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Economic Cost of Surplus Insurance: An Application of the Margrabe Model

1. Introduction

This paper provides estimates of the economic value of a service which shall be called "surplus insurance". In most general terms, "surplus" can be defined as the difference between the market value of assets and the present value of liabilities. "Surplus insurance" means that a financial intermediary (a bank, an exchange, an insurance company, or a financial conglomerate) provides full insurance against a negative surplus [1] for a specific time period. In this paper, the market value of assets and the present value of liabilities are both assumed to be stochastic. The economic value of surplus insurance can be determined by an option pricing model developed by MARGRABE (1978). Surplus insurance has many important applications: deposit insurance, pensions benefit protection, and benchmark insurance in asset management. Section 2 develops the pricing model and gives illustrative values of surplus insurance, and Section 3 discusses some applications.

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2. The economic value of surplus insurance

Let A denote t e market value of assets and L the present value of liabilities. The surplus is then defined by S = A-L; sometimes in the subsequent text, an alternative measure called "funding ratio" will be used which is defined by F = A/L. Surplus insurance as defined in the introduction implies that an intermediary is willing to pay an amount equal to the surplus shortfall in case that A-L is negative; the payoff provided by the intermediary is therefore

$$\min[0, A - L] = \max[0, L - A] \qquad (1)$$

This is equivalent to the payoff of an option to exchange asset A for "asset" L. This option is exercised if the market value of L exceeds the market value of A, i.e. if the surplus is negative. MARGRABE (1978) provides a formula to determine the value of exchange options, W. The expression is given by

$$W = LN[z_1] - AN[z_1] \qquad (2)$$

where

$$z_1 = \frac{\ln\left(\frac{L}{A}\right) + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}} = \frac{\ln\left(\frac{1}{F}\right) + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}$$

Table 1: Cost of Surplus Insurance with Different Asset-Liability-Correlation Coefficients (in % of Asset Value)

	Time Horizon: 1 Year	Corr(A,L)=0.99			Corr(A,L)=0.9			Corr(A,L)=0.3			Corr(A,L)=0			Corr(A,L)=-0.5		
Funding Ratio	Volatility of Liabilities	Volatility of Assets 3%	10%	20%	Volatility of Assets 3%	10%	20%	Volatility of Assets 3%	10%	20%	Volatility of Assets 3%	10%	20%	Volatility of Assets 3%	10%	20%
F=1	3%	0.17%	2.81%	6.79%	0.54%			1.42%			1.69%			2.07%		
	10%	2.81%	0.56%	4.07%	2.96%	1.78%		3.80%	4.72%		4.16%	5.64%		4.70%	6.90%	
	20%	6.79%	4.07%	1.13%	6.91%	4.72%	3.57%	7.69%	7.76%	9.42%	8.05%	8.90%	11.25%	8.62%	10.52%	13.75%
F=1.1	3%	0.00%			0.00%			0.00%			0.54%			0.54%		
	10%	0.27%	0.00%		0.33%	0.03%		0.76%	1.34%		2.96%	1.78%		2.96%	1.78%	
	20%	2.92%	0.92%	0.00%	3.02%	1.34%	0.63%	3.67%	3.73%	5.16%	6.91%	4.72%	3.57%	6.91%	4.72%	3.57%
F=1.2	3%	0.00%			0.00%			0.00%						0.00%		
	10%	0.01%	0.00%		0.02%	0.00%		0.09%	0.29%		0.16%	0.60%		0.28%	1.19%	
	20%	1.13%	0.14%	0.00%	1.19%	0.29%	0.06%	1.63%	1.67%	2.72%	1.84%	2.38%	4.03%	2.20%	3.50%	5.96%
F=1.4	3%	0.00%			0.00%			0.00%			0.00%			0.00%		
	10%	0.00%	0.00%		0.00%	0.00%		0.00%	0.01%		0.00%	0.03%		0.01%	0.14%	
	20%	0.13%	0.00%	0.00%	0.15%	0.01%	0.00%	0.27%	0.28%	0.70%	0.34%	0.55%	1.36%	0.47%	1.07%	2.55%

Corr(A,L): Correlation Coefficient between Assets and Liabilities, ρ_{AL}

$$z_2 = z_1 - \sigma\sqrt{\tau}$$

$$\sigma^2 = \sigma_L^2 + \sigma_A^2 - 2\rho_{AL}\sigma_A\sigma_L$$

1/F is the inverse of the funding ratio, σ is the annualized volatility of the ratio between L and A; σ_A and s_L are the volatilities of assets and liabilities, ρ_{AL} is the correlation coefficient between assets and liabilities, t is the time to maturity (years), and N[.] denotes the cumulative standard normal density. Below, the economic value of the insurance will be expressed as a percentage of the asset value; the following expression will be investigated therefore:

$$\frac{W}{A} = \frac{L}{A}N[z_1] - N[z_2] = \frac{1}{F}N[z_1] - N[z_2] \quad (3)$$

Table 1 provides detailed information about the value of surplus insurance as measured by options prices. The values are based on specific assumptions about volatility (3%, 10%, and 20%), correlation (-0.5, 0, 0.3, 0.9, 0.99), and funding (100%, 110%, 120%, 140%). Insurance costs are symmetric with respect to the volatility of assets and liabilities (see the first cell in the Table). This is apparent from equation (2) where σ_A and σ_L can be interchanged without having any effect on s. Due to this symmetry, only the values on the diagonal axes and below are displayed in the Table.

Insurance costs are negatively related to the funding ratio F as well as to the asset/liability correlation coefficient ρ_{AL}. The effect of the funding ratio needs no further explanation. The correlation effect is also immediate: Insurance is least expensive, if assets perfectly "track" (or immunize) liabilities. This occurs if the assets are perfectly positively correlated with the liabilities and volatilites are equal. Surplus exhibits no volatility in this case, and insurance is obsolete. If the correlation between assets and liabilities decreases, the riskiness of the surplus increases and insurance becomes more valuable.

The effect of volatilities on insurance costs is more tricky to analyze. The relationship is unambigously positive if the volatility of assets and liabilities is increased by the same amount. This is apparent from the diagonal elements in the cells. The effect ambigous, however, if one volatility is held fixed while the other is increased. In most cases, the relationship is still positive, but in general the sign depends on (i) the relative mangnitude of σ_A with respect to σ_L, and (ii) the size of the asset/liability correlation coefficient. Consider, for example, a correlation coefficient of 0.99, a funding ratio of F = 1.1 and a volatility of σ_L = 20%. Increasing the volatility of assets from 3% to 20% causes insurance costs to decrease from 2.92% to less than 0.00%. The same observation emerges if correlation is 0.9 (insurance costs decrease from 3.02% to 0.63%), but the reverse occures if correlation is as low as e.g. 0.3 (insurance costs increase from 3.67% to 5.16%). As is apparent from the figures, an inverse relationship between volatility and insurance costs only occurs if the respective volatility is below the volatility which is held constant. Again, this observation can be explained by the immunisation effect: For sufficiently high correlations (such as 0.99 or 0.9), adjusting the volatility of assets (liabilities) towards the volatility of liabilities (assets) improves the tracking effectiveness of assets (liabilities), reduces surplus volatility and hence decreases the cost of insurance. For low correlations, however, this immunisation effect is not effective: the "volatility effect" dominates the "correlation effect", or to put it differently: an increase in the asset volatility towards σ_L in fact worsens the hedging effectiveness of assets and increases insurance costs.

Table 2 shows the impact of time to maturity on insurance costs. Throughout the table, an asset/liability correlation of 0.3 is assumed for simplicity. Throughout the table, time to maturity increases the expenses for insuring the surplus. At a first glance, this is not a surprising result since a longer time to expiration means an enhanced protection of the surplus[2]. For example, if the funding ratio is 1.1 and volatilities are 10%, one year insurance costs 1.34%, two year insurance 2.82%, and five year insurance 6.14%. These are theoretial costs. In practice, long term insurance may well be (even) more expensive than indicated by these figures. Hedging and replicating long term options is complicated by illiquid (or even missing) hedging instruments or by unexpected changes in the volatility in the underlying markets. Therefore, the figures are at best indicative.

3. Applications

Surplus insurance has several applications. Three more recent ones are discussed in this section.

3.1 Deposit Insurance [3]

In the case of deposit insurance, "liabilities" may be interpreted as protected deposits. Within this framework, the "surplus" corresponds to the bank equity capital. In most banks, deposits are rather short term and therefore exhibit a low market risk. On the other hand, assets are possibly more risky, and are not necessarily highly correlated with liabilities (respectively, with interest rates). This is even true if mortgages represent a major fraction of assets, because political reasons may restrict a continuous adjustment of mortgage rates to market interest rates. A typical set of parameters could therefore be $(\sigma_A, \sigma_L, \rho_{AL}) = (10\%, 3\%, 0.3)$. Most banks are required to hold a minimum equity share of 5% of their assets. These requirements are mostly based on accounting numbers. A funding ratio of 110% could be appropriate in many cases. This implies that the annual insurance fee is 0.76% of the asset value. This figure may appear high. But it is assumed that the entire stock of deposits (i.e. all liabilities of the bank) are protected by the insurance contract. This is, however, not allways true in practice. Insurance is either restricted to certain deposit categories (for example, savings accounts), and/or to a maximum amount of money (for example, 25'000 dollars on XYZ accounts). Typically, this drastically reduces the economic cost of insurance.

3.2 Pension Benefit Protection

Most pension funding systems exhibit some kind of protection against underfunding of individual plans. However, underfunding is not allways realized because pension accounting is not based on market values of assets and liabilities. If a plan is not able to meet the promised payments, some kind of ex

Table 2: Cost of Surplus Insurance with Different Time Horizon (in % of Asset Value)

	Corr(A,L)=0.3	Time horizon: 1 Year			Time horizon: 2 Years			Time horizon: 3 Years			Time horizon: 4 Years			Time horizon: 5 Years		
Funding Ratio	Volatility of Liabilities	Volatility of Assets 3%	10%	20%	Volatility of Assets 3%	10%	20%	Volatility of Assets 3%	10%	20%	Volatility of Assets 3%	10%	20%	Volatility of Assets 3%	10%	20%
F=1	3%	1.42%	3.80%	7.69%	2.00%			2.45%			2.83%			3.17%		
	10%	3.80%	4.72%	7.76%	5.38%	6.67%		6.58%	8.16%		7.60%	9.42%		8.49%	10.52%	
	20%	7.69%	7.76%	9.42%	10.86%	10.96%	13.29%	13.28%	13.41%	16.24%	15.31%	15.46%	18.71%	17.10%	17.25%	20.87%
F=1.1	3%	0.00%			0.05%			0.15%			0.28%			0.42%		
	10%	0.76%	1.34%		1.82%	2.82%		2.75%	4.07%		3.59%	5.16%		4.35%	6.14%	
	20%	3.67%	3.73%	5.16%	6.44%	6.53%	8.65%	8.64%	8.75%	11.37%	10.51%	10.64%	13.66%	12.16%	12.31%	15.69%
F=1.2	3%	0.00%			0.00%			0.00%			0.01%			0.03%		
	10%	0.09%	0.29%		0.50%	1.07%		1.03%	1.91%		1.57%	2.72%		2.12%	3.50%	
	20%	1.63%	1.67%	2.72%	3.74%	3.82%	5.59%	5.59%	5.68%	7.98%	7.22%	7.33%	10.04%	8.69%	8.82%	11.89%
F=1.4	3%	0.00%			0.00%			0.00%			0.00%			0.00%		
	10%	0.00%	0.01%		0.02%	0.12%		0.11%	0.36%		0.25%	0.70%		0.44%	1.07%	
	20%	0.27%	0.28%	0.70%	1.20%	1.25%	2.31%	2.31%	2.37%	3.96%	3.42%	3.50%	5.52%	4.49%	4.59%	6.98%

post measures are typically taken to prevent social damage to the beneficaires. However, basic economics tells that in a risky world welfare can be improved by trading contingent claims or writing insurance contracts allowing an ex ante allocation of risk. Therefore, in many countries view emerges that pension plans should buy insurance against underfunding. This is even true for public pension plans where underfunding is mostly (and explicitely) allowed, but taxpayers are less and less willing to act as residual claimholders of the public funding system.

In the United States, for example, the Pension Benefit Guaranty Corporation (PBGC) acts as a nonprofit independent government corporation established to insure defined benefit plan pension benefits in the event of plan determinations[4]. Fund sponsors pay a premium which are related to the level of funding (funding ratio, surplus); there is also a cap on the premium. However, to limit the adverse effects from moral hazard, the cap should be eliminated, and the premium should be related to the riskiness of the pension portfolio, or with stochastic liabilities, to the riskiness of the surplus. The results in the previous section clearly indicate how sensitive "fair" premiums are with respect to the market risk parameters σ_A, σ_L, and ρ_{AL}. For example, assume a funding ratio of 1.1 and an asset/liability correlation of 0.3. Depending on the volatilities of assets and liabilities, insurance costs vary from less than 0.00% ($\sigma_L = \sigma_A = 3\%$) to 5.16% ($\sigma_L = \sigma_A = 20\%$). Or given an asset volatility of 10% and a liability volatility of 3%, insurance costs vary from 0.27% (ρ_{AL}=0.99) to 1.33% (ρ_{AL}=-0.5), depending on the correlation between assets and liabilities. Of course, this correlation can be determined by the pension fund asset management by selecting an appropriate investment strategy which closely tracks the liabilities. These examples demonstrate that the impact of the market risk of a pension plan as measured by σ_A, σ_L and ρ_{AL} can be easily inferred from the Tables in the previous section. If pension benefit protection premiums do not reflect the individual risk characteristics of plans, then the insurance system is likely to produce a socially inefficient allocation of risk.

3.3 Benchmark Insurance

In modern asset management it has become common to evaluate the performance of portfolios and funds vis-a-vis a so-called benchmark. A benchmark can be thought as a "normal" portfolio or an index representing the agreed-upon investment strategy. Many portfolio managers are able to guarantee a certain benchmark return to their customers. The benchmark thus takes the function of the liability in our setting.

"Indexing" is probably the most popular example among these strategies. Here, the portfolio manager tries to track a generally accepted stock or bond index as closely as possible. Competition among asset managers may create incentives, however, to "beat" the index and to create a positive surplus, while underperforming the index causes sponsors to withdraw capital. Asset managers will therefore wish to guarantee a minimum return represented by an appropriate index while trying to generate a maximum surplus. The basic tradeoff for the portfolio manager is, of course, that superior performance can only the reached by a portfolio composition which deviates from the index. This creates specific or idiosynchratic risk which implies a shortfall risk against the index. "Portfolio insurance" represents a class of strategies which are designated to cut the shortfall risk of a portfolio while preserving maximum upside exposure.

In a few cases, portfolio insurance can be implemented by adding traded options contracts to a portfolio. In this case, strategy implementation is relatively simple and insurance costs are immediately observable through option prices. In most cases, however, portfolio insurance strategies involve dynamic trading in the underlying benchmark (possibly through index futures) and the portfolio. This requires a financial intermediary to provide investment advice and execution[5]. Option pricing theory provides not only the analytical framework to implement the strategy, but also (theoretical) option prices which can be used to estimate the economic cost of benchmark insurance. This is typically measured by the participation rate _ which indicates the percentage by which the insured portfolio moves in the "favorable" direction after accounting for insurance costs. If, for example, the correlation between a portfolio and its benchmark is 0.9, both volatilities are 20% and the funding ratio is 110% (i.e. the value of the portfolio is 10% above the benchmark), the cost of surplus insurance is 0.63% (see Table 1). This implies a participation rate of the insured portfolio of

$$\wp = \frac{1}{1+\frac{W}{A}} = \frac{1}{1+0.0063} = 0.9937 \qquad (4)$$

i.e. the investor holds a participation of 99.37% in the underlying portfolio. If tracking is closer to the benchmark index, i.e. if the correlation is 0.99, the value of the underlying option is less than 0.00% and the participation ratio is almost 1. However, the chance of "beating the index" by over- and underweighting individual stocks relative to the index is also extremely limited thereby. As a conclusion, the values in Tables 1 and 2 can be used to estimate the costs of benchmark protection when insurance is dynamically manufactured against a stochastic benchmark.

4. Conclusions

Surplus insurance is a service provided by financial intermediaries in the financial system in areas such as commercial banking (deposit insurance), pensions (pension benefit protection), and asset management (benchmark insurance). Option pricing theory provides a unified framework to evaluate the economic costs of these contracts. The present paper uses a model developed by MARGRABE (1978) to estimate these costs.

Footnotes

[1] Of course, any level of insurance (i.e. any ratio between assets and liabilities) can be investigated. Here, we assume a surplus of zero as the relevant level of insurance.

[2] This is, however, not necessarily true if protection is provided by ordinary European put options. There, the present value of the exercise price to be received decreases with time to maturity, and therefore, the net effect on the put option price is ambigous.

[3] Deposit insurance contracts were originally priced as put options by MERTON (1977). A put option is a special case of an exchange option where the value of

the liabilities (i.e. the "asset" to be received) is non-stochastic. By setting $\sigma_L=\rho_{AL}=0$ and $A=Xe^{-r\tau}$ equation (3) collapses to the standard pricing formula for a European put option on "asset" L with exercise price X.

[4] See LOGUE (1991), p. 43.

[5] Topics related to the dynamic manufacturing of financial assets and the role of financial intermediation id discussed by MERTON (1992).

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