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Forecasting Volatility in Swiss Financial Markets

1. Introduction

The volatility of asset returns, typically measured by their standard deviation, is an important magnitude in various fields of finance. Examples are risk measures in portfolio management as well as asset pricing in general and option pricing in particular. Frequently, a forecast of volatility is required. The most prominent case are option values which depend crucially on the standard deviation of returns on the underlying asset expected over the remaining lifetime of the option.
Recent empirical evidence shows that the traditionally made assumption of constant volatility over time may not be warranted. Volatility seems to change in a partially predictable way because it is positively serially correlated. Figures 1 to 3 provide an illustration using daily exchange rates for the SFr. versus the Deutschmark over the period from 1974 to 1990. The level of the exchange rate, shown in Figure 1, is well described by a random walk without drift, implying autocorrelated. Squared relative changes provide a measure of volatility. Figure 3 indicates that that its rate of change, pictured in Figure 2, is not clustering occurs in the sense that periods of highly volatile days are followed by more tranquil sequences of observations.
In this paper, a number of recently developed models are used to estimate and forecast such changes in volatility for different time series characterizing Swiss financial markets. Especially so-called Autoregressive Conditional Heteroskedasticity or ARCH-models that have become very popular are investigated in this study.[1] Interest rates for different maturities, returns on a stock market index and different precious metals as well as relative changes in exchange rates are used for this purpose. In the literature, a number of similar studies are available, investigating different models and data sets. Overall, the results are not clear cut in the sense that one or a few forecasting techniques dominate the others in terms of accuracy.[2]
A related literature, focussing on the predictive power of volatility estimates incorporated in option prices, is not considered in this study. The available evidence indicates that the success of this approach is mixed.[3]
The paper starts with a description of the data and their stochastic characteristics in the next section. The models and estimation results are presented in

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Figure 1: Exchange Rate SFr./100 DM

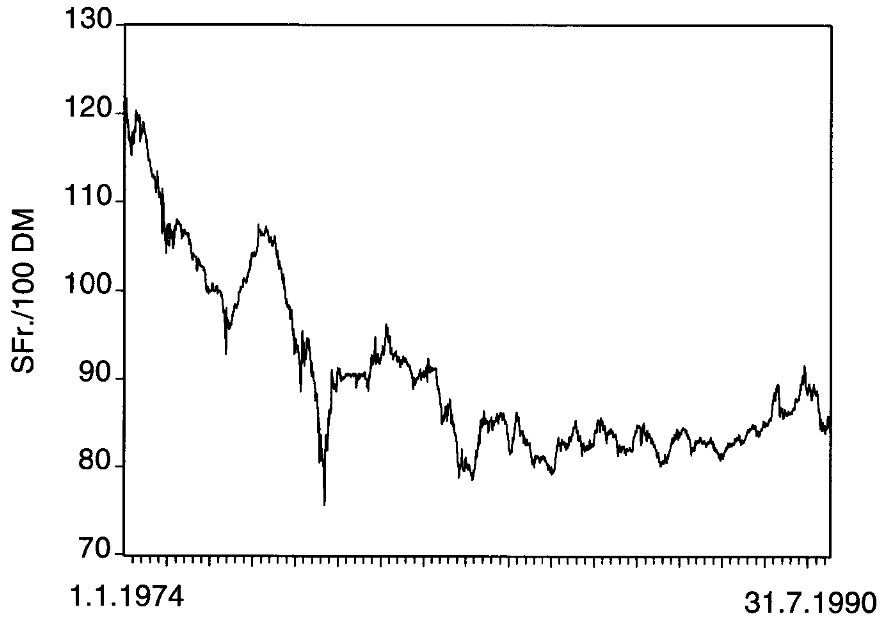


Figure 2: Relative Changes in Exchange Rate SFr./100 DM

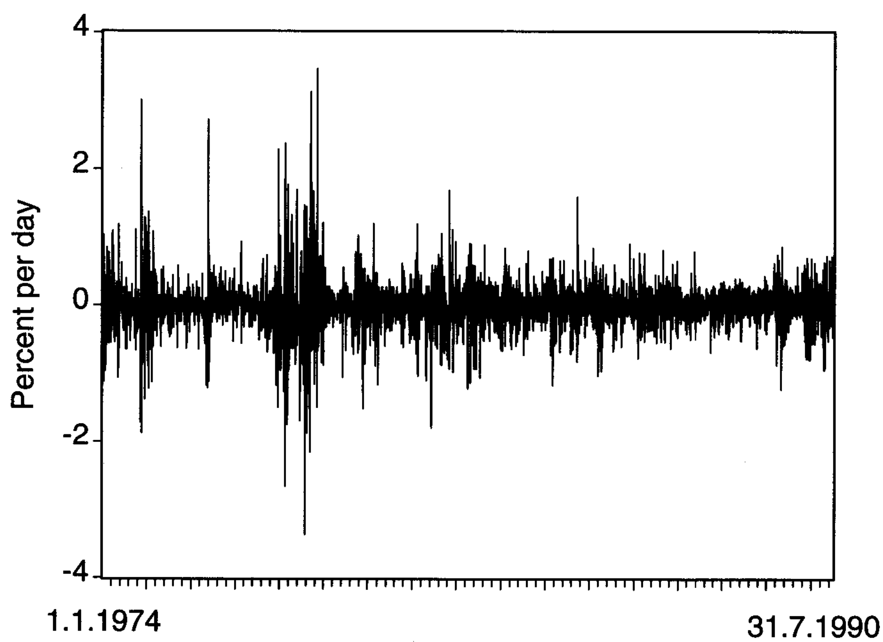
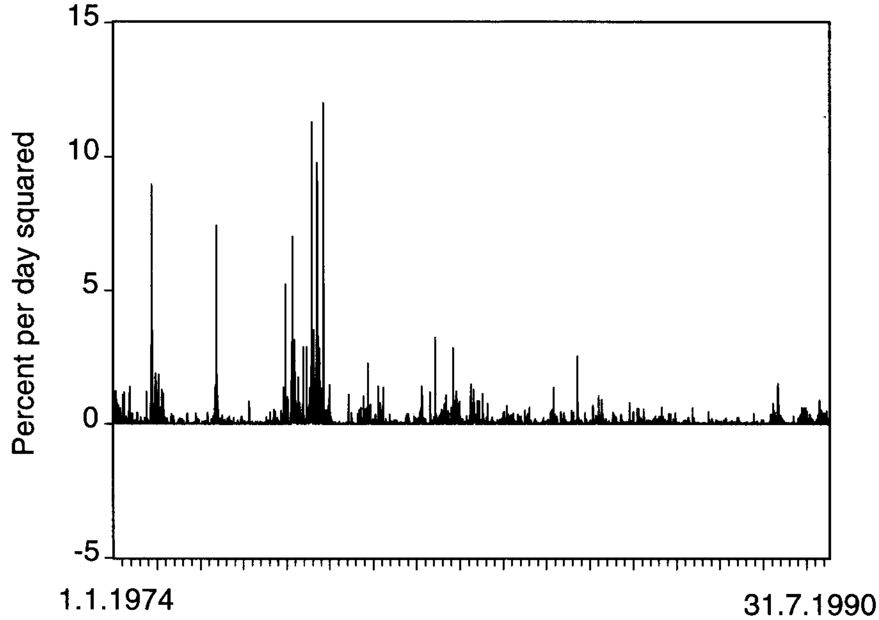


Figure 3: Relative Changes in Exchanges Rate SFr./100 DM squared



section 3. The relative forecasting power of the different frameworks is evaluated in sections 4 and 5. Some conclusions complete the paper.

2. Data and Stochastic Characteristics

Time series containing monthly as well as daily observations for interest rates, returns on stocks and precious metals as well as relative changes in exchange rates are used in the empirical work. The observation period covers the years 1974–1992 which is sufficiently long for a detailed analysis of all models considered. Furthermore, the period includes intervals with different characteristics, that is episodes with both high and low volatility.

The individual time series and their stochastic characteristics are presented in Table 1 for the full sample. Several subperiods are also distinguished according to both economic and statistical criteria but no substantial differences in relevant outcomes emerge from this investigation. Consequently, only results for the total period are reported.

The levels of interest rates for different maturities exhibit a number of patterns which are familiar from the literature. They are all non-stationary based on various standard unit root tests. Consequently, first differences are used in the empirical work. The average change is close to zero for all series. Volatility, as measured by the standard deviation, shows the usual decreasing shape with increasing time to maturity. The Jarque-Bera statistic indicates that normality must be rejected, typically in favor of distributions with much higher kurtosis. Changes in interest rates are only weakly serially correlated, confirming the random walk hypothesis for levels. The significantly positive serial correlation in the squared changes provides a first indication

Table 1: Stochastic Properties of Data

Variable	Obs. period	No. of obs.	Mean	Standard deviation	Jarque Bera	Autocorrelations of variables Lag 1	Lag 2	Lag 3	LB (12)	Autocorrelations of squared variables Lag 1	Lag 2	Lag 3	LB (12)
Interest rates (Euro SFr. deposits, percent per year, first differences)													
Daily data													
1 day	74–90	3'964	–0.002	1.410	40'290	–0.16	–0.14	–0.04	226	0.21	0.27	0.13	1'051
1 month	74–90	4'128	0.000	0.301	250'919	0.03	–0.04	–0.07	66	0.09	0.05	0.17	350
2 months	74–90	4'128	0.000	0.220	160'940	0.01	–0.03	–0.04	32	0.11	0.05	0.20	403
3 months	74–90	4'128	–0.001	0.197	65'507	–0.03	0.00	–0.07	34	0.38	0.26	0.30	1'760
6 months	74–90	4'128	0.000	0.158	22'719	–0.04	0.01	–0.06	37	0.34	0.10	0.13	785
12 months	74–90	4'128	0.000	0.137	32'043	–0.10	–0.01	–0.01	64	0.37	0.16	0.08	742
Govt. bond	74–90	4'075	0.000	0.020	9'906	0.23	0.20	0.14	853	0.17	0.11	0.11	458
Monthly data													
1 month	74–92	223	–0.003	1.006	264	–0.19	–0.06	0.07	22	0.17	0.20	0.11	42
3 months	75–92	211	–0.001	0.776	248	–0.12	0.04	0.04	18	0.20	0.12	0.10	45
Stock returns (Swiss Market Index – SMI, SFr., percent per period)													
Daily data													
Index	88–91	1'003	0.021	1.159	10'783	0.00	–0.01	–0.03	7	0.18	0.06	0.04	41
Monthly data													
Index	70–92	267	0.524	4.998	179	0.07	–0.07	0.01	18	0.14	–0.05	0.07	14
Exchange rates (relative changes, percent per period)													
Daily data													
SFr./DM	74–90	4'128	–0.008	0.355	32'311	0.05	0.00	0.01	25	0.36	0.16	0.21	1'410
SFr./$	74–90	4'128	–0.022	0.814	4'274	–0.01	0.00	0.01	21	0.19	0.11	0.10	686
Monthly data													
SFr./DM	74–92	219	–0.155	1.726	38	0.12	–0.05	0.05	10	0.23	0.11	0.12	28
SFr./$	74–92	219	–0.426	3.816	4	0.03	0.11	0.04	14	0.10	0.06	0.02	13
SFr./FF	74–92	219	–0.456	2.158	41	0.12	0.02	0.01	14	0.10	0.17	0.09	21
SFr./£	74–92	219	–0.488	3.061	6	0.10	0.01	0.02	10	0.12	0.00	0.03	8
SFr./Yen	74–92	219	–0.016	3.096	1	0.02	0.01	0.12	17	0.14	0.06	0.03	29
Prices of precious metals (SFr., relative changes, percent per period)													
Monthly data													
Silver	71–92	264	–0.043	10.468	2'496	0.08	0.01	–0.06	16	0.03	0.04	0.25	34
Gold	71–92	264	–0.450	6.169	64	0.07	0.01	–0.06	16	0.03	0.04	0.25	32

No. of obs.: Number of observations. Jarque-Bera: Test statistic for normality, underlined values indicating that the normal distribution is rejected. LB(12): Ljung-Box Q-statistic for the joint significance of the first twelve autocorrelations. Underlined autocorrelation coefficients are significantly different from zero on the 5 %-level.

for clustering in volatility. This result is much stronger for daily than for monthly observations.
Stock returns, as measured by the Swiss Market Index, relative changes in exchange rates and returns on precious metals exhibit similar patterns. Almost no serial correlation is observed in the variables themselves, indicating that the random walk describes the evolution of levels reasonably well. Persistence in volatility, as measured by the autocorrelations in the squared variables, appears to be quite strong for daily exchange rates but surprisingly weak for daily stock returns. Monthly observations contain virtually no systematic volatility pattern implying that clustering is essentially a very short-term phenomenon.

3. Models and Estimation Results

Different models are used to estimate and forecast the persistence in volatility. The recently developed and widely used ARCH and GARCH frameworks are compared to a number of much simpler techniques. In the remainder of this section, ARCH and GARCH models are briefly described and respective estimation results are presented.
The „Autoregressive Conditional Heteroscedastic" or ARCH model of order q for a stationary time series y starts with the formulation of the mean which may depend on past information. The goal is to reduce y to white noise. In equation (1), a standard ARMA model, including a constant term γ_0, is chosen for that purpose.

$$y_t = \gamma_0 + \gamma_1 y_{t-1} + \ldots + \gamma_k y_{t-k} + \varepsilon_t \quad (1)$$

The serially uncorrelated stochastic innovation ε_t is assumed to be normally distributed with a mean of zero and a standard deviation of σ_t. The persistence in volatility is parametrized by assuming a linear dependence of the variance σ_t^2 on past ε^2's, including a constant term α_0, as shown in equation (2).

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 \quad (2)$$

Equation (2) is essentially an application of the univariate modeling process for y in equation (1) to the variance of the residuals. This can be shown by rewriting equation (2) as

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 + [\varepsilon_t^2 - \sigma_t^2] \quad (3)$$

The last term in square brackets is white noise under rational expectations because the expected value of ε_t^2 is equal to σ_t^2. Consequently, ε_t^2 follows an autoregressive process of order q. Note further that a time series of ε and therefore of ε^2 can be made observable by estimating the residuals in equation (1). The degree of autocorrelation in the estimated ε^2's then informs about the persistence in volatility, as discussed in the previous section.
The unconditional variance of ε, σ^2, is obtained by setting the squared ε's equal to their unconditional expected values, which is σ^2. The result is

$$\sigma^2 = \frac{\alpha_0}{1 - \alpha_1 - \ldots - \alpha_q} \quad (4)$$

Equation (4) implies that the sum of the parameters, $\alpha_1 + \ldots + \alpha_q$, must be smaller than one in order for σ^2 to exist.
In empirical applications, the above ARCH model often involves a relatively large number of parameters and is therefore cumbersome to estimate and use. A more parsimonious framework is the Generalized ARCH or GARCH model of order p, q which consists of the same process for the mean of y and a formulation for the conditional variance, σ_t^2, which also includes the influence of lagged conditional variances. The result is

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_p \sigma_{t-p}^2 \quad (5)$$

Table 2: Estimation Results

Variable	ARCH (q)			GARCH (1,1)	
	q	Sum	SIC	Sum	SIC
Interest rates (respective currency, percent per year, first differences)					
Daily data (Euro SFr. deposits)					
1 day	10	1.400	9'304	1.001	8'908
1 month	5	1.054	–776	1.011	–726
2 months	5	0.913	–2'711	1.001	–2'850
3 months	5	0.803	–3'721	0.927	–3'817
6 months	7	0.955	–5'119	0.994	–5'204
12 months	8	0.919	–5'824	0.994	–5'874
Govt. bond	10	0.825	–21'522	0.972	–21'622
Monthly data					
1 month	2	0.854	547	0.955	554
3 months	5	1.056	440	0.980	424
Stock returns (Swiss Market Index - SMI, SFr., percent per period)					
Daily data					
Index	3	0.672	2'965	0.726	3'018
Monthly data					
Index	4	0.808	1'563	0.316	1'610
Exchange rates (relative changes, percent per period)					
Daily data					
SFr./DM	7	0.859	1'401	0.978	1'311
SFr./$	10	0.971	8'941	0.995	8'896
Monthly data					
SFr./DM	5	1.504	1'274	1.030	1'285
SFr./$	4	2.384	1'625	1.100	1'596
SFr./FF	3	0.702	1'516	0.988	1'494
SFr./£	4	0.599	1'789	0.999	1'722
SFr./Yen	4	1.988	1'555	1.008	1'481
Prices of precious metals (SFr., relative changes, percent per period)					
Monthly data					
Silver	3	0.656	1'802	0.969	1'836
Gold	3	0.464	1'611	0.905	1'622

The various time series are described in Table 1.
q: Order of ARCH model equal to the number of lagged volatility terms included.
Sum: Sum of the estimated parameters.
SIC: Schwarz information criterion.

The most popular alternative in this class is the GARCH (1, 1) model which describes financial time series very well in many cases. It is also used in this study. Setting p = q = 1 yields

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (6)$$

The restriction $\alpha_1 + \beta_1 < 1.0$ ensures that the unconditional variance exists. By adding and subtracting the same terms in equation (6) yields equation (7) which reveals that the observable ε^2 follows an ARMA (1, 1) process.

$$\varepsilon_t^2 = \alpha_0 + [\alpha_1 + \beta_1] \varepsilon_{t-1}^2 + [\varepsilon_t^2 - \sigma_t^2] - \beta_1 [\varepsilon_{t-1}^2 - \sigma_{t-1}^2] \quad (7)$$

If $\alpha_1 + \beta_1 = 1.0$, then the model is Integrated GARCH (1, 1) or IGARCH (1, 1). In this case, ε^2 is non-stationary and is described by an ARIMA (0, 1, 1) model.

Estimation results for the above models are summarized in Table 2. A maximum likelihood procedure is used for this purpose. The process for the mean, described by equation (1) only includes a constant term for most series. Exceptions are the daily interest rate and the yield to maturity of government bonds, both in first differenced form, where autoregressions of order 4 and 3 are respectively estimated to reduce the series to white noise.

A reliable identification of the number of lags in an ARCH model can be difficult because the likelihood surface may be quite flat.[4] Furthermore, these frameworks require a large number of observations for robust estimation. A maximum number of ten lags for daily and five lags for monthly data is considered. The final choice of q is then made based on a likelihood ratio test. Table 2 shows that the maximum lag is selected only in a few cases suggesting that parsimonously parametrized models capture the characteristics of volatility reasonably well. The sum of the parameters is very close to one in most cases and even exceeds one in a few. The latter outcome indicates severe problems with the chosen framework because the unconditional variance does not exist under these conditions.

Fewer estimation problems typically arise in the context of the GARCH (1, 1) model. However, the results show that the sum of the parameters is also essentially one throughout implying that the variance process may be integrated. The two classes of models can be compared using the Schwarz information criterion, SIC, a lower value indicating a better fit.[5] The results included in Table 2 show that the values for SIC are generally very close together for ARCH and GARCH models. The ranking moreover depends on the series considered. Consequently, a clear superiority of one model class over the other can not be established based on the estimation results. An evaluation of the out-of-sample forecasting ability, which is also the relevant information for most practical situations, may shed further light on this issue.

4. Forecasting Methodology

The models discussed in the previous section are used to predict volatility out-of-sample. The accuracy of the forecasts is then evaluated and compared to the results obtained with a number of very simple techniques. Based on the evidence reported above, only daily data are used for this purpose. Non-overlapping forecasting periods of 40 and 10 days are chosen. In all cases, the first 500 observations in the respective time series are used to estimate the model. The available series for equity index returns is too short and is therefore not included in this part of the investigation. Further details of implementation are presented below. The results are discussed in section 5.

The ARCH and GARCH models allow a prediction of volatility on a daily basis. However, the actual variance must be measured using several observations. For a forecasting period covering T days, from t + 1 to T, only information at the beginning of the period is used to derive the expected volatility. This leads to a sequential procedure. In the case of the ARCH model, the forecasted variance for the first day, t + 1, based on information available in t, $E_t(\sigma_{t+1}^2)$, is implied by equation (2) as

$$E_t\left[\sigma_{t+1}^2\right] = \alpha_0 + \alpha_1\ \varepsilon_t^2 + \ldots + \alpha_q\ \varepsilon_{t-q+1}^2 \quad (8)$$

Subsequent forecasts involve ε^2's, which are not yet realized at the beginning of the forecasting period. They are replaced by their conditional predictions. As an example, the expected variance for t+2 is given by

$$E_t\left[\sigma_{t+2}^2\right] = \alpha_0 + \alpha_1\ E_t\left[\sigma_{t+1}^2\right] + \ldots + \alpha_q\ \varepsilon_{t-q+2}^2 \quad (9)$$

The remaining forecasts until T, $E_t(\sigma_{t+T}^2)$, are recursively calculated in an analogous way. Afterwards, the variances forecasted for t + 1 to t + T are averaged, that is summed and divided by T, in order to get a variance prediction for the forecasting period as a whole. Finally, the resulting variance forecast is transformed to a standard deviation by taking the square root which can be compared to the actual volatility over the period. Information in disturbances after t are therefore not taken into account in the prediction of volatility during the forecasting period. For this reason, forecasting over short horizons may produce more reliable information than longer term predictions. The empirical results presented in section 5 confirm this expectation.

The same procedure is applied to derive the expected volatility from the GARCH(1, 1) model. In this case, the relevant expression for t + 1 is

$$E_t\left[\sigma_{t+1}^2\right] = \alpha_0 + \alpha_1\ \varepsilon_t^2 + \beta_1\ \sigma_t^2 \quad (10)$$

As above, expectations beyond t + 1 involve predicted values of σ^2 which have to be replaced by information available in t. Expected future values of ε_{t+j}^2 are again set equal to the prediction of σ^2 for the same point in time, in this case a specific day. The respective expression for t + 2 becomes

$$\begin{aligned} E_t\left[\sigma_{t+2}^2\right] &= \alpha_0 + \alpha_1\ E_t\left[\varepsilon_{t+1}^2\right] + \beta_1\ E_t\left[\sigma_{t+1}^2\right] \\ &= \alpha_0 + (\alpha_1 + \beta_1)\ E_t\left[\sigma_{t+1}^2\right] \\ &= (1 + \alpha_1 + \beta_1)\ \alpha_0 + (\alpha_1 + \beta_1)\ \alpha_1\ \varepsilon_t^2 \\ &\quad + (\alpha_1 + \beta_1)\ \beta_1\ \sigma_t^2 \end{aligned} \quad (11)$$

The second line of equation (11) indicates the recursive nature of the predictions. In the empirical implementation, the initial estimates of the parameters in the ARCH and GARCH models are calculated selecting the first 500 observations. The first prediction is then based on this estimate.

Two procedures are alternatively used to update the parameter estimates over time. With the first procedure, denoted ARCH I and GARCH I respectively, the observations of the most recent forecasting period are added to the sample to get new parameter estimates. Consequently, the sample size increases from one prediction interval to the next. This technique allows more precise estimates provided the observations are generated by the same stochastic process.

The second updating procedure, denoted by ARCH II and GARCH II respectively, relies on a constant sample size including the most recent 500 data points immediately preceding the forecasting period. The same number of observations is therefore deleted at the beginning of the sample as are added at the end before each prediction. This offers the chance to capture a change in parameters which will however materialize only slowly in the estimates.

Three simple techniques serve as benchmarks in order to evaluate the relative forecasting performance of ARCH and GARCH models. The first alternative, shown as „Standard Deviation I“ is the realized standard deviation of the previous forecasting period. This procedure assumes that volatility evolves according to a random walk without drift.

In analogy to the ARCH II and GARCH II frameworks, the estimated standard deviation based on the most recent 500 observations before the respective prediction period provides the second alternative in this class. The abbreviation „Standard Deviation II“ is used for this procedure.

The last technique, denoted by „Standard Deviation III“, is similar to ARCH I and GARCH I in the sense, that the estimated standard deviation is calculated selecting all the available observations up to the beginning of the prediction period. The number of data points used in calculating the forecast is therefore increased as time passes.

In order to assess the accuracy and rationality of the different techniques, the forecasts are compared to the standard deviation of the ε's from t + 1 to T, $\sigma_{t+1,t+T}$, realized during the forecasting period. The ε's are determined as described above. T symbolizes the length of the forecasting period measured in days. The actual standard deviation is given as

$$\sigma_{t+1,t+T} = \sqrt{\frac{\varepsilon_{t+1}^2 + \ldots + \varepsilon_{t+T}^2}{T}} \qquad (12)$$

5. Forecasting Results

Results for predictions over periods with a length of alternatively 40 and 10 days are presented in sections 5.1 and 5.2 respectively.

5.1 Forecasts over 40 Days

Table 3 includes the evidence for a prediction period with a length of 40 days. The relatively complicated ARCH frameworks yield disappointing results for all time series investigated. In many cases, the predictions are severely biased as shown by the mean error. The root mean squared forecast error, RMSE, is typically much higher than for competing techniques. The regressions of the actual on the predicted standard deviation are characterized by slope parameters, beta, significantly smaller than the value of 1.0 implied by rational forecasting. Furthermore, the explanatory power of the regressions is low as indicated by R^2. The relatively high proportions of the RMSE allocated to bias and difference of beta from unity are consistent with the above results. Furthermore, the forecast errors are heavily autocorrelated indicating that past errors would provide useful information to achieve more rational predictions for subsequent periods.

The two GARCH models yield somewhat better results. However, they are also frequently dominated by the much simpler forecasts based on actual standard deviations estimated over past periods. Especially, the random walk predictor „Standard Deviation I“, using the actual standard deviation of the previous forecasting period, is often superior to the other techniques. It is moreover simple to calculate and therefore associated with little cost. However, all procedures suffer from severe autocorrelation in the forecast errors.

5.2 Forecasts over 10 Days

It has already been noted above that the ARCH and GARCH frameworks seem especially relevant for short forecasting horizons because they deliver a different prediction for every data point. The results for a much shorter forecasting period of 10 days are presented in Table 4. As expected, both the ARCH and GARCH models provide much more meaningful results in this case. But rather disappointingly, they still fail to beat the simple alternatives. The random walk model for the standard deviation remains the best alternative for most time series investigated. Consequently, the relatively high costs associated with the implementation of sophisticated techniques such as ARCH and GARCH do not seem to pay off in terms of higher accuracy.

However, fully satisfactory results are not obtained through any technique. The estimated

Table 3: Forecasting 40 days

Variable, Model	Mean error	RMSE	Beta	R^2	Proportions of error (%)			Autocorrelations	
					Bias	Diff.	Res.	Lag 1	Lag 2
Interest rate SFr. 1 day (percent per year, first differences)									
ARCH (q) I	$-6.14*10^3$	$1.61*10^4$	0.00	0.02	14	86	0	0.63	0.30
ARCH (q) II	$-2.48*10^6$	$1.54*10^7$	0.00	0.03	3	97	0	0.07	0.05
GARCH (1,1) I	–0.28	0.83	0.47	0.35	11	37	52	–0.28	0.08
GARCH (1,1) II	–46.65	310.29	0.00	0.01	2	98	0	0.30	0.07
Std. dev. I	0.00	0.68	0.58	0.34	0	21	79	–0.52	0.30
Std. dev. II	–0.19	0.74	0.56	0.16	6	10	84	0.45	0.44
Std. dev. III	–0.64	0.97	0.93	0.04	44	0	56	0.54	0.54
Euro SFr. rate 1 month (percent per year, first differences)									
ARCH (q) I	–15.22	27.53	0.00	0.08	31	69	0	0.70	0.53
ARCH (q) II	–335.50	$2.56*10^3$	0.00	0.01	2	98	0	0.00	0.00
GARCH (1,1) I	–0.36	0.42	0.12	0.08	71	24	5	0.25	0.37
GARCH (1,1) II	–0.40	1.67	0.02	0.18	6	94	0	0.12	0.57
Std. dev. I	0.00	0.10	0.45	0.20	0	28	72	–0.39	0.02
Std. dev. II	–0.04	0.13	0.17	0.02	10	33	57	0.55	0.37
Std. dev. III	–0.17	0.20	0.19	0.02	70	7	23	0.54	0.43
Euro SFr. rate 2 months (percent per year, first differences)									
ARCH (q) I	–1.88	3.64	0.01	0.09	27	73	0	0.82	0.72
ARCH (q) II	–12.32	76.30	0.00	0.00	3	97	0	0.09	0.07
GARCH (1,1) I	–0.42	0.65	0.06	0.15	43	56	1	0.40	0.42
GARCH (1,1) II	–0.75	4.42	0.00	0.00	3	97	0	0.25	0.03
Std. dev. I	0.00	0.08	0.46	0.21	0	27	73	–0.49	0.18
Std. dev. II	–0.03	0.09	0.32	0.06	9	21	70	0.45	0.37
Std. dev. III	–0.11	0.13	0.38	0.04	66	4	30	0.46	0.44
Euro SFr. 3 months (percent per year, first differences)									
ARCH (q) I	–0.12	0.15	0.12	0.02	64	17	19	0.74	0.64
ARCH (q) II	–0.03	0.08	0.33	0.11	12	30	58	0.57	0.34
GARCH (1,1) I	–0.07	0.10	0.45	0.12	52	8	40	0.30	0.16
GARCH (1,1) II	–0.02	0.07	0.50	0.20	6	19	75	0.38	0.11
Std. dev. I	0.00	0.06	0.58	0.33	0	21	79	–0.28	–0.20
Std. dev. II	–0.02	0.07	0.38	0.12	9	25	66	0.54	0.28
Std. dev. III	–0.10	0.12	0.35	0.04	67	4	29	0.58	0.39
Euro SFr. rate 6 months (percent per year, first differences)									
ARCH (q) I	–0.18	0.19	0.36	0.19	84	7	9	0.65	0.53
ARCH (q) II	–1.44	6.84	0.00	0.02	4	96	0	0.68	0.36
GARCH (1,1) I	–0.06	0.08	0.71	0.36	51	4	45	0.25	0.26
GARCH (1,1) II	–0.19	1.08	0.01	0.02	3	97	0	0.28	0.14
Std. dev. I	0.00	0.05	0.69	0.48	0	15	85	–0.24	–0.09
Std. dev. II	–0.01	0.06	0.65	0.26	4	9	87	0.54	0.27
Std. dev. III	–0.05	0.08	0.81	0.03	40	0	60	0.66	0.48

Variable, Model	Mean error	RMSE	Beta	R^2	Proportions of error (%)			Autocorrelations	
					Bias	Diff.	Res.	Lag 1	Lag 2
Euro SFr. rate 12 months (percent per year, first differences)									
ARCH (q) I	–0.09	0.13	–0.22	0.03	56	23	21	0.66	0.52
ARCH (q) II	–0.12	0.48	0.02	0.03	6	92	2	0.43	0.18
GARCH (1,1) I	–0.03	0.06	0.65	0.13	23	3	74	0.40	0.21
GARCH (1,1) II	–0.02	0.11	0.23	0.22	4	72	24	0.05	0.14
Std. dev. I	0.00	0.05	0.59	0.34	0	21	79	–0.32	–0.14
Std. dev. II	–0.01	0.06	0.63	0.23	3	9	88	0.45	0.23
Std. dev. III	–0.04	0.07	1.34	0.05	32	0	68	0.55	0.39
Swiss government bond yield (percent per year, first differences)									
ARCH (q) I	0.01	0.01	0.22	0.06	62	16	22	0.36	0.24
ARCH (q) II	0.65	5.23	0.00	0.02	2	98	0	0.05	0.04
GARCH (1,1) I	0.01	0.10	0.49	0.10	44	6	50	0.04	0.13
GARCH (1,1) II	0.01	0.01	0.22	0.18	12	65	23	0.46	0.28
Std. dev. I	0.01	0.01	0.34	0.14	0	38	62	–0.27	–0.15
Std. dev. II	0.01	0.01	0.57	0.13	4	8	88	0.33	0.12
Std. dev. III	0.01	0.01	0.34	0.01	28	1	71	0.41	0.23
Exchange rate SFr./DM (relative changes, percent per day)									
ARCH (q) I	–0.30	0.45	0.09	0.02	45	40	15	0.57	0.45
ARCH (q) II	–5.97	27.59	0.00	0.00	5	95	0	0.33	0.23
GARCH (1,1) I	–0.15	0.25	0.43	0.35	35	31	34	–0.18	–0.02
GARCH (1,1) II	–0.15	0.43	0.18	0.22	12	74	14	0.38	0.40
Std. dev. I	0.00	0.16	0.62	0.38	0	19	81	–0.18	–0.19
Std. dev. II	–0.03	0.19	0.36	0.06	3	16	81	0.61	0.38
Std. dev. III	–0.10	0.20	0.70	0.02	24	0	76	0.59	0.36
Exchange rate SFr./$ (relative changes, percent per day)									
ARCH (q) I	–0.75	0.92	0.16	0.07	66	23	11	0.26	0.42
ARCH (q) II	–1.95	6.17	0.00	0.00	10	90	0	0.55	0.56
GARCH (1,1) I	–0.13	0.34	0.49	0.25	14	23	63	–0.38	0.05
GARCH (1,1) II	–0.16	0.52	0.19	0.09	9	57	34	0.28	0.46
Std. dev. I	0.00	0.33	0.47	0.23	0	27	73	–0.36	–0.08
Std. dev. II	–0.04	0.35	0.12	0.00	2	17	81	0.50	0.35
Std. dev. III	–0.03	0.32	0.67	0.02	1	0	99	0.45	0.30

The forecasting periods are 40 days long and not overlapping. The models and the estimation procedures are described in the text. RMSE: Root mean squared forecast error. Beta, R^2: Slope parameter and coefficient of determination respectively in a regression of the actual standard deviation on a constant and the predicted standard deviation. Proportions of error: Percentage of mean squared forecast error respectively attributable to bias, difference of beta from unity, Diff., and residual variance, Res., as defined by Theil (1958). Autocorrelations: Autocorrelation coefficients of forecast errors with underlined values indicating significance on the 5 %-level.

Table 4: Forecasting 10 days

Variable, Model	Mean error	RMSE	Beta	R^2	Proportions of error (%)			Autocorrelations	
					Bias	Diff.	Res.	Lag 1	Lag 2
Interest rate SFr. 1 day (percent per year, first differences)									
ARCH (q) I	–4.20	7.41	0.04	0.08	32	67	1	0.43	0.52
ARCH (q) II	–8.08	34.71	0.01	0.14	5	95	0	0.55	0.44
GARCH (1,1) I	–0.14	0.79	0.61	0.33	3	16	81	–0.22	–0.05
GARCH (1,1) II	–0.43	1.41	0.27	0.21	9	61	30	0.36	0.21
Std. dev. I	0.00	0.88	0.48	0.24	0	26	74	–0.47	0.03
Std. dev. II	–0.26	0.88	0.57	0.14	9	7	84	0.43	0.39
Std. dev. III	–0.73	1.12	1.14	0.04	43	0	57	0.47	0.43
Euro SFr. rate 1 month (percent per year, first differences)									
ARCH (q) I	–0.42	0.50	0.14	0.11	69	26	5	0.22	0.30
ARCH (q) II	–0.27	0.74	0.06	0.13	14	84	2	0.46	0.33
GARCH (1,1) I	–0.12	0.20	0.25	0.07	36	26	38	–0.28	0.05
GARCH (1,1) II	–0.08	0.18	0.28	0.12	19	37	44	0.08	0.11
Std. dev. I	0.00	0.15	0.26	0.07	0	38	62	–0.52	0.10
Std. dev. II	–0.06	0.15	0.21	0.02	13	19	68	0.39	0.41
Std. dev. III	–0.18	0.23	0.19	0.01	64	5	31	0.35	0.38
Euro SFr. rate 2 months (percent per year, first differences)									
ARCH (q) I	–0.24	0.28	0.22	0.11	70	19	11	0.25	0.28
ARCH (q) II	–0.09	0.27	0.07	0.03	10	77	13	0.35	0.40
GARCH (1,1) I	–0.12	0.18	0.27	0.11	45	26	29	0.01	0.06
GARCH (1,1) II	–0.06	0.17	0.15	0.05	11	57	32	0.28	0.27
Std. dev. I	0.00	0.12	0.30	0.09	0	35	65	–0.45	–0.07
Std. dev. II	–0.03	0.11	0.35	0.04	9	11	80	0.34	0.26
Std. dev. III	–0.12	0.16	0.34	0.02	56	3	41	0.32	0.25
Euro SFr. 3 months (percent per year, first differences)									
ARCH (q) I	–0.08	0.11	0.63	0.08	54	1	45	0.29	0.39
ARCH (q) II	–0.02	0.08	0.49	0.11	7	11	82	0.38	0.38
GARCH (1,1) I	–0.04	0.08	0.76	0.24	23	2	75	–0.18	0.11
GARCH (1,1) II	–0.01	0.07	0.76	0.21	2	3	95	0.10	0.20
Std. dev. I	0.00	0.08	0.44	0.20	0	28	72	–0.47	0.02
Std. dev. II	–0.03	0.09	0.40	0.09	10	15	75	0.47	0.43
Std. dev. III	–0.10	0.13	0.31	0.02	60	4	36	0.48	0.44
Euro SFr. rate 6 months (percent per year, first differences)									
ARCH (q) I	–0.09	0.12	0.53	0.16	53	6	41	–0.17	0.05
ARCH (q) II	–0.06	0.18	0.16	0.14	11	72	17	0.41	0.42
GARCH (1,1) I	–0.03	0.09	0.55	0.19	13	12	75	–0.27	0.00
GARCH (1,1) II	–0.03	0.09	0.44	0.22	8	29	63	0.24	0.30
Std. dev. I	0.00	0.09	0.38	0.14	0	31	69	–0.48	0.01
Std. dev. II	–0.02	0.08	0.64	0.16	6	5	89	0.28	0.24
Std. dev. III	–0.06	0.10	0.83	0.02	35	0	65	0.36	0.33

Variable, Model	Mean error	RMSE	Beta	R^2	Proportions of error (%)			Autocorrelations	
					Bias	Diff.	Res.	Lag 1	Lag 2
Euro SFr. rate 12 months (percent per year, first differences)									
ARCH (q) I	–0.06	0.10	0.64	0.03	41	1	58	0.15	0.31
ARCH (q) II	–0.02	0.08	0.42	0.15	8	23	69	0.18	0.31
GARCH (1,1) I	–0.02	0.08	0.75	0.12	10	2	88	–0.01	0.14
GARCH (1,1) II	–0.01	0.07	0.62	0.21	2	9	89	–0.05	0.06
Std. dev. I	0.00	0.09	0.29	0.09	0	35	65	–0.54	0.12
Std. dev. II	–0.02	0.07	0.63	0.14	5	5	90	0.20	0.26
Std. dev. III	–0.05	0.09	1.38	0.03	29	0	71	0.26	0.32
Swiss government bond yield (percent per year, first differences)									
ARCH (q) I	–0.01	0.01	0.56	0.11	47	4	49	–0.05	0.20
ARCH (q) II	–0.01	0.02	0.15	0.11	18	66	16	0.46	0.47
GARCH (1,1) I	–0.01	0.01	0.66	0.18	24	4	72	–0.20	0.12
GARCH (1,1) II	0.00	0.01	0.59	0.22	11	11	78	–0.06	0.10
Std. dev. I	0.00	0.01	0.41	0.16	0	29	71	–0.20	–0.02
Std. dev. II	0.00	0.01	0.56	0.09	11	5	84	0.29	0.26
Std. dev. III	–0.01	0.01	0.11	0.00	32	1	67	0.34	0.31
Exchange rate SFr./DM (relative changes, percent per day)									
ARCH (q) I	–0.14	0.25	0.54	0.16	32	8	60	–0.25	0.19
ARCH (q) II	–0.13	0.39	0.17	0.09	10	64	26	0.23	0.54
GARCH (1,1) I	–0.07	0.22	0.52	0.24	9	19	72	–0.42	0.06
GARCH (1,1) II	–0.04	0.23	0.42	0.20	3	31	66	–0.28	0.11
Std. dev. I	0.00	0.22	0.47	0.22	0	27	73	–0.44	0.00
Std. dev. II	–0.05	0.23	0.34	0.04	5	12	83	0.50	0.44
Std. dev. III	–0.12	0.24	0.78	0.02	25	0	75	0.45	0.39
Exchange rate SFr./$ (relative changes, percent per day)									
ARCH (q) I	–0.24	0.42	0.64	0.30	31	9	60	–0.15	–0.06
ARCH (q) II	–0.17	0.52	0.30	0.13	11	40	49	0.38	0.31
GARCH (1,1) I	–0.06	0.33	0.72	0.35	3	7	90	–0.16	–0.22
GARCH (1,1) II	–0.06	0.35	0.68	0.30	3	9	88	0.02	–0.07
Std. dev. I	0.00	0.36	0.57	0.32	0	22	78	–0.25	–0.22
Std. dev. II	–0.08	0.41	0.27	0.01	3	9	88	0.59	0.38
Std. dev. III	–0.07	0.39	1.09	0.03	3	0	97	0.55	0.32

The forecasting periods are 10 days long and not overlapping. The models and the estimation procedures are described in the text. RMSE: Root mean squared forecast error. Beta, R^2: Slope parameter and coefficient of determination respectively in a regression of the actual standard deviation on a constant and the predicted standard deviation. Proportions of error: Percentage of mean squared forecast error respectively attributable to bias, difference of beta from unity, Diff., and residual variance, Res., as defined by Theil (1958). Autocorrelations: Autocorrelation coefficients of forecast errors with underlined values indicating significance on the 5 %-level.

values of beta all lie substantially below 1.0 and significantly positive autocorrelation in the forecast errors remains throughout. These characteristics are similar to the ones reported in other studies.

The results of this study are partly similar and partly different compared to the available literature.[6] As here, other authors also conclude that a future volatility cannot be predicted accurately no matter which technique is used. Moreover, the ranking of models with respect to their forecasting power is not uniform. The superiority of the random walk documented here is however rather unusual. Quite frequently, ARCH and specifically GARCH models perform somewhat better than simple alternatives. But the differences in accuracy are usually small relative to the size of the unexpected component in volatility.

6. Conclusions

A number of recently developed models designed to explain and forecast volatility are empirically investigated using data on interest rates, stock returns, exchange rates and commodity prices from Swiss financial markets. Using daily observations, it is shown that volatility changes systematically over short intervals in the sense that clustering of high and low volatility is present in the data. This phenomenon however disappears almost completely as the observation frequency is decreased from one day to one month.

The estimation results indicate that ARCH and GARCH models are able to describe the observed positive autocorrelation in volatility reasonably well. Rather surprisingly, these techniques do however not deliver very accurate predictions out-of-sample even if the forecasting interval is relatively short. Simple alternatives, especially the standard deviation realized over the immediately preceding forecasting period, provide substantially better predictions.

Footnotes

[1] BOLLERSLEV, CHOU and KRONER (1992) provide a survey of these techniques and their application in finance.

[2] Relevant studies include AKGIRAY (1989), BRAILSFORD and FAFF (1996), CUMBY, FIGLEWSKI and HASBROUCK (1993), DAY and LEWIS (1993), HEYNEN and KAT (1994), JORION (1995), NOH, ENGLE and KANE (1994) and WEST and CHO (1995).

[3] Recent examples using options based techniques are CANINA and FIGLEWSKI (1993), DAY and LEWIS (1993), HARVEY and WHALEY (1992), LAMOUREUX and LASTRAPES (1993) and NOH, ENGLE and KANE (1994).

[4] FIGLEWSKI (1994) discusses these issues in detail.

[5] The Schwarz information criterion is based on the log likelihood function including a penalty for the number of parameters estimated.

[6] The relevant literature is listed in footnotes 2 and 3.

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