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Portfolio management in the 20th century: An overview

1. Introduction

The simplest strategy for an investor is to keep all his money in a bank account. It is safe, requires no expertise and little effort. But more and more investors are not satisfied anymore with the returns earned on a savings account, and agree to take risks on the financial markets against the potential reward of higher expected returns.

But today, money investing has become a difficult task; on the one hand, the opening of European frontiers and the legal harmonization; on the other hand, internationalization, globalization, and sophistication. The investor must face thousands of stocks, bonds and other derivative securities to choose from. Experts recommendations are often conflicting. The apparent knowledge required to select a single security is intimidating. Hence, more and more investors tend to entrust their savings to professional managers, directly (if they can afford the cost of a private portfolio management) or indirectly (through investment clubs and mutual funds). Professional managers benefit from lower costs and better information, which give them a considerable competitive advantage over private individuals.

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But how did portfolio management itself evolve over time? A casual comparison of current portfolio management practices with those twenty-five years ago is enough to note the impact of financial theories such as efficient markets, portfolio selection, risk analysis, option pricing, etc. An art has become a science. Harry MARKOWITZ, William SHARPE, John LINTNER and Jan MOSSIN established forty years ago what we know represent the foundations of portfolio management techniques. Modern portfolio theory has become an established practice with both professional fund managers and institutional clients. Private investors also are becoming increasingly familiar with it or at least with its concept of diversification.

What will portfolio management look like twenty five years from now? What can be done is try to identify the existing theoretical contributions that will significantly affect the future of portfolio management. This is what this paper does, keeping things simple and comprehensive while still preserving the concepts intact.

This paper is organized as follows: section 2 briefly exposes finance theory developments over the last decades and their applications in today's portfolio management; its reviews the early days of portfolio management, the normative theory of MARKOWITZ and the subsequent positive theories of how stocks should be priced if all investors diversify. Section 3 focuses on what we believe to be the most significant recent theoretical contributions to tomorrow's portfolio management, namely, the introduction of the intertemporal and dynamic dimension in finance theory. The implication of these new models for investment management and performance measurement are then discussed. Section 4 concludes.

2. Portfolio management: the early years

2.1 "Traditional" portfolio theory

It is obvious that in the early 1950s, portfolio management was an easy task. On the one hand, most portfolios were made of first quality long term bonds that were purchased at issue price and kept until maturity. As interest rate volatility was low, bonds were traditionally considered as safe. On the other hand, most stocks were held by individuals, which had purchased them at much lower prices in the 1930s or 1940s. Selling those stock to cash in capital gains would have implied paying a huge level of taxes. Thus, here again, buy and hold was the general rule, and investors did not modify much their portfolios over time. Performance was generally expressed as a "total portfolio return since purchase" number. The quick development of pension funds and the retirement of the 1929 traumatized investors progressively modified the game.

Portfolio management slowly became an art based on impressions, inside information, and flair. Buy low, sell high was the common rule. For instance, John TRAIN (1984) reports that Benjamin GRAHAM, one of the leading gurus of Wall Street, declared: "I am no longer an advocate of elaborate techniques of security analysis in order to find superior value opportunities. One should buy stocks at less than their current or intrinsic values as indicated by one or more simple criteria. The criterion I prefer is seven times the reported earnings for the past twelve months". Benjamin GRAHAM's rules for selling were even more simplified: "Sell after your stock has gone up 50 percent, or sell it after two years, whichever comes first; sell if the dividend is omitted; sell when earnings decline so far that the current market price is 50 percent over the new target buying price". Other Wall Street gurus, such as Warren BUFFET, Paul CABOT, T. ROWE PRICE, John HARTWELL, Gerry TSAI or John TEMPLETON were following similar rules.

On the academic front, portfolio management was considered as a specific topic in finance, which was itself only a minor area of research for micro-economists. Portfolio management was for long a descriptive discipline with its major focus on institutional, accounting and legal matters, and the subject was not very popular. For instance, at the Harvard Business School, the portfolio management course was given at the worst time of the day – noon – and was surnamed "Darkness at noon", in reference to the dark book of Arthur KOESTLER. Between 1951 and 1955, less than 4% of the graduates would go to work for a Wall Street firm; and the bull market of the late 1950s only raised this percentage to 6.6%. Despite this, the first academic works related to portfolio management were starting. For instance, MACAULAY (1938) analyzed bond prices variations as a function of the interest rates variations, Graham (1952) proposed a classification and a set of "scientific" methods for the selection of safe securities, undervalued securities, growth stocks, and "near term opportunities" (i.e. short term trading), GORDON and SHAPIRO (1956) developed a stock valuation model based on the discounting value principle and assuming perpetual growth of the stock dividends. Of course, most of these results were not applied in the portfolio management industry, where rumors and noise traders were leading the market.

2.2 "Modern" portfolio theory

The starting point of "portfolio theory" started with ROY (1952) and his safety first principle. As he was very concerned with the behavior of investors, ROY developed a complete model of portfolio selection based on the goal of minimizing the chance of disaster, where disaster is defined as missing a threshold return. Despite his innovative solutions, ROY's paper remained forgotten until the end of the eighties, when Martin LEIBOWITZ developed the "shortfall risk optimization" theory for pension funds and asset/liability management.

MARKOWITZ (1952) has to be considered the founding father of the so-called "modern portfolio theory". He noticed that uncertainty is a salient feature of security investment; thus, investors should not only focus on maximizing the discounted value of future returns, but should also consider the risk of their investments. He then proposed the expected mean return-variance of return (or: mean-variance) rule, which is adequate for investors who "consider that expected returns as a desirable thing and variance[1] of returns as an undesirable thing". The major idea is to build a portfolio in order to minimize the variance of changes in wealth for a given level of final expected wealth change, or equivalently, to minimize the portfolio return variance for a given level of final expected return. Thus, the investor acts as if he had a mean-variance utility function.

Much in the spirit of MARKOWITZ (1952), the portfolio selection problem can be stated as[2]:

$$\min \sigma_P^2 = \overline{x}^T \cdot V \cdot \overline{x}$$
$$\text{s.t.} \begin{cases} \overline{x}^T \cdot \overline{1} = 1 \\ \overline{x}^T \cdot \overline{R} = R_P \end{cases} \tag{1}$$

where $\overline{x}$ is the n-vector of assets weights in the portfolio, V is the $n \times n$ variance-covariance matrix of the asset returns, $\overline{R}$ is the n-vector of expected returns, and $\overline{1}$ is an n-vector of 1. Intuitively, the problem can be understood as minimizing the portfolio variance σ_P^2 subject to two constraints: first, the portfolio weights must sum up to unity (i.e. all the wealth is invested); second, the portfolio must earn an expected rate of return equal to R_P. Technically, we are minimizing a convex function with respect to a set of linear constraints. Solving (1) gives us the n-vector of portfolio weights $\overline{x}$ that minimize the portfolio variance for a given mean-return:

$$\overline{x} = V^{-1}\left[\overline{R}\ \overline{1}\right]\left(\left[\overline{R}\ \overline{1}\right]^T V^{-1}\left[\overline{R}\ \overline{1}\right]\right)^{-1}\left[R_P 1\right]^T \tag{2}$$

This defines the set of minimum variance portfolios frontier (also called the *efficient frontier*) as a parabola in the mean-variance space; it is the graph of the highest returns that can be attained for a given level of risk. Its equation can be derived analytically (see MERTON (1972)). All the portfolios that lie on this frontier provide the best risk-return combination, and are candidates for the optimal portfolio.

An important implication of mean-variance utility functions is the *separation property* (TOBIN (1958), BLACK (1972)): there exist two funds made of the available assets, determined by the joint distribution of random returns and independent of any investor's preference, such that every risk-averse investor will select a portfolio that is a combination of these two funds. Rather than adapting the composition of their portfolio of risky assets to their risk aversion, all investors can use the same portfolio of risky assets and adjust the risk level by borrowing or lending.

The Capital Asset Pricing Model of SHARPE (1964), LINTNER (1965) and MOSSIN (1966) is a direct consequence of the two fund separation at equilibrium. The three authors analyzed the demand for assets assuming that all investors seek mean-variance efficiency, have the same beliefs, and can lend all they have or borrow all they want at the risk-free rate; rather than focusing on correlation or covariance between all assets, it can be proved that a single number called beta would be a sufficient statistic to convey all the required information. At equilibrium, we should have

$$E(R_i) = R_F + \beta_i\left[E(R_M) - R_F\right] \tag{3}$$

where R_F denotes the risk-free rate, R_M the return on the market portfolio[3], $E(\cdot)$ is the expectation operator and

$$\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)} \qquad (4)$$

can be seen as a measure of sensitivity of return on the i^{th} asset to the return on the market portfolio. Intuitively, the expected return of asset i is given by the risk-free rate, plus the exposition to the market risk (β_i) times the market risk premium. The specific risk of individual assets gets trivially diversified away in a portfolio, and only the systematic or market risk remains and should be priced. From there, TREYNOR (1965), SHARPE (1966) and JENSEN (1968) derived a set of performance ratios adjusting return for risk.

At that time, most of the academics were discussing their theories in small collegial circles and published them in scientific reviews. Most investors were laughing at those articles filled up with Greek symbols and written by people who had never managed a portfolio in their life. Judgment about the performance of a security or a fund manager were expressed in terms of returns alone: how much money did you make or lose? Risk had nothing to do with it. Thus, despite their attractive simplicity, their theoretical tractability and their strong empirical implications, these models were not adopted immediately by practitioners.

- the application of MARKOWITZ method requires the computation and the inversion of a large variance-covariance matrix (for instance, for n = 500 assets, we need to compute about 125'000 values and invert a 500×500 matrix). In the 1950s, the technology was clearly unable of solving such large scale problems.
- the CAPM makes it possible to skip over the whole problem of calculating covariance among the individual securities. But its first empirical applications yielded mixed results. It says nothing about the market risk premium $E(R_M)-R_F$, which must be estimated; but all expectations and risk measures are not directly observable, nor are they stable over time. When past performance of securities are used as input, the output of the analysis is portfolios which performed well in the past. Furthermore, the results are very sensitive to the choice of the market portfolio: it should include every asset in the economy (stocks, bonds, real estate, human capital, etc.).
- many factors are typically omitted in mean-variance models, such as inflation, interest rates, liquidity and relative size of the investment in a particular asset class or security, etc.

The limitations of the CAPM led ROSS (1976) to develop an alternative model known as Arbitrage Pricing Theory. The APT considers systematic risk as covariability with not only one factor (the market return, in the case of CAPM), but with several -a priori non specified – economic factors. As in the CAPM, specific risk can be diversified away and is not compensated. But the two models differ by several aspects: the CAPM requires the economy to be in equilibrium, while the APT requires no arbitrage opportunities; the APT makes no assumptions about utility functions, except for monotonicity and concavity, while the CAPM requires quadratic utility functions or normally distributed returns. An obvious difficulty in the APT is that it provides no guidance as to what the factors may be.

It took the crash of 1973–1974 to convince investors that they should focus on risk as well as on returns. While the Standard and Poor's 500 fell by 43% over two years, aggressive performance-oriented mutual funds and portfolio managers generally fell by more than that. Furthermore, traditional portfolio theories were not any more adapted to manage the large trades from pension funds: a portfolio of one billion with twenty stocks must invest fifty millions in each stock, which might be considerable relative to the daily traded volume. All this combined with the progressive recognition of market efficiency contributed to promote modern portfolio theory. The recurring evidence that few active managers were able to consistently outperform the market provided the intellectual impetus for a rapid growth of indexing and the use of passive strategies.

3. Portfolio management: what's next?

Today, diversification has become a true religion among investors: diversification across assets, currencies, maturity, issuers, sectors, and even managers. An increasing level of markets integration forces investors to find non-correlated products, such as exotic emerging markets, non-correlated hedge funds, or catastrophe linked derivative products. And the explosion of indexed mutual funds allowed small investors to diversify as well. Only fools and prophets do not diversify anymore.

The development of finance theory in the last decades has had an irreversible impact on the practice of investment management. Harry MARKOWITZ. William SHARPE. John LINTNER. Jan MOSSIN. These four names represent without any doubt yesterday's theoretical foundations of today's portfolio management techniques. What is next? What are today's theoretical breakthrough that will impact tomorrow's portfolio management? In my opinion, the major contribution – in fact, contributions – to portfolio management for the next twenty five years were made by Robert C. MERTON in the late 1960s.

Most of MERTON's contribution can be summarized in one question: "Why should we restrict ourselves to only one period?" If we all agree on uncertainty being the central element that influences financial economic behavior, there exists a second important dimension that we have omitted up to this point: *time.*[4] There are three time horizons involved in the consumption-portfolio problem: the trading horizon (minimum length of time between possible successive transactions), the decision horizon (length of time between two decisions), and the planning horizon (maximum length of time for which the investor gives any weight to its utility function). Why should these three time-horizons be equal?

Using the mathematical tools of continuous probability à la Norbert WIENER and Kiyoshi ITÔ, Robert MERTON developed the mathematics and economic theory of finance from the perspective of models in which agents can revise their decisions continuously in time[5]. This opened the doors to a new era in Finance.

3.1 Intertemporal models for portfolio selection

MERTON (1969, 1971) assumes that there exist an economy with a risk free asset with return r_t and n risky assets with price per share $P_i(t)$ moving randomly according to a set of Itô processes. i.e.

$$\frac{dP_i}{P_i} = \mu_i(x_t, t) \cdot dt + \sigma_i(x_t, t) \cdot dZ(t) \qquad (5)$$

where dZ(t) is a WIENER process. In other words, each asset i has an instantaneous return that can be decomposed in two components: a trend $\mu_i(x_t, t)$ that is predictable, and a variation $\sigma_i(x_t, t)$ around this trend that is a random component; this trend and variation depend themselves on time (t) and on a s-vector of state variables x_t, which we will specify later on.

At each instant, an agent must select its instantaneous consumption C_t and the instantaneous weights w_t of his portfolio in order to maximize his utility over its entire life. His utility is not defined anymore as the utility of his terminal wealth, but as the utility he derives from its *intertemporal consumption* C_t, plus a bequest function $B(W_T,T)$ at the final date T. In other words, the agent's problem is the choice between an immediate consumption and a future consumption, with his wealth subject to a set of risks. His wealth enters only indirectly in the problem, as it only allows him to differ his consumption.

Note that:

- first, MERTON's model implies that the investor is not totally indifferent to the status of his children after he dies at time T. In that sense, the bequest function can be considered as the utility he derives from leaving some money for them.
- second, although we will consider the final time T as known, MERTON (1971) shows that the analysis is essentially the same for an uncertain lifetime (i.e. when T is a random variable.

In this framework, the portfolio selection problem for a given investor can be set up as [6]

$$\underset{C_t, w_t}{\mathrm{Max}} E_0\left[\int_0^T U(C_t, t)\cdot dt + B(W_T, T)\right] \tag{6}$$

where $E_0(\cdot)$ is the expectation operator at time 0 conditional on the current investor's wealth W_0, $U(C_t,t)$ is the strictly concave VON NEUMANN-MORGENSTERN utility function of the agent, C_t is the consumption flow of the agent at time t, w_t is the n-vector containing the weights of the assets in the portfolio. The problem can be solved using the BELLMAN principle (also called the PONTRYAGUINE stochastic maximum principle).

When the investment opportunity set is considered constant (i.e. when $\mu_i(x_t,t) = \mu_i$, $\sigma_i(x_t,t) = \sigma_i$ and the instantaneous interest rate r_t=r are all constant), MERTON derives the intertemporal optimal allocation and consumption of the investor and proves the following theorem.

Theorem 1: If the return dynamics are described by (5) with $\mu_i(x_t,t)$, $\sigma_i(x_t,t)$ and r_t constant, then, there exist 2 mutual funds constructed from linear combinations of the available n + 1 securities such that, independent of preferences, wealth distribution, or planning horizon, individuals will be indifferent between choosing from linear combinations of just these 2 mutual funds or linear combinations of all n risky securities and the risk-free security.

Of course, here again, possible mutual funds are the risk-free asset and the market portfolio. Furthermore, if all agents have homogeneous expectations, he shows that the following relationship holds

$$\mu_i - r = \beta_i(\mu_M - r) \tag{7}$$

where

$$\beta_i = \frac{\mathrm{Cov}(\mu_i, \mu_M)}{\mathrm{Var}(\mu_M)} \tag{8}$$

β_i is the covariance of the return on the i^{th} asset with the return on the market portfolio. Thus, in an intertemporal context, investors behave as if they were single-period maximizers, and an optimal portfolio will be a mix between a risk-free asset and the market portfolio. But the simple log-normality assumption underlying (5) is now a sufficient condition to obtain the same results as the restrictive mean-variance single-period portfolio selection model of MARKOWITZ.

The problem is that generally, it is very hard to believe that the opportunity set is constant. For instance, the $\mu_i(x_t,t)$ and $\sigma_i(x_t,t)$ can themselves be defined as stochastic processes, or depend on a s-vector of state variables x_t. The simplest example of what a state-variable could be is the risk-free interest rate r_t, the price of an asset from the opportunity set (see JARROW (1988)), the inflation rate, the productivity rate, or the expected market return (see BREEDEN (1987)). The major advantage of using the interest rate r_t as a state variable is that it is observable, it evolves stochastically though time, and it is effectively a good explanatory factor for the return on risky assets; to simplify, let us consider first that x_t = $\{r_t\}$, that is, $\mu_i(x_t,t) = \mu_i(r_t)$ and $\sigma_i(x_t,t) = \sigma_i(r_t)$.

In this context, MERTON solves again the portfolio problem, obtains the optimal allocation and consumption of the agent, and proves the following theorem:

Theorem 2: If the return dynamics are described by (5) where x_t contains only one state variable, then, there exist 3 mutual funds constructed from linear combinations of the available n + 1 securities such that, independent of preferences, wealth distribution, or planning horizon, individuals will be indifferent between choosing from linear combinations of just these 3 mutual funds or linear combinations of all n risky securities and the risk-free security.

Of course, two possible mutual funds are the risk-free asset and the market portfolio. But what is the third one? MERTON shows that it is the portfolio (denoted H) with the highest possible correlation with the state variable considered. Furthermore, if all agents have homogeneous expectations, he shows that the following relationship holds:

$$\mu_i - r_t = \beta_i(\mu_M - r_t) + \beta_i^{(H)}(\mu_H - r_t) \qquad (9)$$

where β_i is defined as in (8), and

$$\beta_i = \frac{Cov(\mu_i, \mu_M)}{Var(\mu_M)} \qquad (10)$$

The first beta is similar to the traditional beta in (4) in the single period CAPM. The second beta refers to the returns of a particular portfolio that we denoted H. How can we interpret this? *As the economy changes over time, the statically optimal decisions that individual makes in one-period models are generally not intertemporally optimal.* As a consequence, risk averse individuals making both portfolio and consumption decisions will take into account the possibility of hedging against adverse shifts of the opportunity set. For the investor, the risk is that future investment opportunities may not be as much attractive as present investment opportunities.. Thus, he will try to smooth his consumption by investing in a *hedge portfolio* H. The best hedge portfolio is the one that is the most correlated with the state variable considered.

Now, what happens if we have more that one state variable underlying the economy? In a more general context, when x_t is an s-vector of state variables following diffusion processes, MERTON derives the following theorem:

Theorem 3: If the return dynamics are described by (5), then, there exist (s + 2) mutual funds constructed from linear combinations of the available n + 1 securities such that, independent of preferences, wealth distribution, or planning horizon, individuals will be indifferent between choosing from linear combinations of just these (s + 2) mutual funds or linear combinations of all n risky securities and the risk-free security. Each hedge portfolio will have the highest possible correlation with one of the state variable considered.

If in general the intertemporal CAPM differs from the static CAPM to account for hedging possibilities, one may wonder whether the latter model is still valid. In fact, under certain conditions, the portfolio selection behavior of intertemporal maximizers will be "as if" they were one period maximizers: if no asset is correlated with any of the state variables (i.e. it is not possible to create any hedge portfolio), if investors have a logarithmic utility function, i.e. are totally myopic and do not think about what can happen in the next period, or if all state variables evolve deterministically, i.e. without uncertainty.

Why do we believe MERTON's intertemporal portfolio selection model is so important? At least five essential contributions need to be identified:

- first, the dynamic generalization of the static mean-variance theory is achieved by considering both the consumption and the portfolio selection problem over time.
- second, the unrealistic quadratic expected utility function assumption of the static CAPM is dropped. The quadratic utility is not very appealing, since it implies increasing absolute risk aversion (risky assets are inferior goods) and satiation (an increase in wealth beyond the satiation point decreases utility), which are economically counter-intuitive. Furthermore,

mean-variance or mean-variance-skewness portfolio selection and portfolio performance measures should always be considered cautiously as they are the results from Taylor series truncations. In particular, the moment ordering is not necessarily identical between the truncated series and the "complete" functions. Ignoring theses facts may conduct to situations in which risk-averse agents appear to behave like risk-lovers in their portfolio selection problem, as evidenced by LHABITANT (1997).

- third, the model assigns zero probability of negative asset prices (as it uses (5), which translates into a log-normal non-negative distribution)
- fourth, it allows for distinct planning horizons
- and finally, it evidences the requirement of explicit hedging opportunities, which clearly shows the limits of the traditional modern portfolio theory. This opens the door to new series of passive "hedge funds", i.e. funds that would be exposed to only one factor in the economy and hedged against all other factors. And this also allows the investor to specify his individual risk profile through the use of adequate state variables. Shouldn't one hedge his portfolio against what he is afraid of?

MERTON's model relies on heavily mathematical computations; furthermore, it does not specify the relevant state variables nor their number [7] and only requires these state variables to follow Brownian motions. Depending on these assumptions, analytical solutions are difficult to obtain, and lengthy numerical algorithms have to be applied to get a solution. This has considerably slowed down practical applications of the model.
To our knowledge, the first effective application of the intertemporal portfolio selection problem was made by BRENNAN, SCHWARTZ and LAGNADO (1997). They considered the case of an investor who can invest in bonds, stocks and cash when there is time variation in expected returns of the asset classes. The time variation is driven by three state variables (short term interest rate, long term interest rate, and dividend yield on a portfolio of stocks). The stochastic process for the state variable is estimated from empirical data and the optimization problem is solved numerically over the 1982–1991 period. Then, the optimal portfolio for an investor solving (6) with a long time horizon (10 years) is compared to the optimal portfolio for an investor solving repetitively (1) at the beginning of each month. The resulting proportion of assets is significantly different, and the results provide encouraging evidence of potential significant improvements in portfolio returns. More recently, BREITLER, HEGI, TUCHSCHMIDT and REYMOND (1997) developed and solved numerically a similar problem in an international (two countries) context. The same conclusions were obtained.

3.2 Dynamic strategies and option pricing

But MERTON did not stop there. A buy and hold strategy is a strategy where an investor will determine a desired mix of assets for the portfolio, buy these assets, and hold them over the duration of the investment period. If agents can revise their decisions continuously in time, they are not confined anymore to the buy and hold strategy. They can enter into dynamic trading strategies, such as "rebalancing", "constant equity exposure", stop loss", "sell stocks as the market falls and buy stocks as the market rises", etc. This gave MERTON (1973) the idea of a replicating portfolio to price options: if one can replicate an option in every state of nature by a pre-specified dynamic trading strategy, then, the price of this option will be equal to the price of its replicating portfolio. Otherwise, arbitrage opportunities would exist.
This allowed MERTON to co-derive with BLACK and SCHOLES (1973) the famous option pricing formula for the values of European call and put options on a stock:

$$C_t = S_t N(d_1) - Ke^{-r(T-t)}N(d_2)$$
$$P_t = Ke^{-r(T-t)}N(-d_2) - S_t N(-d_1) \quad (11)$$

where

$$d_1 = \left[\ln\left(\frac{S_t}{K}\right) + (r + 0.5\sigma^2)(T-t) \right] / \sigma\sqrt{T-t}$$

and $d_2 = d_1 - \sigma\sqrt{T-t}$, C_t and P_t are the call and put option prices at time t, K is the exercise price (or strike price) of the option, S_t is the underlying security price at time t, T is the maturity date of the option, t is the current date, T – t is the time to maturity of the option, r is the annualized continuously compounded risk-free interest rate, σ is the annualized volatility (or standard deviation) of the underlying asset rate of return, and N() is the cumulative normal distribution. In fact, the late Fisher BLACK (1988) wrote:
"A fundamental point on the article I wrote with M. SCHOLES on option pricing was the demonstration of the formula by the arbitrage. Robert gave us this idea. In fact, the paper should probably be called the BLACK, MERTON, and SCHOLES paper".
In his seminal paper, MERTON (1973) also derived the elliptic partial differential equation plus a set of fixed boundary conditions for the value of *any* derivative security whose price depend only on the current value of S_t and on t. This allowed him to clarify and extend the original option pricing model by relaxing most of its assumptions (fixed or proportional dividends, no constant interest rates, no constant volatility, allowance for jump components in the underlying stock price) as well as to provide estimates of the sensitivity of the model to mispecifications of the input parameters (see MERTON (1973, 1976a, 1976b)).
MERTON also applied his dynamic strategies results in other fields of finance that are important for portfolio management. For instance, he was the first to provide a parametric and a non-parametric test of the performance of market timing strategies. Most conventional measures of risk such as those of SHARPE (1966), TREYNOR (1965) and JENSEN (1968) assume that the risk level of the portfolio under consideration is stationary through time, as if a buy and hold strategy was followed and exclusively focus the security selection skill. But a macro-forecaster or market timer will in fact change the risk level of its portfolio according to the expectations he may have regarding the behavior of the market return in the next period. If he expects the market return to be high, he will increase the beta of its portfolio; if he expects the market return to be low, he will decrease the beta of its portfolio. Indeed, a market timer will switch from more risky to less risky securities (and vice-versa) in an attempt to outguess the movement of the market. But the stationarity of systematic risk (β) through time is crucial to the conventional performance measures, as the non-stationarity is a violation of the ordinary least squares model specification[8]. Fortunately, MERTON (1981) and MERTON and HENRIKSSON (1981) proposed an augmented ordinary least squares regression that can be used to check for market timing abilities as well as stock selection abilities.
Furthermore, MERTON (1974) launched most of the current theoretical research on modeling the term structure of interest rates and the yield curve. According to its initial model, yield curve movements were due to variations in one single underlying factor, the short term interest rate. This factor can be modeled as an arithmetic Brownian motion

$$dr_t = \mu \cdot dt + \sigma \cdot dZ_t \quad (12)$$

that is, the short term rate has a trend which is predictable and a variation around this trend which is unpredictable.. In this framework, MERTON derived the entire yield curve and the price of a bond subject to interest rate risk. Equation (12) was simply extended by VASICEK (1977) to allow mean-reversion in the short term rate behavior and by HULL and WHITE (1993) to allow for time varying coefficients. Later on, MERTON

integrated his works on option pricing and interest rate derivatives by providing a pricing formula for a risky-coupon bond, e.g. a bond subject to default risk. This allowed him to define and examine the risk-structure of interest rates, by analogy with the term structure of interest rates.
Unfortunately, as such a survey cannot be exhaustive, we will stop here. Corporate finance, insurance theory, financial intermediation, bank deposit guaranties and financial institutions regulations are also some of the many fields that we did not consider to which MERTON added some essential contributions as well.

4. Conclusions

This paper exposes some important results of finance theory and their application in portfolio management. The primary capital market models in finance were the one-period mean-variance model of MARKOWITZ and TOBIN, and its equilibrium version of the SHARPE-LINTNER-MOSSIN CAPM. These models are now established in the portfolio management. Unfortunately, they omit a fundamental dimension: time. A good portfolio is more than a long list of stocks and bonds. It is a balanced whole, providing the investor with protections and opportunities with respect to a wide range of contingencies that exist and vary dynamically over time.
MERTON introduced the time dimension in portfolio management theory and solved some of the major recent academic problems in finance in domains such as intertemporal portfolio selection, option pricing, dynamic strategies, performance evaluation, debt pricing, financial intermediation, etc. He provided the groundwork for just about all of the theoretical finance that followed, and also supported a variety of applications.
Unfortunately, MERTON's work tends to be highly technical. Its powerful analytics are a temptation to strong focus on mathematical rigor, which is the source of blames from the practitioners community. We have to remember that a theory should not be judged by the complexity of its derivations or by the restrictiveness of its assumptions [9], but rather by the contribution of its conclusions to improve our understanding of the real world and the robustness of these conclusions. Similar objections were already raised with the original MARKOWITZ model; despite this, even the attacks on MARKOWITZ have triggered new concepts and new applications that might have never come about without his innovative contributions. Its option pricing work has become the paradigm for the relationship between the academic and applied worlds in investment (and awarded him the Nobel prize in 1997).
Thus, we think that despite the critiques, MERTON's contributions have been immense. His 1990 book "Continuous Time Finance" contains most of his important articles, from the original intertemporal portfolio selection problem to his recent works on managing university endowment funds. Commenting it, Stephen ROSS said: "Modern Finance has much to do, but it can not do better than to add to what MERTON has already done".
The theoretical foundations are ready for the applications of intertemporal models in investment management. Of course, there still remains much resistance to the idea that asset allocation is a mechanical and impersonal process. Managers would often like to generate the thrust of a portfolio strategy. But whims and tips are no longer valid input for managing portfolios in the next century; instead, return forecasts must be combined with risk measures to quantitatively assess intertemporal portfolio risk.

Footnotes

[1] Variance is a statistical measure of how much asset returns move around their average. Covariance and correlation are statistical measures of how much the returns on two assets swing in the same direction over time.

[2] This formulation is an extension of MARKOWITZ (1952), which only considers three securities and does not allow short sales, as he solves the problem by geometric methods. Note that the model can also be extended in order to include inequalities restrictions (see RUDOLF (1994))

[3] A market portfolio is defined as a portfolio that holds all available securities in proportion to their market capitalization.

[4] The time dimension was already present in Economics for many years, for instance in repeating games such as the Saint-Petersburg paradox. In Finance, RAMSEY (1931) proposed the first intertemporal portfolio selection problem, but without uncertainty. MOSSIN (1969), HAKANSSON (1974) and others attempted to develop multi-period models of portfolio selection, but the general consensus was they were simply a series of one-period models.

[5] The use of continuous time mathematics in finance started in fact in 1900, when BACHELIER (1900) completed his "Théorie de la spéculation", which came to light only by accident more than 50 years after he wrote it.

[6] The problem assumes times-additive and state independent utility of consumption. The case of non time-separable preferences and habit formation is discussed in SUNDARESAN (1989).

[7] Note that the consumption CAPM of BREEDEN (1979) synthesizes all the uncertainty in Merton's model in one single variable: consumption.

[8] By allowing dynamic strategies and evaluating their performance using static-models measures, we obtain incorrect results. In particular, as evidenced by LHABITANT (1997b), some strategies will dominate and even appear to systematically beat the market!

[9] The assumption to derive the one period CAPM are very restrictive, but they have been relaxed. For instance, BLACK (1972) relaxed the assumption of the existence of a risk-free asset. SOLNIK (1974) and STULTZ (1980) extended it in an international context. On the intertemporal front, MERTON's model has also be extended: CONSTANTINIDES (1986) and DUMAS and LUCIANO (1991) considered the impact of transaction costs, DETEMPLE and ZAPATERO (1992) introduced habit formation, while KARATZAS, LEHOCZKY, SHREVE and XU (1991) examined the case of incomplete markets (e.g. markets in which some states of nature cannot be insured)

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