic" event that the underlying asset becomes worthless is equal to the initial value of the stock portfolio minus the maximum possible gain of the put option contract, in sum:

$$MPL = s_0 - \alpha_1(x - p). \quad (2)$$

If the put option and the underlying stock are correctly priced (e.g. under BLACK/SCHOLES assumptions) the MPL declines when the hedge ratio $(0 \leq \alpha_1 \leq 1)$ and/or the exercise price increases.[7] In the case of the put purchased being in-the-money $(x > s_0)$, it is possible that MPL becomes negative, which is a minimum achievable gain. However, the investor has to pay a higher put premium if the exercise price or the hedge ratio are increasing. Hence, this corresponds with somewhat larger losses when stock prices decline is moderate and smaller gains when the stock price increases. In general, the protective put position (i.e. $\alpha_1 > 0$) leads to a higher return than the straight stock as long as $S_1 < x - p$.

The following diagrams portray the return characteristic for the protective put as a function of the stock price at the end of the investment period. The figure on the left hand side shows the fully protective put if the exercise price is below (above) the current stock price, $x_1 < s_0$ $(x_2 > s_0)$, and the figure on the right hand side shows the return characteristic if the hedge ratio varies $(\alpha_1 = 1 / 0{,}5)$ and the put is at-the-money $(x = s_0)$.

A closer description of the examined put option buying strategy can be carried out, if the probability distribution of the protective put is compared with the straight stock position. Figure 2 illustrates the probability distribution $f(R_P)$ of the return of two protective put strategies and the original return distribution $f(R_S)$ of the straight stock. There are two effects: First, the density

Figure 1: Pay-Off-Characteristics of the Protective Put-Strategy

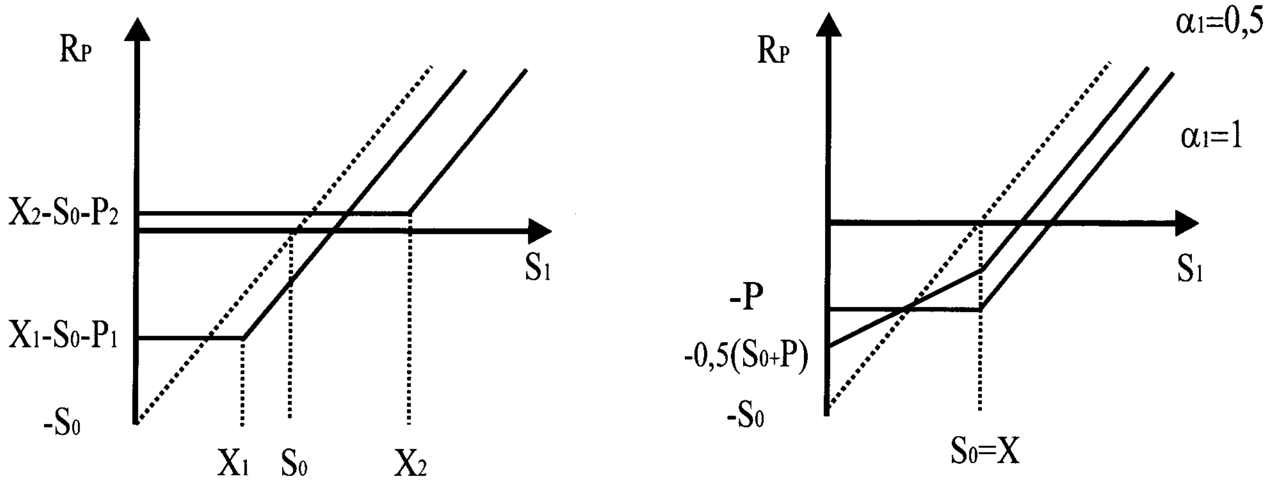
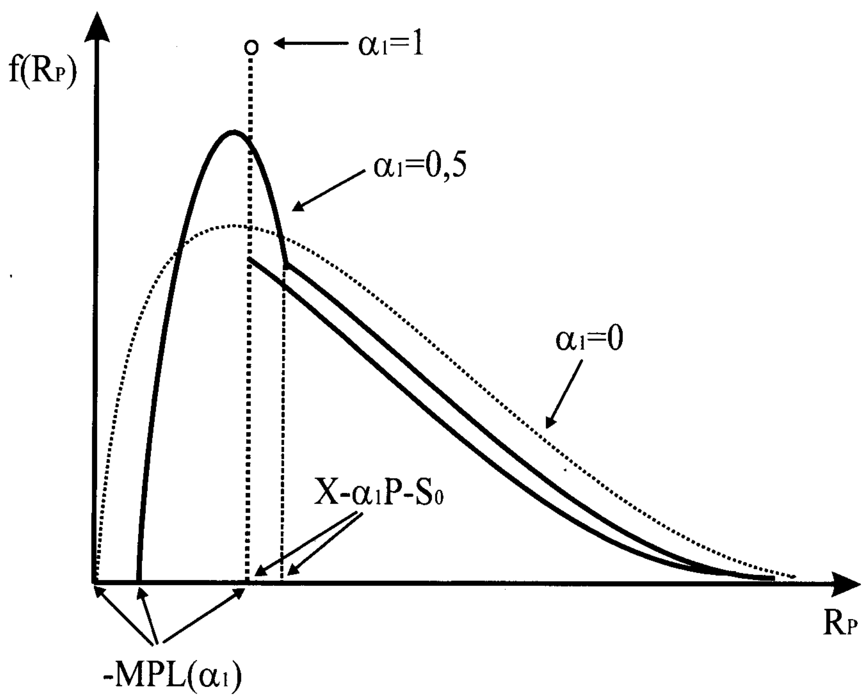


Figure 2: Probability-Distribution of two Protective Puts and the Straight Stock Position



function of the straight stock is shifted to the left by the net financing costs of the option premium, i.e. $\alpha_1 p$. Second, because the long put position limit the losses if the price of the underlying stock declines, the distribution is truncated. Depending upon the hedge ratio, the effect of this truncation will distribute the downside probability $P(R_P < -MPL)$ among the intervall $(-MPL, x - \alpha_1 p - s_0)$. In the case of a fully protective put ($\alpha_1 = 1$), this downside probability is concentrated into $x - p - s_0$ (= MPL). Obviously, the probability distribution of the return of the protective put is complex, distinctly asymmetrical and far from normal.

3. The Covered Short Call Strategy

The covered short call strategy assumes that at the beginning of the investment period a specific number of stocks is purchased and European type call options are sold on the same stocks with excercise price y, time to maturity equal to the end of the investment period and price $c_0(y)$. The hedge ratio $0 \leq \alpha_2 \leq 1$ equals the number of call options written divided by the number of stocks purchased. The option premiums received are used for an investment in treasury bills with the interest rate r, maturing at the end of the investment period. Defining $c = c_0(y) \cdot \exp(r)$ this results in a repayment amount of $\alpha_2 \cdot c$. Hence, the return (in DM) for the covered short call strategy amounts to:

$$R_C = S_1 - s_0 + \alpha_2 \left[c - \max(S_1 - y, 0)\right]. \tag{3}$$

Because the return is a nondecreasing function of the stock price at time to maturity, this option strategy is "bullish" in the underlying stock. The return of the covered short call position is superior to the straight stock position, if the stock either declines or does not increase too much, in general if $S_1 < c + y$. These larger returns are earned at the cost of larger gains the investor renounces whenever the stock price rises above the strike price ($S_1 > y$). In this case, the investor does not participate ($\alpha_2 = 1$) or only partly participates ($0 < \alpha_2 < 1$) in stock price increase. The MPL is given in the following equation:

$$MPL = s_0 - \alpha_2 c. \tag{4}$$

Due to the call premium received, the covered short call strategy leads to a lower MPL relative to the underlying asset, in the case of the underlying asset becoming worthless. The MPL is lower the more calls are sold and the lower the chosen exercise price is. In summary, the writer of a covered short call will reduce his loss exposure if the market value of the underlying stock decreases. This loss reduction is attained at the cost of a reduction of possible gains if stock prices increase.

The following pay-off diagram portrays the return position for a covered short call strategy as a function of the stock price at the end of the investment period. The figure on the left hand side shows the fully covered short call for different exercise prices $y_1 < y_2$, and the figure on the right hand side shows the pay-off characteristic if the hedge ratio varies.

Figure 4 illustrates the transformation of the probability distribution $f(R_C)$ created with a covered short call strategy. The received call option premiums shift the original density function of the straight stock position to the right. Because the short call position caps (fully or partly) the possibilities of gains if the price of the underlying stocks increases, the density function is truncated. That means the whole ($\alpha_2 = 1$) or some part ($0 < \alpha_2 < 1$) of the upside probability $P(R_C > y + \alpha_2 c - s_0)$ is transferred into ($\alpha_2 = 1$) resp.closer to ($0 < \alpha_2 < 1$) the truncation point $y + \alpha_2 c - s_0$. As in the case of the protective put, the probability distribution of the return of the covered short call is distinctly asymmetrical and the assumption of a normal distribution is unquestionably not valid.

Figure 3: Pay-off-Characteristics of Covered Short Call Strategies

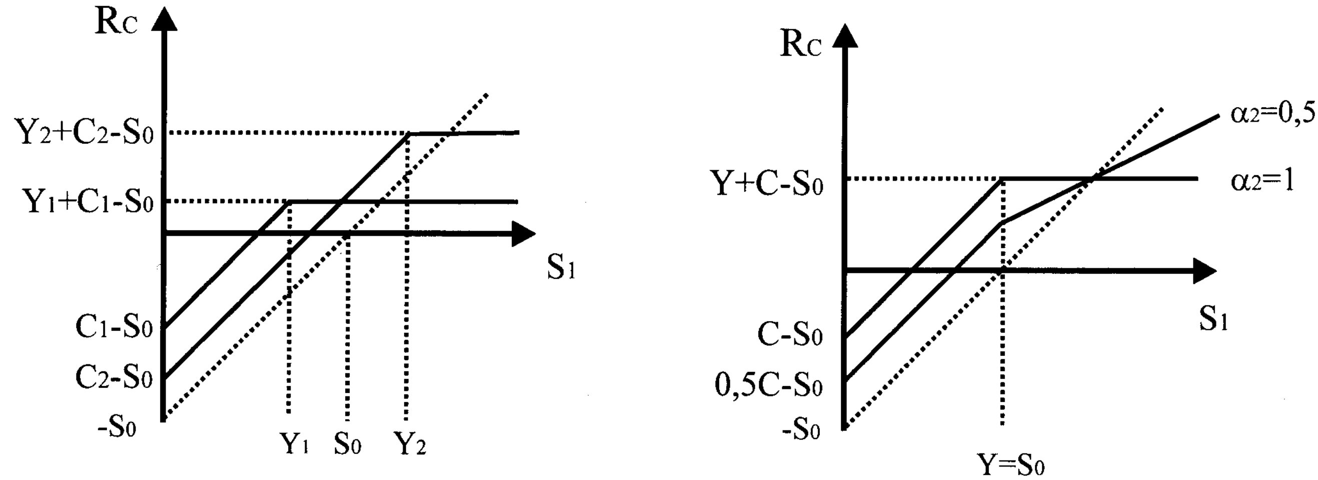
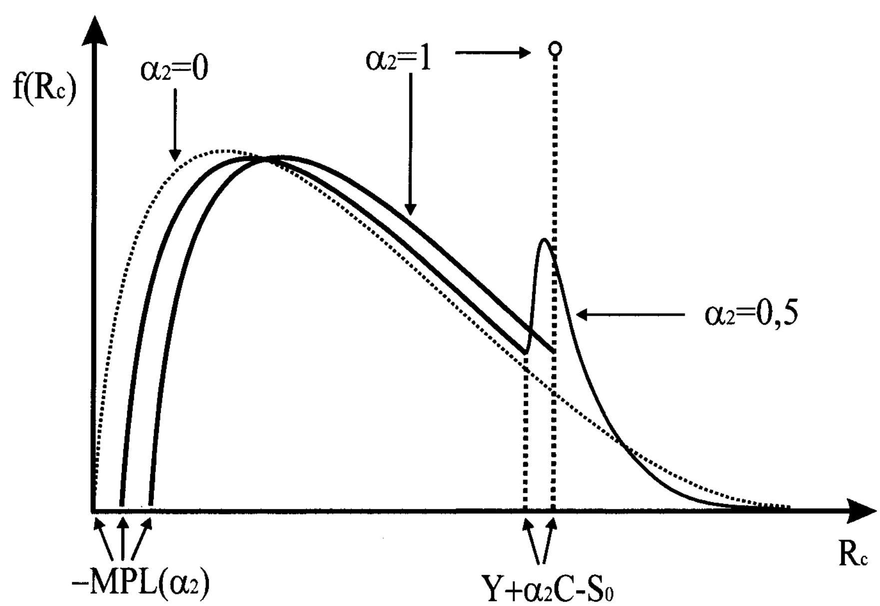


Figure 4: Probability-Distribution of two Covered Short Calls and the Straight Stock Position



4. Conceptualizing and Measuring Risk

It should be noted that there is no single best strategy for all investors, if stocks and options traded on financial markets are priced correctly. It is well known, for example, that the BLACK/SCHOLES model describes an equilibrium in financial markets. In equilibrium, there cannot be any security that is dominated by or dominates other securities from the viewpoint of all investors. However, this does not imply that a particular investor is indifferent concerning his choice among option strategies, since investors have different objectives resulting in an individual trade-off between risk and return.[8] For example, the covered short call and the protective put have the one feature in common that they are bullish on the underlying stock and "insure" the downside risk if stock prices decline, but they do this in different ways. The protective put strategy includes a floor on losses and does not limit gains, while the covered short call (partly) caps potential gains and reduces losses by the option premiums received. Therefore, each type of strategy results in completly different return distributions. Hence, the protective put strategy may be preferred by one investor while the covered short call is preferred by another.

To be able to compare the different option strategies on a quantitative basis it is necessary to introduce a formal risk/return framework for investment decision making. In this section we propose to evaluate the presented option portfolio strategies within several risk-value models.[9] Letting X and Y denote the (random) returns of two different investments with distribution functions F_X and F_Y, a preference comparison between these two investments within a risk-value model can be made by:

$$X \succ Y \text{ if and only if } f[R(X), V(X)] > f[R(Y), V(Y)]. \tag{5}$$

In contrast to other models of choice, e.g. the expected utility model or the stochastic dominance analysis[10] where risk is defined only indirectly through the shape of the utility function, the comparison within a risk-value model depends on an explicit measure of risk R, an explicit measure of value V and a function f which reflects the trade-off between value and risk. Clearly, a higher value and a lower risk is more desirable for the investor. A portfolio of financial assets is efficient if it maximizes the value for a given level of risk, and minimizes the risk for a given level of value.

Although there is a certain amount of agreement[11] that (for a continuous random return X with distribution function F) the expected return

$$E(X) = \int_{-\infty}^{+\infty} x\,dF(x) \tag{6}$$

is a good measure of value, there is little consensus on how risk should be measured adequately. The major problem is that risk is not an objective feature of a decision alternative, but rather a subjective construct[12] peculiar to the individual concerned. In the economic and psychological literature on decision-making, there have been various attempts to develop formal features (axioms) of a good risk measure.[13] Although recently a growing interest can be seen in the financial literature to adapt these streams of research for financial risk measurement (c.f. PEDERSEN/ SATCHELL 1998), no consensus concerning the features to be found. From a practical point of view, BALZER (1994) pointed out that a good risk measure should accord with intuition and should be capable of standing alone without the introduction of additional concepts and constructs.

Probably the best known risk value model in the investment industry is obtained by defining the expected return as the measure of value and the variance of return

$$Var(X) = E[X - E(X)]^2 \tag{7}$$

as the measure of risk, i.e. the classical mean-variance model that backs up the modern portfolio selection theory as developed by MARKOWITZ and others. To adjust the dimension of the risk measure to that of the expected return, the square root of the variance is taken, i.e. the standard deviation $STD(X)=\sqrt{Var(X)}$, which is also referred to as volatility. Notice that the volatility is location-free[14] and that the volatility of a sure investment is zero. Both features are generally desirable for a financial risk measure.[15] On account of these characteristics and of their computational convenience, variance and standard deviation are widely used as measures of risk in finance and management science. In our case it is obvious that for the protective put and the covered short call the effect of truncation leads to a smaller variance of returns compared to the variance of the straight stock. However, if the stocks and options are priced correctly, the lower variance should also lead to a lower expected return for both option strategies.

Although the variance and the standard deviation are widely used as measures of risk, their adequacy has been increasingly questioned. The criticism focuses directly on the definition of variance as the mean quadratic deviation from the expected value. Positive and negative deviations from the mean contribute equally to variance, therefore an overperformance relative to the mean is penalized just as much as an underperformance. However, most investors consider such a feature of a risk measure to be counter-intuitive. Instead, in accordance with other fields of research, as well as with conventional wisdom, risk is associated with the possibility of "something bad happening"[16]; in other words, the return remains under the expected value or any other predetermined target. In the case of a sufficiently symmetrical distribution, the variance is indeed a legitimate measure of risk. Unfortunately, option strategies are distinctly asymmetrical because of their specific non-linear payoff characteristics. Therefore BOOKSTABER and CLARKE (1985) draw the conclusion that "variance is not a suitable proxy for risk in these cases because option strategies reduce variance asymmetrically".

In order to capture the fundamental asymmetrical form of the return distribution of option strategies, some researchers suggest incorporating the skewness

$$\gamma(X)=\frac{M_3(X)}{Var(X)^{3/2}} \tag{8}$$

as the standardised third central moment $M_3(X) = E[X - E(X)]^3$ in the risk judgement.[17] In the case of a positive (negative) skewness the distribution is called "right-skewed" ("left-skewed"), and whenever the skewness is zero the distribution is called "symmetrical" around the mean. The more left-skewed a distribution is, the lower is the probability-weighted sum of cubed positive deviations from the mean relative to the negative ones. In general, investors prefer positive and dislike negative skewness. Given equality of the mean and the variance of return, those distributions which are more skewed to the left are said to have a higher downside risk. For the protective put, the effect of truncation leads to a more positive-skewed return distribution compared to the straight stock. The covered short call leads to a reduction in the positive skewness that generally is a feature of stock returns.

The major practical shortcoming of skewness and of any other risk conception which explicitly incoporates higher moments than skewness is that it does not provide financial analysts with a stand alone risk measure in accordance with intuition. If the variances of two return distributions differ and the distribution with the higher variance is also more skewed to the right, it is necessary to introduce a function which reflects the trade-off between the two components of risk. Therefore JIA and DYER (1996) provide a simple way to combine skewness with variance into the risk measure $R_{JD}(X) = Var(X) - cM_3(X)$. Higher variances increase risk, and if the parameter c > 0 is positive (negative), skewness reduces (increases) risk. However, the JIA/DYER risk measure has theoretical as well as practical problems. For example, if the skewness is positive, the risk measure could be negative, which implies that the financial decision-maker is (partly) a risk-seeker. Additionally, the dimension of the risk measure is $DM^2 - DM^3$ and does not lead to an intuitive interpretation, which professional financial analysts in general are interested in.

The focus of lower partial moments as risk measures is the possibility of getting a return that is below some critical target specified by the investor. Returns below the target (losses) are considered to be undesirable or risky, while returns above the target (gains) are desirable or nonrisky. In this sense, lower partial moments are called "relative" or "pure" measures of risk.[18] The mathematical formula of these risk measures is:

$$LPM^n(t,X) = E[\max(t - X, 0)^n] \qquad (9)$$

where t is the target return and $n \geq 0$ determines the weights attached to negative deviations from the target. Only if the target return is equal to or below the maximum possible loss, i.e. $t \leq MPL$, is the risk of an investment strategy zero. FISHBURN (1977) shows that by varying the degree n of the lower partial moments, the utility function defined with a mean-LPM model can accurately reflect the preferences of an individual towards risk for below-target returns. In general, for $n < 1$ the investor is a risk-seeker and for $n > 1$ he or she is a risk-averter for below-target returns. Additionally FISHBURN shows that there is a strong relationship between the mean-LPM set of efficient investment strategies and (depending the degree n) the efficient set using stochastic dominance analysis. This is important, because stochastic dominance analysis does not make any distributional assumptions of returns and belongs to a very general class of utility functions.

For the special cases n = 0, 1, 2 we obtain the shortfall probability, the shortfall expectation and the shortfall variance. The shortfall probability only takes into consideration the probability but not the amount of negative deviations from the target return. If the same investment strategy can be repeated many times, the shortfall probability answers the question "how often" and not "how badly" a loss occurs.[19] Therefore, the shortfall probability is an appropriate risk index if even small negative deviations from the target lead to drastic consequences for the decision-maker, e.g. losses in the investment portfolio of a bank or insurance company lead to liquidity problems or possibly bankruptcy. However, if small negative deviations from the target are relatively harmless, the shortfall probability is an incomplete risk index due to its neglect concerning the extent of the loss. The other shortfall risk measures take into consideration both the probability and the amount of losses. The shortfall expectation n = 1 is the sum of losses weighted by their probabilities, therefore it is a measure of the average loss amount. In the case of the target semivariance, n = 2, which is the most widely used measure of financial shortfall risk, the negative deviations from the target returns are taken into account quadratically, consequently the higher losses are more strongly weighted than the lower ones.

5. Risk and Return of the Protective Put and the Covered Short Call: A Case Study

5.1 Assumptions

In the case study below we implement the risk measures that we presented in the previous section. From an ex ante viewpoint, a concrete calculation of the expected return and (with the exception of MPL) the risk metrics requires a specification of the mathematical form of the distribution of the underlying asset. In the case of the standard assumption that the price process of the underlying asset follows a geometric WIENER process, it follows that the DM-return of the straight stock is lognormally distributed in the intervall [$-s_0$, ∞). In this case, the analytical formulas which are developed in ADAM/MAURER/MÖLLER (1996) and ADAM/MAURER (1997), can be used to calculate the expected value, the variance, the skewness and the first three lower partial moments for the option strategies under consideration. For simplification, the time to maturity has been standardised to one year, the investment budget s_0 chosen is 100 DM, the expected return of the unmanaged stock portfolio is 10,52 DM, the volatility of the return is 11,08 DM[20], and the sure repayment of a treasury bill with a current market value of 100 DM one year in the future is 105,13 DM.[21] The option prices at the beginning of the period of time examined are formulated using the BLACK/SCHOLES formula.

The setting of the target (benchmark) return, which translates the returns of an investment into gains or losses, is of decisive importance when employing lower partial moments as risk measurements. In general, the target return is a subjective matter related to the objective of the investor. Because investors usually have different objectives, it is appropriate to consider different target returns in shortfall analysis. Although somewhat arbitrarily, we have chosen in the following three target returns which we believe to be reasonable candidates in shortfall risk analysis: The first and very basic target return is zero (t = 0), i.e. risk is understood as loosing the status quo of capital. The second target is the sure return of treasury bills (t = 5,13), which means a comparison of the random return of the option strategies with a complete hedge of the straight stock position by using a short forward contract (fair cost of carry prices and no cross hedge risk presupposed). The return on T-Bills is a natural reference point in risky investment decision situations because it represents the return that could be achieved with certainty if all risky investments were to be rejected. The third benchmark is the expected return (t = E(R)) of an investment strategy, which makes the shortfall risk measures (as volatility and skewness) location free.

Due to the large number of possible variations of the hedge ratio as well as the exercise price, an exhaustive examination of the strategies considered is not feasible. Hence we have limited our study to four different hedge ratios ($\alpha_1 = \alpha_2 =: \alpha$ = 0,25 / 0,5 / 0,75 / 1) and five exercise prices for the protective put and the covered short call strategies, respectively. The exercise prices of the put options are selected in such a way that we obtain two in-the-money positions (x = 90 / 95), one at-the-money position (x = 100)

Table 1: Expected Return (in DM) of Protective Put and Covered Short Call Strategies

		Exercise Price Put (Call)				
		90 (130,5)	95 (123,6)	100 (117,4)	105 (111,85)	110 (106,75)
	0	10,52				
Hedge Ratio	0,25	10,47	10,39	10,23	10,01	9,76
	0,5	10,43	10,26	9,94	9,5	9,01
	0,75	10,38	10,13	9,65	8,99	8,26
	1	10,34	10	9,37	8,48	7,5

and two out-of-the-money positions (x = 105 / 110). To be able to compare the hedging effectiveness between the protective put and the covered short call directly, we have done the following: the exercise prices of the covered short call strategies have been chosen in such a way that identical expected values of the protective put strategies result whenever the hedge ratios are the same for the call as well as the put strategies. Then the relative hedging effectiveness can be quantified using the respective risk measurement.
Table 1 presents the expected return (in DM) for the underlying asset ($\alpha = 0$), the protective put and the covered short call strategies.
It can be seen that for the protective put, both an increase in the chosen exercise price and an increase in the hedge ratio leads to a reduction of the expected return. All values are below the expected return of the straight stock position. This result fits the notion that a higher insurance component, i.e. an increasing hedge ratio or an increasing exercise price, coincides with a reduction of the expected return. Similarly, all variants of the covered short call reveal a lower expected return than the straight stock position. The latter result is in contrast to the commonly held opinion that the writing of covered call options automatically leads to a surplus for the creator of the option. In the case of the covered short call, the reduction of the expected return is higher the higher the hedge ratio is chosen and the lower the exercise price is chosen, respectively.

5.2 Risk Reduction and Relative Hedging Effectiveness

Table 2 contains the maximum possible loss of the straight stock position ($\alpha = 0$) as well as the put and the call strategies presented above.
The MPL of all option strategies lies below that of the straight stock position. The amount of the MPL decreases with an increasing hedge ratio and for the protective put (covered short call) with an increasing (decreasing) exercise price. Moreover, for in-the-money put options and a hedge ratio of one, the protective put produces a negative MPL, i.e. a minimum achievable gain. Furthermore, the protective put shows a reduction of the MPL that is significantly higher than the one of the covered short call. Therefore, compared to the covered short call, the protective put is the more effective hedge instrument against the "catastrophic" risk that the underlying assets become worthless.
Table 3 shows the results of the volatility of all considered strategies.

Table 2: Maximum Possible Loss (in DM) of Protective Put and Covered Short Call

		Exercise Price Put (Call)									
		90	(130,5)	95	(123,6)	100	(117,4)	105	(111,85)	110	(106,75)
	0	100									
Hedge Ratio	0,25	77,56	(99,98)	76,44	(99,94)	75,48	(99,82)	74,73	(99,58)	74,20	(99,18)
	0,5	55,12	(99,97)	52,89	(99,88)	50,96	(99,64)	49,46	(99,16)	48,40	(98,35)
	0,75	32,68	(99,95)	29,33	(99,82)	26,45	(99,46)	24,19	(98,74)	22,61	(97,53)
	1	10,24	(99,94)	5,77	(99,76)	1,93	(99,28)	–1,07	(98,32)	–3,19	(96,71)

Table 3: Volatility (in DM) of Protective Put and Covered Short Call

		Exercise Price Put (Call)									
		90	(130,5)	95	(123,6)	100	(117,4)	105	(111,85)	110	(106,75)
Hedge Ratio	0	11,08									
	0,25	11,04	(10,92)	10,94	(10,67)	10,72	(10,26)	10,36	(9,75)	9,89	(9,25)
	0,5	11,00	(10,77)	10,80	(10,29)	10,38	(9,50)	9,69	(8,52)	8,79	(7,51)
	0,75	10,96	(10,64)	10,68	(9,95)	10,08	(8,83)	9,09	(7,43)	7,81	(5,96)
	1	10,92	(10,52)	10,56	(9,65)	9,80	(8,26)	8,58	(6,55)	7,02	(4,76)

As expected, the volatility of all option strategies is lower than that of the pure stock position and decreases with an increasing hedge ratio. Furthermore, for a protective put (covered short call) a lower volatility corresponds with a rising (falling) exercise price. Additionally, for an identical expected return, the covered short call in all cases has a lower volatility than the protective put. In this sense, the covered short call dominates the protective put. The same result led BOOKSTABER and CLARKE (1985, p. 50) to their criticism of the exclusive use of mean-variance analysis for appropriate investment decisions.

Table 4 gives the values for the skewness coefficients.

Inspection of this table demonstrates that the skewness coefficient for the protective put increases if both exercise price and hedge ratio rise and are above the skewness of the straight stock position. A covered short call strategy, on the other hand, has a skewness that always lies below the skewness of the underlying asset and decreases as the hedge ratio rises. However, the sensitivity of the skewness as regards the exercise price of the covered call is not monotonous, as can be seen in the case of $\alpha = 0{,}25$. Let us make the common assumption that investors have a preference for right-skewed distributions. By using a combined risk measurement of volatility and skewness, the protective put strategy appears

Table 4: Skewness of Protective Put and Covered Short Call

		Exercise Price Put (Call)									
		90	(130,5)	95	(123,6)	100	(117,4)	105	(111,85)	110	(106,75)
Hedge Ratio	0	0,3									
	0,25	0,33	(0,21)	0,37	(0,12)	0,45	(0,02)	0,55	(–0,04)	0,63	(–0,03)
	0,5	0,35	(0,13)	0,44	(–0,04)	0,59	(–0,26)	0,80	(–0,45)	1,02	(–0,53)
	0,75	0,37	(0,07)	0,49	(–0,18)	0,71	(–0,51)	1,04	(–0,89)	1,45	(–1,27)
	1	0,39	(0,02)	0,54	(–0,29)	0,81	(–0,71)	1,23	(–1,26)	1,81	(–2,01)

Table 5: Shortfall Probability (in %) of Protective Put and Covered Short Call

		Exercise Price Put (Call)									
		90	(130,5)	95	(123,6)	100	(117,4)	105	(111,85)	110	(106,75)
	Target t = 0 / Unprotected Position: LPM_0 = 17,11										
Hedge Ratio	0,25	17,27	(17,07)	17,63	(16,95)	18,42	(16,63)	16,38	(16,00)	14,80	(14,99)
	0,5	17,43	(17,03)	18,15	(16,79)	19,79	(16,16)	14,97	(14,94)	10,78	(13,04)
	0,75	17,59	(16,99)	18,69	(16,63)	21,21	(15,70)	11,21	(13,93)	3,28	(11,25)
	1	17,75	(16,94)	19,24	(16,47)	22,68	(15,25)	0	(12,96)	0	(9,64)
	Target t = 5,13 / Unprotected Position: LPM_0 = 32,64										
	0,25	32,85	(32,58)	33,33	(32,42)	34,39	(31,99)	36,23	(31,13)	35,28	(29,71)
	0,5	33,07	(32,53)	34,04	(32,20)	36,16	(31,35)	39,91	(29,65)	40,71	(26,89)
	0,75	33,29	(32,47)	34,74	(31,99)	37,96	(30,71)	43,65	(28,20)	51,92	(24,19)
	1	33,50	(32,42)	35,45	(31,77)	39,78	(30,08)	47,41	(26,77)	58,27	(21,63)
	Target t = E(R) / Unprotected Position: LPM_0 = 52,00										
	0,25	52,06	(51,78)	52,26	(51,30)	52,78	(50,27)	53,87	(48,55)	55,70	(48,73)
	0,5	52,13	(51,56)	52,52	(50,60)	53,57	(48,54)	55,73	(45,09)	59,32	(42,16)
	0,75	52,19	(51,34)	52,78	(49,90)	54,35	(46,80)	57,57	(41,64)	62,83	(34,49)
	1	52,26	(51,12)	53,04	(49,20)	55,13	(45,06)	59,38	(38,22)	66,20	(28,99)

more favourable in comparison to the isolated risk measurement of volatility.

The following table 5 presents the results using the shortfall probability as a risk measure.

For the targets t = 5,13 and t = E(R), the shortfall probability of the protective put strategies increases with a rising hedge ratio and is always higher, often significantly so, than that of the straight stock position. When assessing the hedging effectiveness according to the level of shortfall probability, the put strategies are classified as riskier – by simultaneously earning a lower expected return – than the underlying asset. In this sense, the straight stock position dominates the protective put strategies. This result, which at first may seem surprising, can be justified by the fact that the protected put leads to a truncated return distribution. The point of truncation $x - \alpha_1 p - s_0$ has to be below the riskless return, i.e. t = 5,13, otherwise there are riskless arbitrage opportunities, which are inconsistent with the BLACK/SCHOLES option pricing model.

For the target t = 0, however, such a clear result cannot be concluded, since for in-the-money puts the shortfall probability decreases with an increasing hedge ratio and is completely eliminated for $\alpha = 1$. This signifies that the floor on losses of the fully ($\alpha = 1$) protective put, i.e. the return of the truncation point described above, is between zero and 5,13 DM. Such a result is obtainable, since the price of European in-the-money put options can be lower than their intrinsic value (see COX/RUBINSTEIN 1985, p. 145). Furthermore, the shortfall probability increases monotonously with an increasing exercise price only if the target t = E(R) is chosen.

Regarding the covered short call, the shortfall probabilities for all strategies and all considered targets are below that of the straight stock position and decline the more calls are sold. Additionally, a decreasing exercise price reduces the shortfall probability for the covered short call (with only one exception). On the whole, compared to the pure stock position and the writing of covered calls, the purchase of put options is inefficient, when the shortfall probability is used to evaluate the hedging effectiveness. Only in the case of in-the-money puts and a target of zero is the inverted result obtained.

The shortfall expectation takes into account both the probability and the amount of losses. Table 6 contains the results for this risk measure:

For the protective put, an increasing exercise price reduces the shortfall expectation. Additionally, the shortfall expectation declines in nearly all cases the more puts are purchased. Only for the target of t = 5,13 and an exercise price of 90 is the opposite result obtained. With the same exception, the protective put shows a systematically lower shortfall expectation in comparison to the unmanaged stock index portfolio. Furthermore, table 6 illustrates that for all covered short call strategies and all targets, both a decrease in the chosen exercise price and an increase in the hedge ratio occurs alongside a reduction of the shortfall expectation. Additionally, the shortfall expectation of these strategies is below that of the straight stock position. A comparison of both option strategies reveals the following: the protective put dominates the covered short call for the lowest target. For the highest target we get the contrary result. For the medium target, it is important to distinguish the following: if the exercise price of the put option is chosen out-of-the-money, the corresponding covered short call with an identical expected return proves to have a slightly lower shortfall expectation. On the other hand, if the put is in-the-money, the assessment depends upon the hedge ratio.

Tabel 6: Shortfall Expectation (in DM) of Protective Put and Covered Short Call

		Exercise Price Put (Call)									
		90	(130,5)	95	(123,6)	100	(117,4)	105	(111,85)	110	(106,75)
Hedge Ratio		Target t = 0 / Unprotected Position: LPM_1 = 0,875									
	0,25	0,868	(0,872)	0,84	(0,86)	0,75	(0,84)	0,62	(0,80)	0,54	(0,74)
	0,5	0,860	(0,870)	0,80	(0,85)	0,62	(0,81)	0,37	(0,73)	0,24	(0,62)
	0,75	0,853	(0,867)	0,77	(0,84)	0,51	(0,78)	0,13	(0,67)	0,03	(0,51)
	1	0,845	(0,864)	0,73	(0,83)	0,40	(0,75)	0	(0,61)	0	(0,42)
		Target t = 5,13 / Unprotected Position: LPM_1 = 2,13									
	0,25	2,133	(2,125)	2,125	(2,11)	2,08	(2,07)	1,96	(1,98)	1,79	(1,86)
	0,5	2,135	(2,120)	2,120	(2,09)	2,04	(2,01)	1,83	(1,86)	1,48	(1,62)
	0,75	2,138	(2,115)	2,117	(2,07)	2,01	(1,95)	1,74	(1,73)	1,29	(1,39)
	1	2,141	(2,110)	2,116	(2,05)	1,99	(1,89)	1,69	(1,61)	1,24	(1,20)
		Target t = E(R) / Unprotected Position: LPM_1 = 4,41									
	0,25	4,40	(4,38)	4,37	(4,31)	4,30	(4,16)	4,16	(3,93)	3,93	(3,64)
	0,5	4,39	(4,34)	4,34	(4,21)	4,20	(3,93)	3,93	(3,48)	3,49	(2,92)
	0,75	4,38	(4,31)	4,30	(4,11)	4,10	(3,70)	3,70	(3,07)	3,09	(2,31)
	1	4,37	(4,28)	4,27	(4,02)	4,00	(3,48)	3,48	(2,69)	2,72	(1,80)

Table 7: Shortfall Volatility (in DM) of Protective Put and Covered Short Call

		Exercise Price Put (Call)									
		90	(130,5)	95	(123,6)	100	(117,4)	105	(111,85)	110	(106,75)
Hedge Ratio		Target t = 0 / Unprotected Position: LPM_2 = 2,73									
	0,25	2,65	(2,726)	2,48	(2,71)	2,21	(2,67)	1,98	(2,59)	1,83	(2,46)
	0,5	2,57	(2,721)	2,26	(2,69)	1,71	(2,61)	1,23	(2,46)	0,96	(2,21)
	0,75	2,51	(2,716)	2,06	(2,67)	1,25	(2,55)	0,49	(2,33)	0,21	(1,98)
	1	2,45	(2,711)	1,89	(2,65)	0,88	(2,50)	0	(2,20)	0	(1,77)
		Target t = 5,13 / Unprotected Position: LPM_2 = 4,71									
	0,25	4,66	(4,703)	4,55	(4,68)	4,32	(4,63)	4,03	(4,51)	3,80	(4,32)
	0,5	4,61	(4,696)	4,40	(4,65)	3,99	(4,54)	3,41	(4,32)	2,91	(3,96)
	0,75	4,57	(4,689)	4,28	(4,63)	3,71	(4,46)	2,91	(4,13)	2,10	(3,62)
	1	4,54	(4,682)	4,18	(4,60)	3,50	(4,38)	2,59	(3,95)	1,66	(3,30)
		Target t = E(R) / Unprotected Position: LPM_2 = 7,52									
	0,25	7,46	(7,48)	7,35	(7,40)	7,10	(7,24)	6,74	(6,97)	6,34	(6,60)
	0,5	7,42	(7,45)	7,19	(7,29)	6,73	(6,97)	6,03	(6,44)	5,23	(5,74)
	0,75	7,37	(7,41)	7,05	(7,18)	6,39	(6,70)	5,40	(5,94)	4,23	(4,96)
	1	7,33	(7,38)	6,92	(7,07)	6,10	(6,44)	4,88	(5,46)	3,48	(4,24)

Finally, the results of the shortfall volatility have been tabulated.

As expected, the protective put and covered short call strategies have, for all variations and for all targets, a lower shortfall volatility than the straight stock position. Furthermore, the risk is reduced in both strategies as the hedge ratio increases. In the case of protective put strategies (covered short call strategies), the risk declines with an increasing (decreasing) exercise price. In conjunction with the results from table 1, a reduction of risk, measured by the shortfall volatility, can only be achieved together with a lower expected return. For the risk measurement we used in this context, the comparison of the two option strategies shows that the protective put is superior to the covered short call in all cases, especially for in-the-money puts with a high hedge ratio.

6. Summary

In this paper the hedging effectiveness of two popular option strategies, the protective put and the covered short call, has been analysed within a downside risk framework. The results of our case study are summarised in table 8:

In comparison to the pure stock position, all protective put and covered short call strategies have a lower expected return. The reduction of the expected return is higher the more the hedge ratio increases and the further the excercise price chosen is in-the-money.

The hedging effectiveness of the considered strategies was evaluated with different risk measures. It can be concluded that in all the examined cases, a reduction of the expected return simultaneously results in a decrease of both the maximum possible loss and the volatility of return.

Table 8: Comparison between the Protective Put and the Covered Short Call

	Sensitivity				Ranking
	Hedge Ratio ↑		Exercise Price ↑		
	Put	Call	Put	Call	
Mean	–	–	–	+	Put = Call
Maximum Possible Loss	–	–	–	+	Put $\succ$ Call
Volatility	–	–	–	+	Put $\prec$ Call
Skewness	+	–	+	+/–	Put $\succ$ Call
Shortfall Analysis with Target Nominal Capital Protection					
Shortfall Probability	+/–	–	+/–	+	Put $\sim$ Call
Shortfall Expectation	–	–	–	+	Put $\succ$ Call
Shortfall Volatility	–	–	–	+	Put $\succ$ Call
Shortfall Analysis with Target Riskless Return					
Shortfall Probability	+	–	+/–	+	Put $\prec$ Call
Shortfall Expectation	+/–	–	–	+	Put $\sim$ Call
Shortfall Volatility	–	–	–	+	Put $\succ$ Call
Shortfall Analysis with Target Expected Return					
Shortfall Probability	+	–	+	+/–	Put $\prec$ Call
Shortfall Expectation	–	–	–	+	Put $\prec$ Call
Shortfall Volatility	–	–	–	+	Put $\succ$ Call

Furthermore, protective put (covered short call) strategies lead to an increase (decrease) of the skewness relative to the pure stock position. The size of the skewness can be controlled by the hedge ratio and the exercise price. Finally, the hedging effectiveness was evaluated using various shortfall risk measurements, which take into account only realisations below the expected return or two other predetermined target returns, namely the nominal capital protection and the riskless return of treasury bills. The covered short call has, in all investigated cases and for all the targets chosen, a lower risk than the pure stock position. Furthermore, for these strategies an increasing hedge ratio and a decreasing exercise price reduces all shortfall risk measures. However, only for the shortfall volatility of the protective put strategy can such clear statements be asserted. The shortfall volatility of the protective put is always lower than that of the straight stock position and the extent of the hedging effect can be controlled by varying the exercise price or the hedge ratio. The results for the other two shortfall risk measures depend on the the target chosen.

In order to be able to compare the hedging effectiveness of both option strategies considered, the exercise price of the covered short call was fixed in the way that, for an equal hedge ratio, an identical expected return to the protective put results. On this condition the protective put reduces the MPL to a significantly higher extent than the covered short call. Furthermore, it may be observed that the covered short call always has a lower volatility than the protective put. This signifies that the covered short call dominates the protective put when measuring risk only according to volatility. However, when taking the third central moment into account, the protective put performs better. For the shortfall measures, the results depend upon two significant parameters: the degree of the lower partial moments considered and the target chosen. Thus, in summary: the protective put looks more impressive the lower the target and the higher the degree chosen.

Footnotes

[1] Dynamic portfolio insurance strategies are described by LELAND 1980, RUBINSTEIN 1985 or BENNINGA 1990.

[2] If the utility function is increasing and concave and the returns are lognormally distributed, the variance is also an appropriate measure of risk, see LEVY 1992.

[3] For a non-parametric examination of increasing downside risk, see MENEZES/GEISS/TRESSLER 1980.

[4] JEAN 1971, 1973, ARDITTI/LEVY 1975, KRAUS/LITZENBERGER 1976 and SIMKOWITZ/BEEDLES 1978 have developed a portfolio theory and capital-asset-pricing based upon the first three (central) moments (expected value, variance and skewness).

[5] See e.g. MARMER/NG 1993, ZIMMERMANN 1994, ALBRECHT/MAURER/STEPHAN 1995 or ADAM/ALBRECHT/MAURER 1996.

[6] See e.g. LEWIS 1990, FIGLEWSKI/CHIDAMBARAN/KAPLAN 1993 or LEE 1993.

[7] Under BLACK/SCHOLES assumptions we get dMPL $/d\alpha_1 = x - p \geq 0$ and because $dp_0/dx \in [0, \exp(-r)]$ (c.f. COX/RUBINSTEIN 1985, p. 221) it follows that $dMLP/dx = \alpha_1[1 - dp_0/dx \exp(r)] \geq 0$.

[8] MERTON/SCHOLES/GLADSTEIN 1978, p. 184.

[9] A survey of risk value models is given in SARIN/WEBER 1993.

[10] For stochastic dominance analysis of option strategies see BROOKS/LEVY 1987 and TRENNEPOHL/BOOTH/TEHRANIAN 1988.

[11] See ALBRECHT/MAURER/MÖLLER 1998 for a discussion of other measures of value.

[12] See BAYERISCHE RÜCK 1993.

[13] For a review of these features see BRACHINGER/WEBER 1997.

[14] A risk measure R is location free if and only if $R(X + a) = R(X)$, i.e. adding a real constant a to the random return X has no influence on the level of risk, see BRACHINGER/WEBER 1997, p. 239.

[15] See for example SARIN/WEBER 1993, p. 137, PEDERSEN/SATCHELL 1998, p. 95 and JIA/DYER 1996, p. 1692.

[16] See BRACHINGER/WEBER 1997, p. 237.

[17] See MENEZES/GEISS/TRESSLER 1980 or BOOKSTABER/CLARKE 1983a.

[18] See FISHBURN 1984 and BRACHINGER/WEBER 1997.

[19] See BALZER 1994, p. 52.

[20] This means, that the drift and the diffusion of the WIENER-Process is 10%, respectively.

[21] This means that the continuously calculated riskless interest rate is 5%.

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