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MARTIN BRUAND*

The jump-diffusion process in Swiss stock returns and its influence on option valuation

1. Introduction

Option pricing modelling in finance is generally associated to the famous work of BLACK and SCHOLES (1973). Their valuation formula revolutionised finance and is used with success in the whole financial community. To obtain their result, that is to say an analytical and easy to use formula, they had to make several assumptions on the parameters influencing the prices of options. One of them is about the riskless interest rate. BLACK and SCHOLES suppose that it is constant and equal for lenders and borrowers. This hypothesis is subject to critics but its influence on short term options is so low that we will maintain it in this paper. However, the hypothesis on normality of returns and thus of constancy of variance has always been strongly criticized. On the side of practitioners, the BLACK-SCHOLES formula is used with a volatility based on agents predictions which could be more or less accurately specified. This technique seems to work but it uses a formula while violating one of the hypothesis on which it rests. Even before the formula has been developed, FAMA (1965) showed that the distribution of financial assets returns had fatter tails than the normal distribution, as well as non constant volatilities. Moreover PRESS (1967) detected jumps in the distribution of returns of US stocks.

Several approaches were developed in order to improve the BLACK-SCHOLES model. COX and ROSS (1976) assume an inverse deterministic relationship between the variance and the price of the stock to develop their CEV (Constant Elasticity of Variance) model. HULL and WHITE (1987) derive a model in which the returns and the variance follow different and independent diffusion processes. DUAN (1995) proposes a GARCH (Generalized Autoregressive Conditional Heteroscedasticity) model to value options. MERTON (1976) supposes that the returns are normally distributed with a constant variance, but he postulates that these returns are in addition subject to discontinuities, and thus to jumps. Under the hypothesis that this jump risk is diversifiable, he develops an analytical formula to value options. NAIK and LEE (1990) develop a general equilibrium model to price index options, in which they remove MERTON's hypothesis. AHN (1992) generalizes this model, in order to price stock options. In this paper, MERTON's model is chosen to be tested on the Swiss options market. The

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objective of the paper is to complete the following set of papers: CHESNEY, GIBSON and LOUBERGÉ (1993), Adjaouté (1993) and SABBATINI (1994) which tested CEV, stochastic volatility and GARCH models' performance on the Swiss market index and stocks written options.
Some empirical studies have detected significant jumps in the returns of several financial assets. BALL and TOROUS (1983) detected significant jumps in the returns of US Stocks. JORION (1988) and POWELL (1989) found significant jumps in exchange rates and crude oil futures returns. This paper will first estimate the different parameters needed to characterize a jump-diffusion distribution, in order to test if there are significant jumps in the returns of Swiss stocks and of the Swiss Market Index. The estimations will be realized on long periods (14 and 5 years), but also on one year periods. These estimations have also the objective to detect the impact of well known jumps: the 1987 and 1989 Crashes and the "Nestle effect" in 1988. It will be shown that these events have a strong impact on the estimated parameters and that they are significant.
MERTON's hypothesis on the fact that jump risk is diversifiable is also tested by estimating the correlation between jumps in the market returns and jumps in individual stock returns. A strong correlation is obtained between these jumps. The estimations are then used to test the performance of MERTON's model and to compare it to the BLACK and SCHOLES model on the Swiss options market. The differences between the three models are small, confirming the results obtained by BALL and TOROUS (1985). A more detailed simulation based comparison of the MERTON and BLACK-SCHOLES models is also proposed to test the influence of some parameters on the differences between both formulas. This allows us to see in which situation a jump-diffusion option valuation model could significantly improve the results obtained with the BLACK-SCHOLES formula.
The paper is organized as follows: section 2 presents the mixed jump-diffusion process which will be used. The results of parameter estimations are presented in section 3 and the correlations between jumps in the stock and index returns are computed in section 4. Section 5 presents the empirical results obtained with these models. Comparisons are made in section 6, before the main summary of the results. The pricing models used are briefly presented in an appendix.

2. The dynamics of stock returns

To determine the price of any contingent claim, we first have to know how the price of the underlying security will change in the future. If we are able to describe this evolution, financial modelling based on the no arbitrage argument and mathematics will more or less easily give us the price of the contingent claim. The most classical and known process is the one proposed by BLACK and SCHOLES: the geometric brownian motion. It is given in the following form:

$$dS/S = \mu dt + \sigma dz \qquad (1)$$

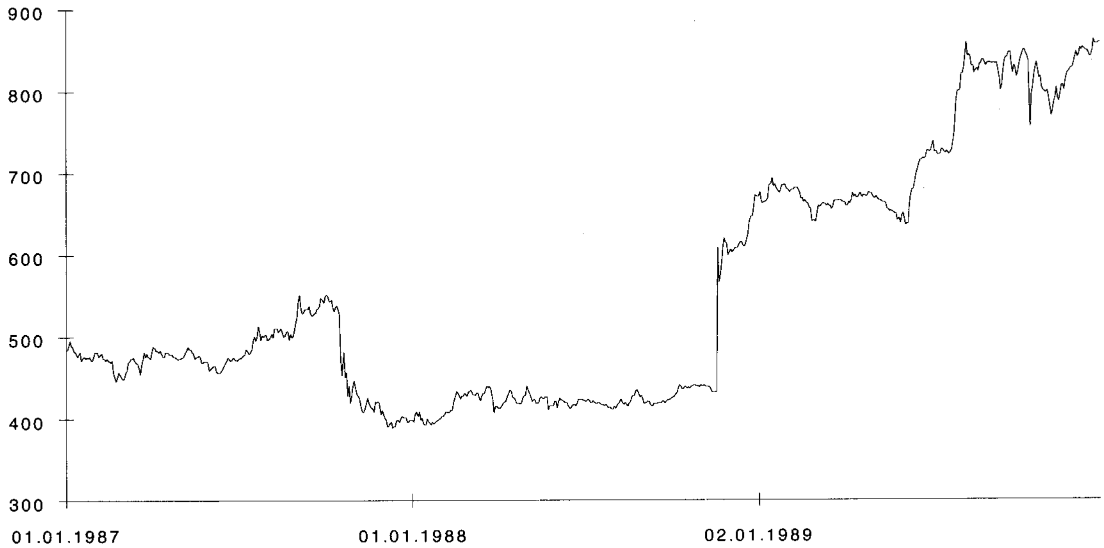
S: price of the underlying
μ: expected instantaneous return on the underlying.
σ: standard deviation of the instantaneous returns.
dz: standard Wiener process.

This process, when used to characterize a financial security's return, is subject to several critics about two main assumptions:

1. The expected return and the standard deviation of returns are constant.
2. The dynamic of returns follows a continuous sample path.

The object of this paper is to verify this second hypothesis, which is not convincing because we can observe some exceptional events in the financial markets. These events are characterized by extra high or low returns, which cannot be integrated in a continuous path. The October 1987 Crash is a classical example of what can happen to

Figure 1: Price of the registered Nestle stock



the world financial markets. Figure 1 illustrates the presence of jumps in the evolution of the price of the Nestle stock.

An alternative process has to be chosen to take these jumps into account. Several processes have been proposed to model these jumps. COX and ROSS (1976) consider a pure jump process. MERTON (1976) proposes a mixed jump-diffusion process, which is chosen in this paper. This process stipulates that the securities are subject to two types of variations:

1. the usual diffusion induced variations which cause a marginal change in the security prices. They are modelled by a geometric brownian motion, like the one proposed by BLACK and SCHOLES.
2. The exceptional variations due to important new informations on a firm or a market segment. These informations induce a big change in the price of securities. The jumps arrival is modelled by a Poisson process. This process is used in insurance mathematics to describe rare events, so that it should be a good choice in our situation.

The MERTON jump-diffusion process:

$$dS/S=(\mu-\lambda k)\, dt + \sigma\, dz + dq \qquad (2)$$

dq: Poisson process.
k: expected size of jumps.
λ: number of jumps per period

Notice that if there are no jumps (λ=0 and dq=0), we are back to the original diffusion process proposed by BLACK and SCHOLES.

Applying a generalized version of Itô's lemma and supposing that the size of the jumps is lognormally distributed, MERTON derives the density of probability of the distribution of returns:

$$f(x) = \sum_{n=0}^{\infty} \frac{e^{-\lambda}\,(\lambda)^n}{n!}\, \varphi(\mu_B + n\,\mu_j,\ \sigma_B^2 + n\,\sigma_j^2) \qquad (3)$$

Φ: density of the normal distribution
μ_B : expected return due to Brownian
σ_B^2 : variance of returns due to Brownian
μ_j : expected return due to jumps
σ_j^2 : variance of returns due to jumps

Notice that we get to a compound Poisson distribution. The normal distribution has parameters depending on the diffusion and jump part of the process.
Following TRAUTMANN and BEINERT (1995), we can obtain the non-conditional expected return, knowing that the unconditional expectation is equal to the expectation of the conditional expectation:

$$E[X] = E[E[X \mid N]] = E[\mu_B + n\,\mu_j] = \mu_B + \lambda\mu_j \quad (4)$$

Next we can get to the non-conditional variance of returns:

$$\begin{aligned} Var[X] &= E[Var[X \mid N]] + Var[E[X \mid N]] \\ &= E[\sigma_B^2 + n\,\sigma_j^2] + Var\,[\mu_B + n\,\mu_j] \\ &= \sigma_B^2 + \lambda(\sigma_j^2 + \mu_j^2) \end{aligned} \quad (5)$$

Notice that if the expected return due to jumps is equal to 0, the distribution is centred at μ_B, otherwise it is biased. Moreover, if there are jumps ($\lambda>0$), the distribution will have fatter tails. Consequently the distribution that we propose could explain the kurtosis observed in the empirical distributions of stock returns.

3. Parameter estimation

In this section we describe the procedure used to estimate the parameters of the distribution of returns. We proposed two alternative hypothesis for the distributions:

1. Normal distribution
2. Compound Poisson distribution

JORION (1988), BALL and TOROUS (1985), TRAUTMANN and BEINERT (1995) used with success the maximum likelihood method to estimate the parameters of the mixed distribution. I logically chose this method. For the normal distribution the arithmetical mean and the variance are the estimates which maximize the likelihood function. The estimation is much more complex for the compound Poisson distribution. First we have to determine the likelihood function by multiplying the densities of each possible event. The function is easier to handle if we use the logarithm of the function, because the products become sums. This transformation doesn't change the result of the maximisation. The loglikelihood function that we get is the following:

$$\ln[L] = -m\lambda - m/2\,\ln(2\pi) + \sum_{i=1}^{m} \ln\left[\sum_{n=0}^{10} \frac{\lambda^n}{n!} \frac{1}{\sqrt{\sigma_B^2 + n\sigma_j^2}} \exp\left(-\frac{(x_i - \mu_B - n\mu_j)^2}{2\sigma_B^2 + 2n\sigma_j^2}\right)\right] \quad (6)$$

Let $P = (\mu_B, \sigma_B^2, \lambda, \mu_j, \sigma_j^2)$ be the vector of parameters to estimate.

We have to solve the following system:

$$\frac{\partial \ln[L]}{\partial P} = 0 \quad (7)$$

This system does not lead to analytical solutions. So we have to solve it numerically. The Fletcher-Reeves and Newton-Raphson algorithms were used to solve the system. The program was run with several starting values, so that we are almost sure not to reach a local minimum. In order to compare both hypothesis on the return distributions, we use the likelihood ratio test. The Δ statistic is defined as follows:

$$\Delta = -2\,(\ln[L]\,(\text{brownian}) - \ln[L]\,(\text{jump-diffusion})) \quad (8)$$

This statistic follows a χ^2 distribution with 3 degrees of freedom (nb of restrictions on the parameters under the null hypothesis).

3.1 Data

The objective of the paper is to test an option valuation model. As a preliminary step, we therefore chose stocks whose options have a large volume of Transition at SOFFEX (Swiss Options and Financial Futures Exchange). These stocks are: CIBA-GEIGY registered, NESTLE registered, SBS bearer and UBS bearer. I also considered the SMI (Swiss Market Index). The price quotations are extracted from DATASTREAM between January 1st 1980 and July 1st 1994. These prices are closing prices. TRAUTMANN and BEINERT (1995) have noticed that daily returns had more significant jump parameters than weekly returns, so that we used daily quotations in this paper.[1] Instantaneous daily returns were computed according to the following formula:

$$X(t)=\ln (S(t)/S(t-1)) \quad (9)$$

In order to test for the stability of parameters I divided de chosen period in three sub periods in the following way:

1. January 1st 1980 – July 1st 1994
2. January 1st 1980 – December 31th 1984
3. January 1st 1985 – December 31th 1989
4. January 1st 1990 – July 1st 1994

Table 1 presents the results. It gives the estimations for each subperiod and each security. It also gives the statistic of the likelihood ratio, as defined earlier.

Table 1: parameter estimates on long periods

This table [2] gives first the size of the sample (m), the estimations in the case of a geometric brownian motion (μ and σ^2), the same estimations computed according to formulas (4) and (5) (μ_C and σ_C^2), the estimations of the mixed process (μ_B, σ_B^2, λ, μ_j and σ_j^2), the respective likelihoods of both processes ([L] Mixed and Brownian), the Δ statistic and finally the probability to accept the normality of returns.

Whole period

Period		Ciba	Nestle	SBS	UBS	SMI (86–94)
	m	3783	3783	3783	3783	1575
	$\mu \times 10^3$	**0.570**	**0.522**	**0.185**	**0.277**	**0.346**
	$\sigma^2 \times 10^3$	**0.177**	**0.121**	**0.147**	**0.127**	**0.104**
	$\mu_C \times 10^3$	0.555		0.167	0.275	0.346
	$\sigma_C^2 \times 10^3$	0.168		0.134	0.119	0.101
80–94	$\mu_B \times 10^3$	**0.404**		**–0.261**	**–0.003**	**0.616**
	$\sigma_B^2 \times 10^3$	**0.041**	N/A	**0.039**	**0.027**	**0.056**
	λ	**0.437**		**0.459**	**0.533**	**0.077**
	$\mu_j \times 10^3$	**0.346**		**0.933**	**0.522**	**–3.509**
	$\sigma_j^2 \times 10^3$	**0.291**		**0.205**	**0.172**	**0.574**
	[L] Mixed	11512.3		11716.1	12071.7	5157.6
	[L] Brownian	10977.5	11694.4	11321.4	11605.5	4986.4
	Δ	1069.6		789.4	932.3	342.4
	Prob(accept normality)	0		0	0	0

First subperiod

Period		Ciba	Nestle	SBS	UBS
80–84	m	1304	1304	1304	1304
	$\mu \times 10^3$	**0.523**	**0.402**	**0.132**	**0.207**
	$\sigma^2 \times 10^3$	**0.059**	**0.039**	**0.079**	**0.058**
	$\mu_C \times 10^3$	0.512	0.401	0.129	0.204
	$\sigma_C^2 \times 10^3$	0.058	0.037	0.079	0.057
	$\mu_B \times 10^3$	**–0.116**	**–0.006**	**–0.662**	**–0.246**
	$\sigma_B^2 \times 10^3$	**0.014**	**0.010**	**0.023**	**0.017**
	λ	**0.689**	**0.531**	**0.628**	**0.424**
	$\mu_j \times 10^3$	**0.911**	**0.767**	**1.260**	**1.060**
	$\sigma_j^2 \times 10^3$	**0.064**	**0.050**	**0.087**	**0.091**
	[L] Mixed	4587.4	4914.5	4380.1	4631.3
	[L] Brownian	4493.8	4763.1	4310.7	4506.4
	Δ	187.2	302.9	138.8	249.7
	Prob(accept normality)	0	0	0	0

Second subperiod

Period		Ciba	Nestle	SBS	UBS
85–89	m	1304	1304	1304	1304
	$\mu \times 10^3$	**0.821**	**0.827**	**0.153**	**0.202**
	$\sigma^2 \times 10^3$	**0.264**	**0.209**	**0.150**	**0.168**
	$\mu_C \times 10^3$	0.748	0.827	0.152	0.202
	$\sigma_C^2 \times 10^3$	0.251	0.142	0.158	0.152
	$\mu_B \times 10^3$	**1.149**	**0.861**	**0.288**	**0.172**
	$\sigma_B^2 \times 10^3$	**0.059**	**0.044**	**0.067**	**0.008**
	λ	**0.403**	**0.127**	**0.173**	**1.442**
	$\mu_j \times 10^3$	**–0.994**	**–0.267**	**–0.781**	**0.259**
	$\sigma_j^2 \times 10^3$	**0.477**	**0.770**	**0.525**	**0.099**
	[L] Mixed	3712.2	4261.3	4011.4	3958.1
	[L] Brownian	3521.3	3674.7	3891.0	3816.3
	Δ	381.9	1173.3	240.6	283.6
	Prob(accept normality)	0	0	0	0

Third subperiod

Period		Ciba	Nestle	SBS	UBS	SMI
90–94	m	1175	1175	1175	1175	1175
	$\mu \times 10^3$	**0.344**	**0.318**	**0.282**	**0.438**	**0.320**
	$\sigma^2 \times 10^3$	**0.210**	**0.114**	**0.201**	**0.157**	**0.104**
	$\mu_C \times 10^3$	0.343	0.318	0.280	0.438	0.320
	$\sigma_C^2 \times 10^3$	0.206	0.113	0.196	0.150	0.101
	$\mu_B \times 10^3$	**0.199**	**0.484**	**–0.730**	**0.441**	**0.586**
	$\sigma_B^2 \times 10^3$	**0.084**	**0.070**	**0.019**	**0.036**	**0.053**
	λ	**0.357**	**0.111**	**1.339**	**0.618**	**0.148**
	$\mu_j \times 10^3$	**0.403**	**–1.499**	**0.754**	**–0.005**	**–1.806**
	$\sigma_j^2 \times 10^3$	**0.341**	**0.392**	**0.131**	**0.186**	**0.317**
	[L] Mixed	3392.7	3742.3	3407.6	3592.3	3831.0
	[L] Brownian	3309.0	3667.5	3332.3	3479.0	3723.1
	Δ	167.3	149.6	150.6	226.5	215.8
	Prob(accept normality)	0	0	0	0	0

Before making any comment on the results, we have to be sure that the estimations[3] of the parameters of the mixed distribution are significantly better than in the case of a geometric brownian motion. The estimations have been computed in both cases with the maximum likelihood method, so that the improvement can be judged by relying on the likelihood ratio test. The Δ statistics are so high that the χ^2 indicates that we can reject at the 99.99% level the null hypothesis of normally distributed returns. We conclude that the compound Poisson distribution is a better representation of daily returns than the normal distribution. The result of the computation of the total expected return and of the total variance from the estimations of the parameters of the jump-diffusion process is given on lines four and five of table 1. These results are very close to the ones obtained while estimating the parameters of the geometric brownian. According to formulas (4) and (5), the results should be equal. The difference is very small, so that we can say that our estimations are fairly accurate. Let us add the following comments:

The level of the estimated parameters is comparable to the results of BALL and TOROUS (1985) and TRAUTMANN/BEINERT (1995) for the US and German markets respectively.

The following table indicates the proportion of variance due to jumps included in the total variance of returns:

	Ciba	Nestle	SBS	UBS	SMI
80–94	76%		71%	77%	45%
80–84	76%	74%	71%	69%	
85–89	77%	69%	58%	94%	
90–94	59%	39%	90%	76%	47%

For stocks there is only one case where the jumps don't "dominate" the variance: Nestle between 1990 and 1994. We see that the jumps have a strong influence on the global variance, but only in the case of individual stocks. In the case of the SMI, the brownian part slightly dominates. This result confirms MERTON's hypothesis, which says that the jump risk is mainly firm specific and therefore should be diversifiable.

The observation of the expected returns due to jumps is very interesting: In the 85–90 period (86–94 for the SMI) and for four securities out of five, this expected return is negative, showing that the estimation probably detected the 1987 and 1989 Crashes and reflected it through these returns.

The main observation concerns the jump intensity parameter. The values that we get are different from the intuition we can have of a jump process. The original idea was to postulate that the returns follow a geometric brownian motion subject to rare jumps due to an exceptional information. The smallest value of the λ parameter (0.08) represents 0.08×360=29 jumps pro year. This corresponds to our intuition but unfortunately the other estimations are close to 1 jump every 2 days. We even obtain more than one jump a day in two cases. This problem leads to several hypotheses:

1. Since we are close to one jump per day, we could suppose that the best process to describe the evolution of returns is a pure jump process.
2. BALL and TOROUS (1985) and TRAUTMANN/BEINERT (1995) also get results attesting that there is more than one jump per day without any specific remark. However this result is difficult to sustain: How could an estimation method detect an event which occurs more often than the number of available data? Do we use a bad estimation method which brings incorrect results. More fundamentally we can criticize the specification of the jumps. MERTON supposed that the jumps are lognormally distributed in order to get an analytical solution. This hypothesis is arbitrary and without any economical justification.
3. If we consider that a λ parameter of more than one is unrealistic, shouldn't we put a constraint in the program in order not to get values of more than one?

In order to get a better understanding of these problems, we have to estimate the parameters on shorter periods.

Table 2: Parameter estimates on one-year periods

This table gives first the size of the sample (m), the estimations in the case of a geometric brownian motion (μ and σ^2), the same estimations computed according to formulas (4) and (5) (μ_C and σ_C^2), the estimations of the mixed process (μ_B, σ_B^2, λ, μ_j and σ_j^2), the respective likelihoods of both processes ([L] Mixed and Brownian), the Δ statistic and finally the probability to accept the normality of returns.

Panel a

Period		80	81	82	83	84
Nestle registered	m	261	261	261	260	261
	$\mu \times 10^3$	**–0.321**	**–0.280**	**–1.054**	**1.048**	**0.393**
	$\sigma^2 \times 10^3$	**0.027**	**0.052**	**0.046**	**0.044**	**0.036**
	$\mu_C \times 10^3$	–0.320	–0.280	–1.054	1.048	0.393
	$\sigma_C^2 \times 10^3$	0.026	0.049	0.042	0.040	0.034
	$\mu_B \times 10^3$	**–0.641**	**–0.967**	**0.184**	**0.742**	**0.781**
		(0.000017)	**(0.000071)**	**(0.000000)**	**(0.000031)**	**(0.000026)**
	$\sigma_B^2 \times 10^3$	**0.004**	**0.018**	**0.000**	**0.008**	**0.006**
		(0.000000)	**(0.000000)**	**(0.000000)**	**(0.000000)**	**(0.000000)**
	λ	**0.853***	**0.138**	**1.761***	**0.510**	**0.660**
		(0.54)	**(0.0088)**	**(2.32)**	**(0.081)**	**(0.19)**
	$\mu_j \times 10^3$	**0.377**	**4.962**	**–0.703**	**0.598**	**–0.589**
		(0.00046)	**(0.032)**	**(0.015)**	**(0.0022)**	**(0.00096)**
	$\sigma_j^2 \times 10^3$	**0.025**	**0.200**	**0.023**	**0.063**	**0.041**
		(0.000000)	**(0.000000)**	**(0.000000)**	**(0.000000)**	**(0.000000)**
	[L] Mixed	1029.9	973.4	967.0	981.3	996.6
	[L] Brownian	1005.1	918.1	933.7	936.4	965.6
	Δ	49.6	110.7	66.7	89.8	62.1
	Prob(accept normality)	0	0	0	0	0

Panel b

Period		85	86	87	88	89
Nestle registered	m	261	261	261	261	260
	$\mu \times 10^3$	**1.554**	**–0.094**	**–0.746**	**0.995**	**0.948**
	$\sigma^2 \times 10^3$	**0.077**	**0.087**	**0.219**	**0.543**	**0.125**
	$\mu_C \times 10^3$	1.649	–0.094	–0.548	2.000	0.948
	$\sigma_C^2 \times 10^3$	0.063	0.087	0.168	0.526	0.125
	$\mu_B \times 10^3$	**1.197**	**1.520**	**0.114**	**1.255**	**0.783**
		(0.000083)	**(0.00013)**	**(0.00033)**	**(0.00023)**	**(0.00022)**
	$\sigma_B^2 \times 10^3$	**0.021**	**0.032**	**0.086**	**0.060**	**0.056**
		(0.000000)	**(0.000000)**	**(0.00000)**	**(0.000000)**	**(0.000000)**
	λ	**0.099**	**0.272**	**0.076**	**0.032**	**0.035**
		(0.008)	**(0.014)**	**(0.0047)**	**(0.0080)**	**(0.0035)**
	$\mu_j \times 10^3$	**4.538**	**–5.913**	**–8.620**	**23.166**	**4.699**
		(0.11)	**(0.010)**	**(0.28)**	**(3.7)**	**(1.8)**
	$\sigma_j^2 \times 10^3$	**0.399**	**0.165**	**0.993**	**13.961**	**1.934**
		(0.00000)	**(0.00000)**	**(0.000047)**	**(0.038)**	**(0.00011)**
	[L] Mixed	922.9	878.2	784.2	849.7	867.7
	[L] Brownian	865.4	851.0	729.9	611.4	799.9
	Δ	114.9	54.5	108.6	476.5	135.6
	Prob(accept normality)	0	0	0	0	0

Panel c

Period		90	91	92
Nestle registered	m	261	261	262
	$\mu \times 10^3$	**–0.830**	**0.794**	**1.142**
	$\sigma^2 \times 10^3$	**0.170**	**0.091**	**0.060**
	$\mu_C \times 10^3$	–0.830	0.794	1.142
	$\sigma_C^2 \times 10^3$	0.156	0.081	0.060
	$\mu_B \times 10^3$	**–0.496**	**–0.978**	**1.398**
		(0.000021)	**(0.000072)**	**(0.00019)**
	$\sigma_B^2 \times 10^3$	**0.005**	**0.017**	**0.050**
		(0.000000)	**(0.000000)**	**(0.000000)**
	λ	**1.595**	**0.585**	**0.028**
		(0.10)	**(0.083)**	**(0.00096)**
	$\mu_j \times 10^3$	**–0.209**	**3.026**	**–8.901**
		(0.0015)	**(0.0027)**	**(0.17)**
	$\sigma_j^2 \times 10^3$	**0.094**	**0.101**	**0.266**
		(0.00000)	**(0.000000)**	**(0.000033)**
	[L] Mixed	789.3	885.0	1049.3
	[L] Brownian	762.8	844.4	901.6
	Δ	53.1	81.2	295.4
	Prob(accept normality)	0	0	0

The standard errors are given in parentheses. The * indicate parameters that are not significantly different from zero (at the 1% level)

3.2 One year estimations

In order to test for the stability of the different parameters, it is interesting to estimate them on several successive periods. These estimations being time consuming, they were realised only for one stock, Nestle. These estimations have been made yearly during the 1980–1994 period. The objective is also to detect some special events like the 1987 Crash, the "Nestle effect" in 1988, the mini-Crash in 1989.

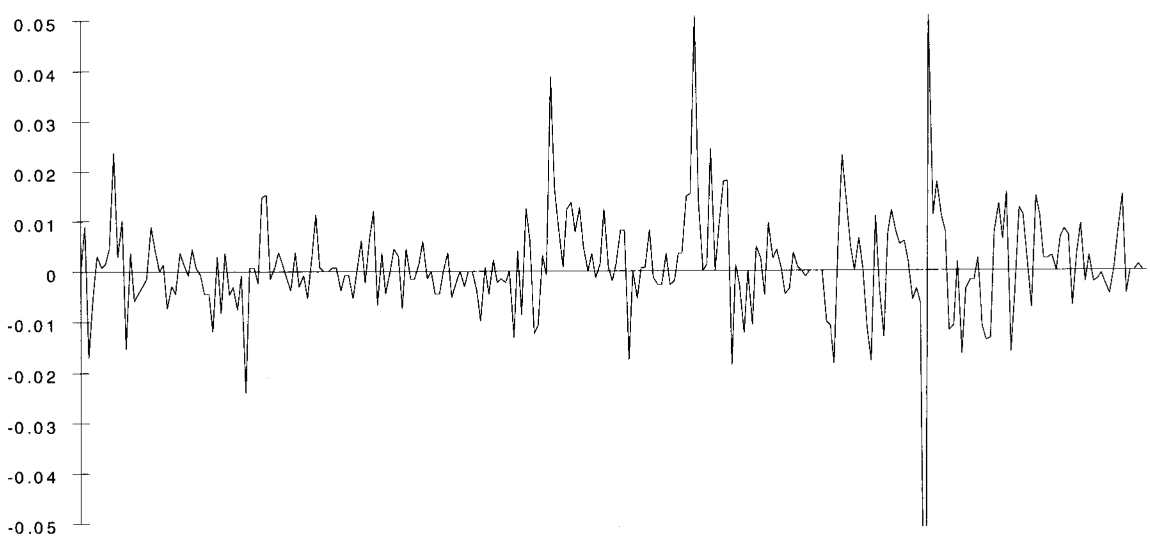
As the data samples are much smaller now, the standard errors of the estimates could be computed. They are obtained from the Hessian (second derivatives) matrix of the loglikelihood function (6) we maximized. The standard errors are the elements of the diagonal of the inverse of the Hessian evaluated for the present estimations.The results are presented in table 2.

Looking at the statistics in table 2, we observe first that the improvement brought by the jump-diffusion process is highly significant in all cases. The only problem concerns year 1982: the variance of the brownian part tends to 0. The program probably reached a local minimum; although we started with alternative values, the result has always been the same. Consequently, year 1982 will not be taken into account for further comments.

Secondly the standard errors of the estimates are very small. Only two parameters are not significantly different from 0 at the 1% level: λ during 1980 and 1982.

As far as the stability of the different parameters is concerned, we first notice that the λ parameter is very unstable. Its values range from 0.02 to 1.59. This fact goes against the specifications of the model, which say that the parameters are constant through time. The trend parameters are also unstable. The values that we get for the mean jump amplitude are very interesting. The highest value occurs in 1988, when Nestle opened its capital to foreigners. On November 18th 1988 the Nestle nominative stock progressed by 35%. This jump had a main influence on the 1988 estimates. We also notice that the mean jump amplitude of 1987 is negative, confirming the presence of the Crash. Surprisingly, this amplitude is positive in 1989, even if a Crash took place. This positive jump amplitude can be understood by looking at the figure 2, which indicates that several positive jumps occurred before and after the Crash itself.

Figure 2: Nestle returns in 1989



The influence of the different events on the estimates can clearly be seen on the parameters of the variance of jumps. Figure 3 shows the evolution of the variances during the observed period. We notice from figure 3 that the Nestle effect had an normous influence on the global variance of the process. The 1987 and 1989 Crashes had an influence, but a much smaller one. This global variance curve shows that the BLACK-SCHOLES assumption regarding the constancy of the variance is not verified if exceptional events are likely to occur. The "jump variance" curve shows that the jump variance parameter has clearly detected the jump induced by the Nestle effect, because of its spectacular amplitude. The jump variance is lower in the case of the 1987 Crash, because more jumps of lower amplitude occurred, which have a smaller influence on the variance; Indeed, the jump variance is strongly influenced by big jumps because of the influence of the squared mean jump amplitude. We notice from the "brownian variance" that the variance of the diffusion part is also high during "big jumps" periods, but not a lot. This means that jumps explain a great part of the instabilities of variance. The last part could be explained by other element other elements should explain it: stochastic volatility or (G)ARCH as alternative modelling of the diffusion part of the process may be considered. JORION (1988) noticed that even when taking on ARCH (1) specification into account, there are still discontinuities in the data. We conclude that a better modelisation could be a combination between a jump model and a GARCH or stochastic volatility model, like the model developed by BATES (1993).

The relationship between the mean number of jumps per day and their mean amplitude (in absolute value) is an interesting fact to study. Even if the mean jump amplitude is reduced by the presence of up and down jumps, we notice on figure 4 that an inverse relationship seems to exist between these two parameters.

Figure 3: This figure shows the decomposition of the variance in its jump and brownian components

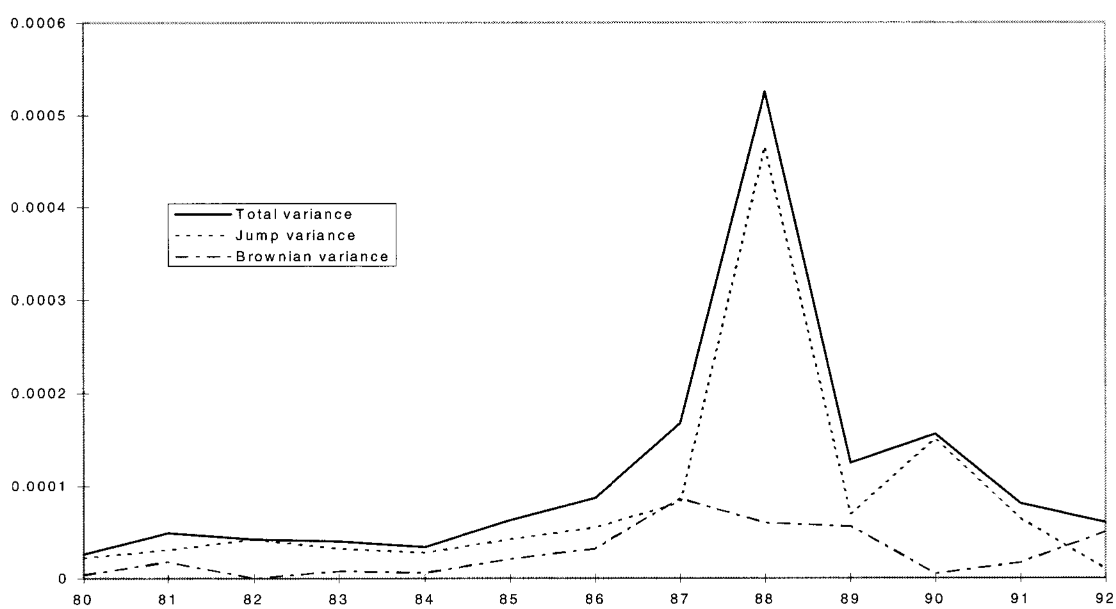
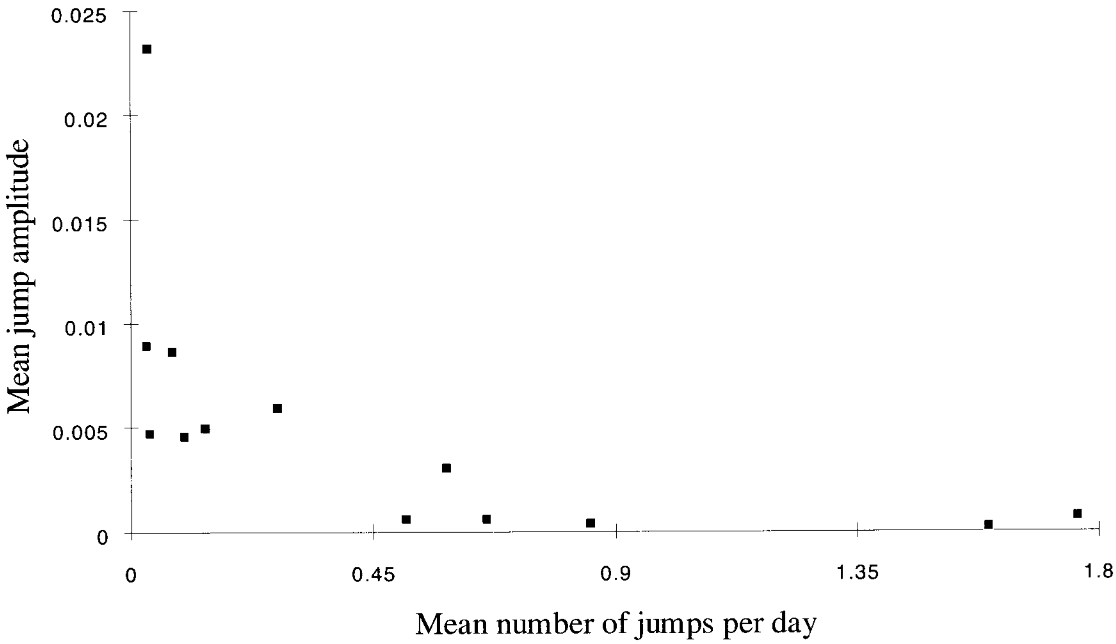


Figure 4: Relationship between the absolute value of the mean jump amplitude and the mean number of jumps per day in the case of the Nestle stock.



This leads to the following conclusion: when there are large jumps, the estimation induces a small λ parameter, because these jumps are easy to distinguish from usual returns. On the other hand, when there are many jumps of small amplitude, the program cannot distinguish a jump from a normal return. Finally, when there is no distinction between a jump and a normal return (there are only jumps or only normal returns), the estimation leads to unrealistic cases where λ gets values of more than one. Consequently, during years when exceptional events took place, the λ parameter is very small. Contrarily, during quiet years we get a high λ parameter. In this case this parameter acts as an indicator of the presence of significant jumps in the returns. This phenomenon is due to the fact that the estimation procedure doesn't know that only "exceptional" returns should be considered as jumps. If it doesn't find exceptional returns, it considers smaller returns as jumps, which explains the high λ and the small mean jump amplitude. The specified process is consequently not perfect to detect jumps. We could even conclude that if λ is close to 1, there are no significant jumps. Other authors had also found such results but didn't comment on their interpretation.

3.3 Data for option valuation

In this section, we present the choice of the study period for option valuation. In order to get meaningful market prices, the essential criterion must be the volume of transactions. Moreover, it is useful to have relatively long series. A third criterion is to choose recent prices, because the Swiss option market is relatively young and therefore the number of transactions still grows with time. Calls expiring in January 1994 were chosen in order to meet all the criteria. All the prices expiring in

January 1994 have been taken into account[4], providing the volumes were not too small. The choice of January is due to the fact that no dividends are distributed during the 6 months preceding it. This allows us to compute the prices of stock options like European ones even if they are American.

The prices and volumes are extracted from DATASTREAM, as well as the prices of underlyings and the interest rates. The 1, 3 and 6 months daily London EUROFRS were chosen and used in the following way:

1. 6 month rate : computations of option prices for July and August 1993.
2. 3 month rate : computations of option prices for September, October and November 1993.
3. 1 month rate : computations of option prices for December 1993 and January 1994.

3.4 Parameter estimation for option valuation purposes

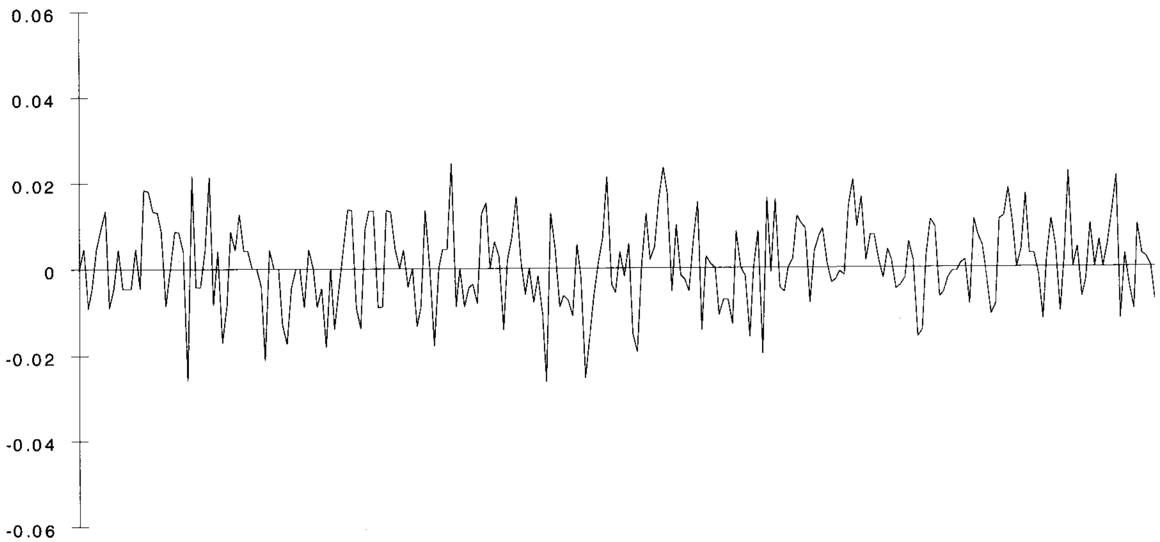
We now turn to the estimation of the parameters which will be used for option valuation. The results obtained over the 5 years periods are not applicable since we cannot accept that these parameters are the same in 1994 than in 1990. However, a fairly large number of observations is necessary to get significant parameters for the jump-diffusion process. Consequently, I used a one year period to estimate the parameters of both processes. As the objective of this paper is to test valuation models and not to predict prices, the model prices are computed within the sample. The estimations are made on the 250 returns preceding the common expiration date of the studied options. The estimations are based on the same methodology as previously described. The results are the presented in table 3.

Table 3: Parameter estimates for option valuation purposes

This table gives first the size of the sample (m), the estimations in the case of a geometric brownian motion (μ and σ^2), the same estimations computed according to formulas (4) and (5) (μ_C and σ_C^2), the estimations of the mixed process (μ_B, σ_B^2, λ, μ_j and σ_j^2), the respective likelihoods of both processes ([L] Mixed and Brownian), the Δ statistic and finally the probability to accept the normality of returns.

Period		Ciba	Nestle	SBS	UBS	SMI
	m	250	250	250	250	250
	$\mu \times 10^3$	**1.605**	**0.919**	**1.765**	**1.932**	**1.442**
	$\sigma^2 \times 10^3$	**0.124**	**0.094**	**0.160**	**0.081**	**0.053**
	$\mu_C \times 10^3$	1.605	0.919	1.765	1.935	1.442
	$\sigma_C^2 \times 10^3$	0.124	0.094	0.161	0.079	0.053
93	$\mu_B \times 10^3$	**–0.937**	**5.329**	**0.172**	**1.499**	**2.781**
		(0.000071)	**(0.00052)**	**(0.00027)**	**(0.000083)**	**(0.00015)**
	$\sigma_B^2 \times 10^3$	**0.015**	**0.079**	**0.058**	**0.019**	**0.033**
		(0.000000)	**(0.00000)**	**(0.00000)**	**0.000000)**	**0.000000)**
	λ	**2.708**	**1.601**	**0.811**	**0.732**	**0.265**
		(0.072)	**(0.022)**	**(0.082)**	**(0.18)**	**(0.0056)**
	$\mu_B \times 10^3$	**0.938**	**–2.754**	**1.963**	**0.595**	**–5.033**
		(0.00015)	**(0.000087)**	**(0.0016)**	**(0.0015)**	**(0.0024)**
	$\sigma_j^2 \times 10^3$	**0.040**	**0.002**	**0.123**	**0.082**	**0.048**
		(0.000000)	**(0.000000)**	**(0.000000)**	**(0.00000)**	**(0.00000)**
	[L] Mixed	772.7	804.3	743.8	839.8	883.4
	[L] Brownian	769.6	803.8	737.9	822.4	876.8
	Δ	6.2	1.1	11.8	34.8	13.2
	Prob(accept normality)	0.10	*0.76	0.01	0.00	0.00

Figure 5: Nestle returns in 1993



We can make the following comments on the estimates: the reduction of the sample has induced a loss of significance. Nevertheless, four securities out of five lead us to consider the improvement of the jump process as significant at the 10% level. The Nestle stock was very problematic because the program found a local maximum. Alternative starting values allowed us to get the above result, but it could be another local maximum. This problem leads us to be careful for further comments on the Nestle results. Notice that the improvement are very significant for the SBS, UBS and SMI underlyings.

The standard errors of estimates are all very small. All the parameters are significantly different from 0 at the 1% level.

It is interesting to have a look at the proportion of the variance due to jumps:

	Ciba	Nestle	SBS	UBS	SMI
93	88%	16%	64%	76%	37%

If we exclude the surprising result of Nestle for the above reasons, we see that the jump part of the variance is dominant in the case of stocks and dominated in the case of the index. This confirms the results obtained on longer estimation periods.

Unfortunately the problem of the high jump intensity persists. Some values are bigger than one, some are close to one. The SMI is the only security which displays a λ with a reasonable value. This confirms MERTON's hypothesis on the diversifiability of jumps, although the jumps seem to be only partly diversifiable. However, these estimations remain far away from our intuition of a mixed jump process.

When we estimated the parameters yearly for the Nestle stock, the results were usually plausible for the value of the λ parameter. In 1993, we notice that the λ parameter is very high (2,7!). Why do we obtain such a difference? A partial answer can be given through the observation of the Nestle returns in 1993, which are displayed in figure 5.

If we compare this graph with the 1989 one, we understand that it is very difficult to determine what a jump is in the 1993 case. The question is then: Are there no jumps or only jumps? This graph allows us to understand that the estimation program looses itself while looking for exceptional returns.

3.5 One year parameter estimation with constraints

Since we found some unrealistic figures for the mean number of jumps parameter, we decided to constrain this parameter in order to find more intuitive results. It is evident that the fact of adding a constraint lowers the significance, but the results are nevertheless interesting.
The first idea was to constraint the λ parameter so that it remains under 1. For the Ciba stock, the program has simply reached the constraint. This result is as unrealistic as the earlier one, and thus not worth pursuing.
The second idea was to set a much stronger constraint, in order to get the intuitive results of a small number of jumps. The constraint was arbitrarily fixed to 0,01, that is to say 3,6 jumps per year. The program reached the constraint again but the variance of the jumps was equal to zero. The estimations seemed to take the direction of a local maximum characterized by a variance of jumps of zero. Then the program was started again with the last estimations but *without any constraint* and reached the expected local maximum. The same thing happened when the above methodology was applied to the other securities. Since the scenario seems to be systematic, it is interesting to explain what happens: we are back to an unconstrained problem with a changed assumption. the size of the jumps is not lognormal anymore but constant. Table 4 shows the results under this new assumption.

Table 4: Constrained parameter estimates for option valuation purposes

This table gives first the size of the sample (m), the estimations in the case of a geometric brownian motion (μ and σ^2), the same estimations computed according to formulas (4) and (5) (μ_C and σ_C^2), the estimations of the mixed process (μ_B, σ_B^2, λ, μ_j and σ_j^2), the respective likelihoods of both processes ([L] Mixed and Brownian), the Δ statistic and finally the probability to accept the normality of returns.

Period		Ciba	Nestle	SBS	UBS	SMI
93	m	250	250	250	250	250
	$\mu \times 10^3$	**1.605**	**0.919**	**1.765**	**1.932**	**1.442**
	$\sigma^2 \times 10^3$	**0.124**	**0.094**	**0.160**	**0.081**	**0.053**
	$\mu_C \times 10^3$	1.605	0.919	1.765	1.935	1.442
	$\sigma_C^2 \times 10^3$	0.124	0.094	0.161	0.079	0.053
	$\mu_B \times 10^3$	**2.009**	**1.381**	**2.087**	**2.410**	**2.294**
	$\sigma_B^2 \times 10^3$	**0.112**	**0.089**	**0.149**	**0.068**	**0.039**
	λ	**0.014**	**0.041**	**0.009**	**0.017**	**0.050**
	$\mu_j \times 10^3$	**–28.048**	**–10.555**	**–33.556**	**–24.955**	**–16.864**
	$\sigma_j^2 \times 10^3$	**0**	**0**	**0**	**0**	**0**
	[L] Mixed	771.1	804.3	738.8	827.3	883.7
	[L] Brownian	769.6	803.7	737.8	822.3	876.8
	Δ	2.9	1.1	1.9	9.9	13.8
	Prob(accept normality)	0.23	0.57	0.38	0.01	0.00

The significance of the improvement relative to the geometric brownian motion has been lowered, as expected. However, the UBS and SMI securities' parameters are very significant. Since the likelihood is higher than in the case of the brownian, it is interesting to analyse the estimations in greater detail.
Notice that having a jump variance of zero doesn't mean that they have no influence on the global variance of the process. According to formula (5), the variance due to jumps is $\lambda\mu_j^2$ if the jump variance is zero. It is interesting to compute again the participation of the jumps in the global variance:

	Ciba	Nestle	SBS	UBS	SMI
93	10%	5%	8%	14%	27%

There are huge differences with the results obtained with the previous assumption. In this case, the variance of the diffusion is strongly dominant. This result is not surprising because our constraint had the following effect: only the really exceptional returns are considered as jumps. The interest of the result is that we really estimate parameters of a geometric brownian motion which is subject to rare jumps of a high amplitude. The high λ parameter that we had when estimating without constraint are due to the fact that the normal distribution is not a good representation of the "normal" returns (which are not jumps). When the mixed distribution is introduced, the method will "choose" between the normal and the compound Poisson distribution. The high λ parameters are due to the fact that the method has chosen the compound Poisson process to be the best representation. The constraint introduces a kind of bias for the normal distribution by limiting the number of jumps. This new representation is closer to intuition but it remains less significative than the unconstrained representation. Another question is whether the chosen Poisson distribution is a good way of modelling stock returns jumps. It is surprising that the variance due to jumps is bigger for the SMI than for individual stocks. This can be explained by the fact that each stock is subject to a small number of non diversifiable jumps. Moreover, the studied stocks have a large weight in the index, so that each jump is reflected as a jump in the index. The high λ parameter of the SMI confirms this fact. The conclusion is that in a very concentrated market like the Swiss one, MERTON's hypothesis on the diversifiability of jumps is not verified.The crucial point underlying these results is the fact that the assumption on the constancy of the size of the jumps is very restrictive. But is it really more restrictive than an arbitrary assumption of lognormality?

4. Correlation between jumps in stocks and jumps in the index returns

The objective of this section is to provide another test of MERTON's hypothesis on the diversifiability of jumps in the market index. This hypothesis was already tested when we showed that the index returns presented a significant jump component. A way to add insight into the test of this hypothesis is to compute the correlation between the jumps of the market and jumps of individual stocks.
In order to be able to compute the correlations, we first have to extract the jumps from the series of returns. The methodology is the following: if the λ parameter tells us that there are n jumps per year, we extract the n highest returns from the series. For each jump extracted from the SMI series, we take the corresponding return of the considered stock. Identically, for each jump in a stock, we consider the corresponding return of the SMI. Then we can directly compute the correlation between the two series. The computations have been made for 1993 and for both methods (with and without constraint). When the λ parameter was above 1, the correlation is simply the correlation of the returns. The results are the following:

Correlations between jumps (unconstrained method)

	Ciba	Nestle	SBS	UBS	SMI
93	69%	71%	66%	71%	100%

Correlations between jumps (constrained method)

	Ciba	Nestle	SBS	UBS	SMI
93	90%	82%	81%	84%	100%

These correlations seem to be very high for each firm. The levels show clearly that the hypothesis saying that the jumps in the market returns and the jumps in the individual stock returns are not correlated is not verified in Switzerland. This result is not surprising if we observe the weights of the studied stocks in the SMI (July 93):

	Ciba	Nestle	SBS	UBS	Total
93	8.2%	21.7%	5.9%	12.6%	48.4%

The weight of each stock is so high that a jump will obviously have an influence on the index. MERTON's hypothesis first says that firm specific informations induce jumps. This part of the hypothesis could be rejected because the main jumps that we noticed (Crash, Nestle effect) were clearly not firm specific. Moreover, even if the jumps were firm specific, they would induce jumps in the market because of the high weights of the considered stocks in the index. Notice that these comments are specific to the Swiss market. In a less concentrated market, the firm specific jumps would certainly be more diversifiable.
We notice some differences between correlations computed with or without the constraint. This is due to the fact that the unconstrained correlations are almost simple correlations of returns, since almost every return is considered as a jump. The constrained correlations are really computed on exceptional returns, which seem to be highly correlated with the market. This confirms the fact that the jumps are not only firm specific on the Swiss market.

The global conclusion of this section thus confirms from a correlation analysis perspective the statistically significant component in the SMI returns already identified in section 3. Both results thus justify the development of a model accounting for "priced" jump risk, like the one developed by AHN (1992).

5. Option prices computation

In this section, the theoretical option prices are computed. The prices are computed according to the BLACK-SCHOLES and MERTON formulas, which are given in the appendix. The objective of the chapter is to test if one of the jump formulas is better suited than the BLACK-SCHOLES one for the purpose of pricing options on the Swiss market. The computations are made according to the formulas given in the former section.
The option prices are computed directly with daily parameters. For example an option expiring in 20 calendar days has a τ parameter of 20. This corresponds to the hypothesis that the information is flowing continuously and not only during trading hours.
The prices were computed in the constrained and unconstrained case. Then they were compared to market prices. The comparisons were made through the computation of the mean and standard deviation of the relative errors. This criterion is justified by the fact that it gives the sign of the bias. As this computation reduces the errors, the operation was repeated with the absolute values of relative errors.

$$\text{Relative error} = \frac{\text{Market price} - \text{Theoretical price}}{\text{Market price}}$$

The computations were not made when the options did not been trade and when the time to expiration was under 5 days, in which case prices are very close to the intrinsec value of the option and volatility is subject to expiration effects. The

Table 5a: Relative errors between model and market prices

		Ciba		Nestle		SBS		UBS		SMI	
		Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
Unconstrained estimations	MERTON	9.7%	11.9%	5.8%	7.0%	7.1%	11.0%	29.1%	10.2%	12.7%	8.6%
	BS	9.7%	12.0%	5.6%	7.0%	**7.7%**	11.3%	**28.1%**	10.0%	12.8%	8.5%
Constrained estimations	MERTON	9.9%	12.2%	5.7%	7.0%	7.9%	11.5%	29.4%	10.6%	12.6%	8.6%
	BS	9.7%	12.0%	5.6%	7.0%	7.7%	11.3%	**28.1%**	10.0%	12.8%	8.5%

The fat values indicate when a model shows a mean error which differs by more then 0.3% from the others (relative to the market prices).

Table 5b: Absolute values of relative errors between model and market prices

		Ciba		Nestle		SBS		UBS		SMI	
		Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
Unconstrained	MERTON	10.8%	10.9%	6.7%	6.1%	9.5%	9.0%	29.1%	10.2%	12.7%	8.6%
estimations	BS	10.9%	11.0%	6.5%	6.0%	**9.9%**	9.5%	**28.1%**	10.0%	12.8%	8.5%
Constrained	MERTON	11.0%	11.2%	6.6%	6.1%	10.0%	9.7%	29.4%	10.6%	12.6%	8.6%
estimations	BS	10.9%	11.0%	6.5%	6.0%	9.9%	9.5%	**28.1%**	10.0%	12.8%	8.5%

The fat values indicate when a model shows a mean error which differs by more then 0.3% from the others (relative to the market prices).

implied volatilities were extracted from the BLACK and SCHOLES formula.
Table 5a, b presents the means and standard deviations of the relative errors for the aggregate sample of option prices written on the five securities. Both formulas give quasi identical results. This is consistent with the results obtained by TRAUTMANN and BEINERT (1995) who mention that the BLACK-SCHOLES and MERTON formulas differ significantly only if the λ parameter is small and the variance is highly influenced by jumps. Recall that our unconstrained estimations gave a very high λ and that our constrained estimations gave a low influence of jumps in the variance. This explains that we have only small differences.

There are small differences between the two kinds of errors. This indicates that we are systematically underestimating the market prices. This problem is essentially due to the fact that our estimations were made on relatively long periods and that we considered them as constant. It is known that agents work on a day to day basis and use implied volatilities, which departs from the hypothesis in our models. Maybe we should rely on a different estimation technique, since the historical estimations that we ran prove to be quite unsatisfactory. The high standard deviations of the errors indicate that the underestimation is not the only problem. This means that the real volatilities could be stochastic and thus not caught by our models.

6. Comparison and analysis of theoretical prices

After having observed that the market prices are systematically underestimated, this first part compares theoretical prices between them. We keep the real data to compare the model prices in a more detailed way. Even if we noticed that the models seemed equivalent when they are used to price options (on the whole sample), it is interesting to study the differences between models as a function of some option specific parameters. We define the following relative error:

$$\text{Relative error} = \frac{\text{Merton price} - \text{Black and Scholes price}}{\text{Merton price}}$$

We also define the rate of "in-the-moneyness" as the ratio between the stock price and the exercise price. Figures 6 to 8 present the relative errors as a function of this rate (for Ciba, Nestle and SMI):

Figure 6 : Relative error as a function of the rate of "in-the-moneyness"

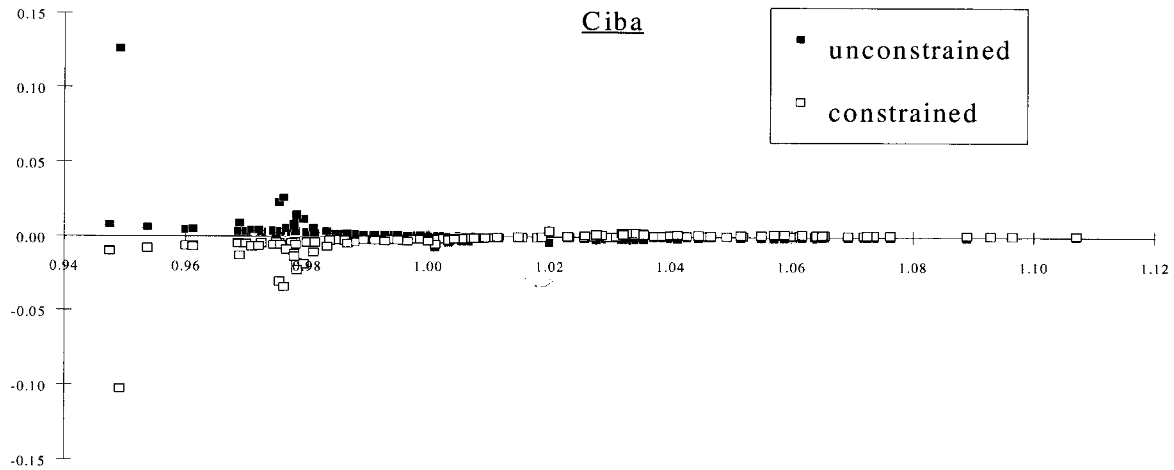


Figure 7: Relative error as a function of the rate of "in-the-moneyness"

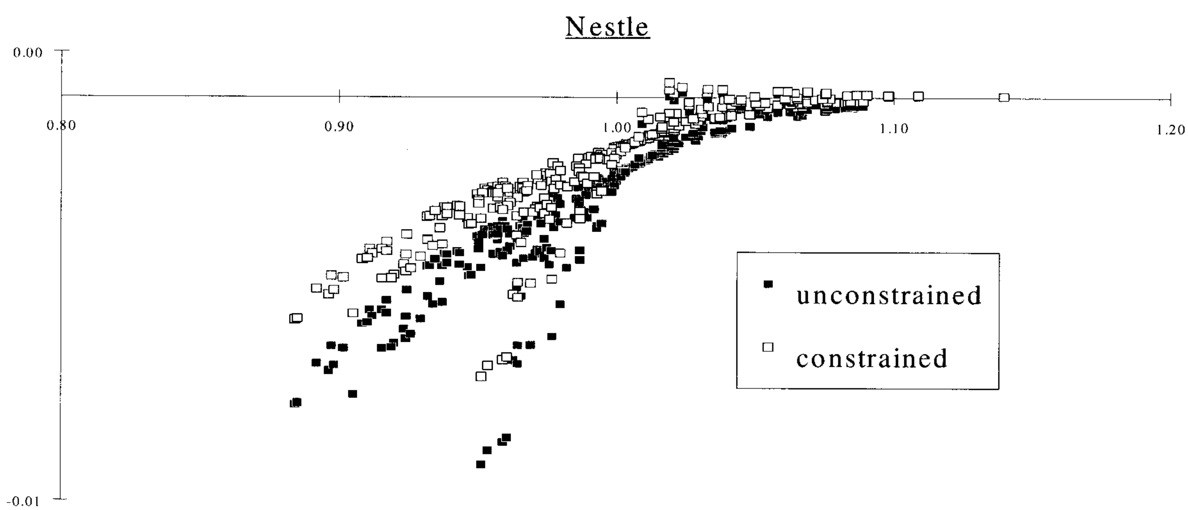
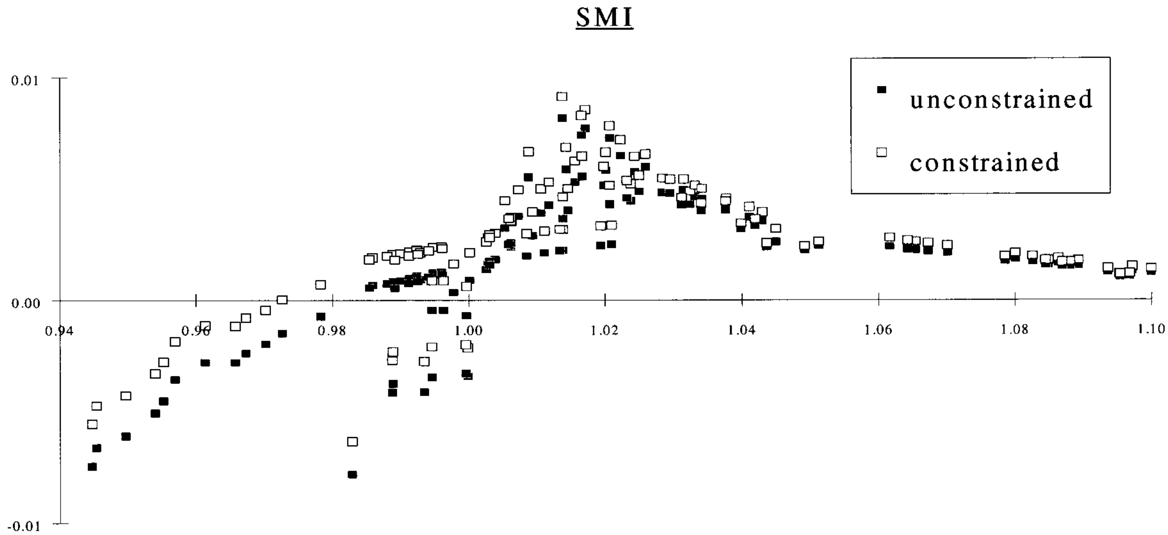


Figure 8: Relative error as a function of the rate of "in-the-moneyness"



By looking at these graphs, we observe that there are small differences between MERTON and BLACK-SCHOLES formulas for in- and at-the-money options. On the opposite, a higher difference can be detected for out-of-the-money options.

The mean jump amplitude has a strong effect. When it is positive, an out-of-the-money options can become in-the-money if a jump occurs. This explains why MERTON gives higher prices than BLACK and SCHOLES (Ciba). The inverse effect can be observed for constrained prices in particular, whose expected jumps are systematically negative.

The SMI graph seems to be different from the others. In fact, it is more detailed for a reason of scale. However, we notice that the curves have similar shapes as the others but they are located higher on the graph. As this phenomenon is common to the constrained and unconstrained result, it should depend on a common parameter. This parameter is the total variance, which is logically lower for the index. As the influence of a change in volatility is stronger on the BLACK-SCHOLES formula than on MERTON's, this explains why MERTON's formula gives higher results in the case of the SMI.

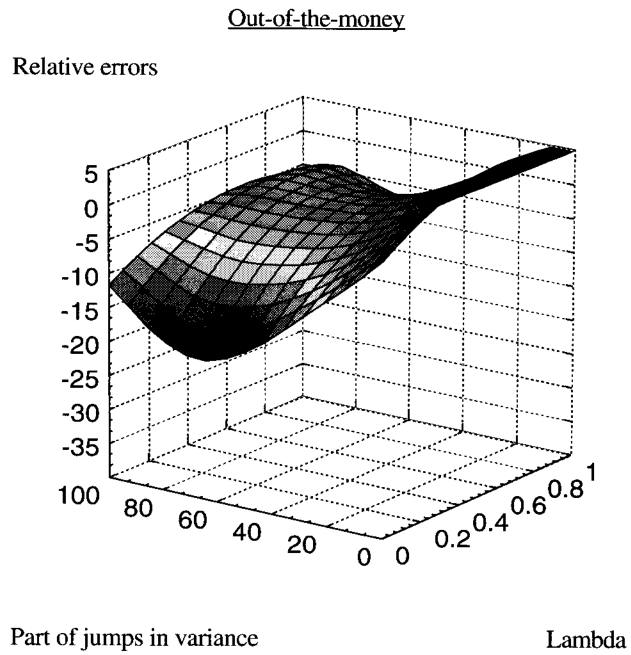
A U shape can be noticed on all graphs (inversed if the mean jump amplitude is negative). This was already noticed by MERTON (1976) and confirmed by TRAUTMANN/BEINERT (1995).

To summarize, the difference is very small between MERTON and BLACK-SCHOLES results. MERTON's model doesn't bring any noticeable improvement to the valuation of the options considered. It could be interesting to determine for which parameters the latter formula provides significantly different results.

Comparison between BLACK-SCHOLES and MERTON' models

In this section, the idea is to compare BLACK-SCHOLES and MERTON prices while changing the parameters. The varying parameters are the mean number of jumps per day and the part of the variance induced by jumps. The other parameters

Figure 9: Relative difference between Black-Scholes and Merton's models for out-of-the-money options



are chosen arbitrarily but the values are kept at a constant, empirically observed, level.

1. Strike price: 1500
2. Lifetime: 30 days
3. Interest rate: 4%
4. Variance of the brownian part: 0.0001 (per day)
5. Variance of the jump amplitude: 0.00005

The computation were made for three different values of the security: 1400 (out-of-the-money), 1500 (at-the-money) and 1600 (in-the-money). Figures 9 to 11 present the relative errors between MERTON and BLACK-SCHOLES as a function of λ (lambda) and the part of the jumps in the total variance (part of jumps in variance).

First we notice on all graphs that the highest price differences occur for out-of-the-money options. They can reach 20%. This confirms the conclusion based on the graphs of the previous section. Moreover, it confirms the analysis made by JORION (1988) who obtains the same result on US stocks and exchange rates.

Looking at figure 9, we notice that the jump participation has a small influence on the differences, provided this participation is not too small. On the opposite, the differences depend strongly on lambda, confirming BALL/TOROUS (1985) and TRAUTMANN/BEINERT (1995) results.

figures 10 and 11 illustrate a different situation. In the case of at- and in-the-money options, the differences are only high if the jump part of the variance is very large. The lambda parameter has also an influence, but a much smaller one.

MERTON's models brings significantly different results than BLACK-SCHOLES only if returns are subject to rare jumps of a high magnitude. But these big jumps don't occur every year. In 1988, for the Nestle stock, the λ parameter is small (0,032) and the jump part of the variance is about 90%. Unfortunately, this case is unique and suggests that it is not correct to estimate parameters over short periods. The jump risk is always present and the parameters are highly dependant on

Figure 10: Relative difference between Black-Scholes and Merton's models for at-the-money options

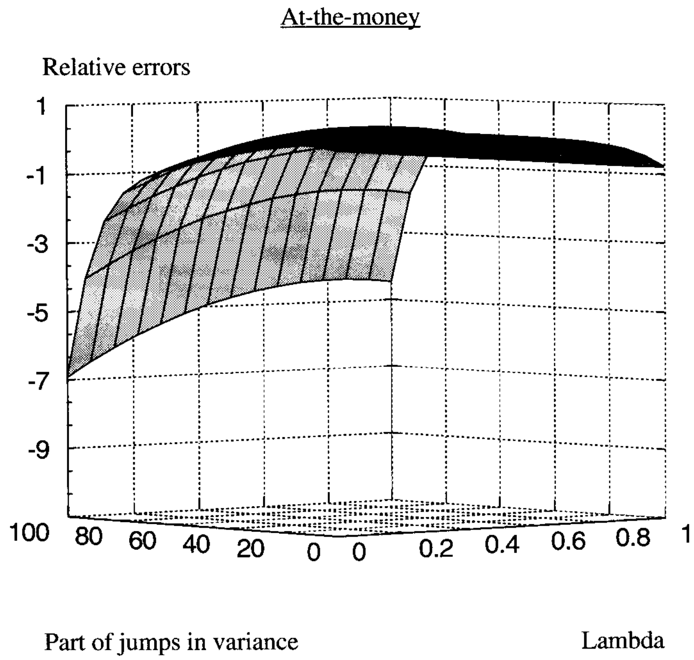
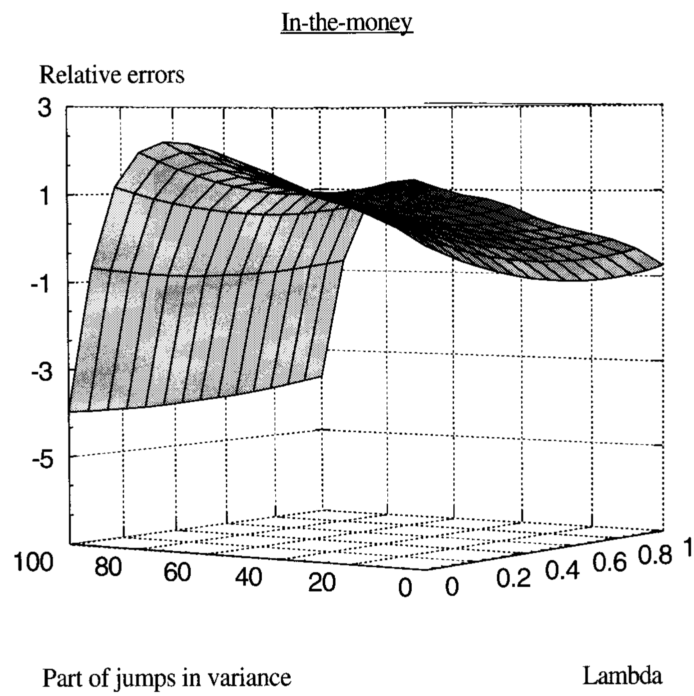


Figure 11: Relative difference between Black-Scholes and Merton's models for in-the-money options



the chosen estimation period: if there are jumps in the estimation period, the estimates don't take the jump risk into account, which is inccorrect.

7. Conclusion

In this study, we have first shown that the presence of jumps in Swiss stock returns is verified. The likelihood ratio test shows that we reject in almost all cases the null hypothesis of normally distributed returns. Hence, it is more likely that returns follow a compound Poisson distribution than a normal one. When it was possible to compute them, the standard errors of the estimates revealed that its parameters were highly significant.

The estimations led sometimes to surprising results. In some cases, we obtained more than one jump per day, which is contradictory given that we used daily data. We conclude that the chosen process is not really adapted. The problem could be induced by the hypothesis on the lognormality of the jumps' size.

Another point is the fact that the instability of the variance is not completely explained by jumps. The evolution of the brownian part of the variance shows that it remains unstable in the periods when jumps occurred. Thus it is possible that stochastic volatility or GARCH phenomena also influence the variance. This leads us to question the relevance of a constant variance parameter. The jump process is a better model than the geometric Brownian motion, but it seems that a stochastic characterisation of the diffusion volatility is also required. A combination between a jump and a GARCH or a stochastic volatility model could produce a good solution.

An interesting relationship was also noticed between the mean number of jumps per period and their mean amplitude: when the jump frequency is high (close to 1 jump per day), the mean amplitude (in absolute value) is small. Hence, a significant and greater than one value of the λ parameter doesn't mean that high amplitude jumps occurred. On the opposite, it means that the estimation method perceived normal returns as jumps. Other authors also obtained high λ parameters but they did not investigate the causes behind such estimates. It could be interesting for further researchto investigate these high λ parameters with the use of intra-daily data.

A computation of the correlations between jumps in individual stocks and those in the stock market index returns showed that jumps are not fully diversifiable. This means that jumps have a systematic risk component which should be priced by the market.

The application of BLACK-SCHOLES' and MERTON's option pricing models to the stock and stock index calls is quite disappointing but in line with the conclusions of BALL and TOROUS (1985): all models perform almost equivalently.

An illustration showed that if the jump part of the variance is really important, MERTON and BLACK-SCHOLES formulas bring different results. They also give different results if the jump frequency is small, but this influence seems to be lower.

The Swiss markets presents significant jumps but they have a small influence on option valuation. This doesn't mean that we should forget them; on the opposite, we have to remember that such events can happen and that they can happen at any time.

A further extension suggested by this study is in determining a better characterisation of the stock's jump amplitude. Indeed, all the literature has so far assumed that it is lognormally distributed, a hypothesis which is obviously not warranted on the Swiss stock market.

Appendix: Option valuation models

BLACK-SCHOLES option valuation model

The price of a call option is given by:

$$C(S,\tau,K,\sigma^2, r) = S\,\Phi(d1) - Ke^{-r\tau}\,\Phi(d2) \qquad \text{(A1)}$$

with:

$$d1 = \frac{\log(\frac{S}{K}) + (r + \frac{\sigma^2}{2})\tau}{\sqrt{\sigma^2\tau}}$$

$$d2 = d1 - \sqrt{\sigma^2\tau}$$

MERTON's option valuation model

MERTON's model is based on a mixed jump-diffusion process. The basic hypothesis are the following:

1. Frictionless markets: no transaction costs, continuous trading, short saling allowed, equal rate for lenders and borrowers.
2. Constant interest rate.
3. No dividends are distributed during the whole life of the option.
4. The stock return dynamics are characterized by a mixed jump-diffusion process: $dS/S = (\mu - \lambda k)\,dt + \sigma\,dz + dq$

Like BLACK and SCHOLES, MERTON tries to create a riskless portfolio. The risk induced by the diffusion can be eliminated in this way but the jump risk cannot. With the hypothesis on diversifiability of individual stock jumps, the portfolio is like a zero-beta security whose return is equal to the riskless rate.

MERTON finds an analytical solution for the jump-diffusion European call option pricing model in the case where jumps are lognormally distributed:

$$C^{MERTON} = \sum_{n=0}^{\infty} \frac{e^{-\lambda'\tau}(\lambda'\tau)^n}{n!}\,[C(S,\tau,K,\sigma_n^2,r_n)] \qquad \text{(A2)}$$

with:

$$\lambda' = \lambda\,(1+k) = \lambda\; e^{\mu_j + \frac{1}{2}\sigma_j^2}$$

$$\sigma_n^2 = \sigma_B^2 + n\,\sigma_j^2/\tau$$

$$r_n = r - \lambda k + n\,(\ln(1+k))/\tau$$

This solution is a weighted sum of BLACK-SCHOLES terms with modified parameters. We can also get European put prices through the put-call parity.

Notes

[1] Notice that the DATASTREAM quotes are adjusted for operations like splits or capital raising but not for dividends. So the data had to be modified at each dividend date to supress the anticipated jumps that dividends induce. Dividend series can be obtained from DATASTREAM.

[2] The estimation of the different parameters requires time consuming computations. Because of database problems I couldn't get results for the Nestle stock over the whole period. I tried several starting values, but the program never converged to satisfactory solutions. For the SMI index, the estimations only start in 1986 because of the availability of the data on DATASTREAM.

[3] The standard errors of the estimates are unfortunately not available for the above estimations, because the number of observations is very high. The computations required in order to get these standard errors exceeded the capacity of the informatic resources available at the University of Lausanne. The standard errors will only be available when the estimations are made on shorter periods.

[4] The choice of a fixed expiry date is subject to critics but it is necessary because the estimations and price computations are long and complex operations. This choice allows us to estimate the parameters only once for all the prices we have to compute.

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