MARC CHESNEY, RAJNA GIBSON AND HENRI LOUBERGÉ*

Arbitrage Trading and Index Option Pricing at SOFFEX: an Empirical Study Using Daily and Intradaily Data.

1. Introduction

Index options, whose popularity has grown steadily since they began trading in 1983, respond on one hand to a demand for simple, inexpensive stock portfolio hedging instruments and on the other hand for highly leveraged investment vehicles. It can be shown that the use of index options to hedge the systematic risk of a portfolio is less costly than the purchase of numerous stock options to hedge each portfolio component separately. As asset allocation instruments, index options allow investors to pursue both market trends and volatility movements based strategies with a relatively small outlay of capital. The index option holder can trade on his/her market expectations while limiting the risk involved to the premium paid for the option. With index option trading came the need for a model permitting to price these complex instruments. Index options have several important characteristics which set them apart from stock options. Firstly, the underlying is not delivered; instead, index options are cash-settled. For American options, the fact that this settlement is based on closing index values introduces a new risk : timing risk. The option holder who decides to exercise before the index settlement value is announced does not yet know what the exact outcome of the exercise will be. The option writer learns of assignment the following day; in the meantime an unfavorable market swing may have deteriorated the value of the index portfolio intended to hedge the option investment. Secondly, as in the case of interest rate derivatives, the underlying instrument is not traded. In order to maintain a perfectly hedged position an index tracking portfolio must continually be rebalanced, which implies high transaction costs. Moreover, in a thin and illiquid market, and in the absence of a stock index futures, such a dynamic hedging strategy may be practically impossible to implement.

*This paper is part of a research project supported by the Swiss National Fund for Scientific Research under grants 12-27584.89 and 12-33761.92. The authors would like to express their gratitude to SOFFEX in particular Mrs. Wagner, Reuters Europe, in particular Mr. Neirynck, and Telekurs, in particular Mr. Cuennet, for the provision of data and information. They are also indebted to J. Pasquier (University of Fribourg), W. Hauser, P. Guerrero, and C. Metz (UBS), L. Arnoux and M. Guillermo (Credit Suisse), and to R. Steiner (SBC) for supplying additional data and information. The support of the computer center (CUI) at the University of Geneva, particularly D. Tsichritzis, C. Pellegrini and X. Pintado, is also gratefully acknowledged.
The paper has benefitted from helpful comments by Thomas Stucki and participants at the European Finance Association meeting in Lisbon, the AMEX Financial Market Research of Fribourg. Last, but not least, the authors are particularly indebted to Charles Christophi, David Miceli and Barbara Schlaffer for their assistance at various stages of this research. Of course, the usual disclaimer applies.
Henri Loubergé, Department of economics, University of Geneva, Uni Mail, 1211 Geneva 4, Tel.: 022/705 82 83, Fax: 022/705 82 93.

Specifications of index options which set them apart from stock options put into question the validity of applying the BLACK and SCHOLES (1973) stock option pricing formula to the problem of index option valuation. Some of the model's assumptions for stock options cannot be considered to extend to index options. For example, if individual stock prices can be considered to be lognormally distributed, then the sum of the stocks' market values making up the index cannot. Can the BLACK and SCHOLES model, already under attack in the context of stock option pricing for simplifying assumptions such as constant interest rates and volatility, stand up under empirical testing of index options? This question drives most of the empirical literature on index option pricing.

One of the first empirical studies on index options was conducted by EVNINE and RUDD (1985), using intradaily prices of options on the S&P 100 and Major Market indexes. They found that option premiums often violated the lower boundary limits and the put-call parity relationship. They concluded that arbitrage obstacles arising in the framework of index option trading could account for the violations. However, they qualified this conclusion by observing that the data is not perfectly synchronous. Even when real time data is accessed directly from the exchanges, it turns out that the market outpaces the data capture system when the number of operations rises sharply.

The BLACK and SCHOLES option pricing model was tested by FIGLEWSKI (1988) using NYSE call index option premiums. He observed significant gaps between theoretical and market values and suggested that this might be accounted for by arbitrage obstacles encountered in the framework of index option trading. In his view, these obstacles lead traders to use strategies other than the arbitrage trading assumed by the BLACK and SCHOLES model. Strategies based on traders expectations, excluded by BLACK and SCHOLES, would regain importance. This point of view is supported by BEINER (1991), who used Swiss Market Index (SMI) options to test the same model. The latter study - a preliminary study to our own - was conducted using daily closing prices for those SMI calls which could be assimilated to European calls during the year 1989. SHEIKH (1991) used S&P 100 calls to study implied volatility patterns generated by different models, and in the process also tested the empirical validity of the BLACK and SCHOLES formula. He found that the market prices of the S&P 100 calls differed systematically from their BLACK and SCHOLES theoretical values. He also found that the observed biases corresponded to biases that arose if market prices incorporated a stochastically changing index volatility.

Along the more recent lines of research, option pricing models have been tested by analyzing the pattern and distributional properties of their implied standard deviation estimates. In particular, DAY and LEWIS (1988), FRANKS and SCHWARTZ (1991) and HARVEY and WHALEY (1991, 1992) pursued such a testing procedure. This enabled the latter authors to emphasize that the non synchronicity of option and stock index closing prices induce spurious negative serial correlation in the implied standard deviation estimates. The phenomenon is reinforced by the bid-ask spread and to a lesser degree (at least in the USA) by the infrequent trading problem.

In this paper, we pursue two main objectives. First, we examine the learning process on the market for SMI (Swiss Market Index) options since the launching of these instruments by SOFFEX (Swiss Options and Financial Futures Exchange) in December 1988. More specifically, we track the observed deviations from rational, no arbitrage pricing boundaries for calls and puts premiums during different periods, in light of the institutional characteristics of the Swiss stock and derivative markets. (see LEFOLL (1994) for a similar study on Swiss stock options). Secondly, we test the performance of the BLACK and SCHOLES (1973) model in explaining the prices of SMI index options after having accounted for alternative volatility estimates and different scenarios on the dividend stream generated by the index.

To analyze the magnitude of the non synchronous prices effect on the performance of the model, the tests are conducted first with daily data, and then with intradaily data. As explained below (section 2.3), our daily data do not provide perfect simultaneity of the options and underlying values. Moreover, an unknown percentage of the recorded option prices do not represent genuine transaction prices. This introduces a possibly severe bias in the results that we intend to correct by conducting the testing procedure with a second data set, based on continuously recorded intradaily transaction prices. As we might suspect, our results show that the non synchronicity in the data is a substantial determinant of the pricing errors. Thus, our observations for Swiss index options confirm HARVEY and WHALEY's (1991) conclusions in a different market context.

Our results document severe violations of the rational pricing boundaries. However, the results improved slightly over the period, thus suggesting a gradual learning experience by SOFFEX market participants. Transaction costs, short selling restrictions and the impact of block trades can partly explain the persistence of these violations. Concerning the performance of the BLACK and SCHOLES model, the results of our empirical tests do suggest that the model's validity is severely questionable for the purpose of closing prices' estimation. Even if the model's performance turns out to be improved when intradaily data is used, it is unclear how the overall performance of the model can be assessed in light of the moneyness and volatility driven pricing biases uncovered by the regression analysis.

On many standardized index options markets, option values actually adjust to the parameters of the "implied spot index" embedded in the more liquid futures price series. Our results show that this is not the case for the SOFFEX and for the period under review (see in particular Table 12). The latter conclusion is corroborated by TAMBURINI and PIROTTE's (1994) study, which finds that the spot SMI index value leads the futures SMI prices.

The paper is structured as follows: In section 2, the statistical characteristics of the SMI index, the institutional aspects of the Swiss derivatives market and the two data sets are described. In section 3, we present and interpret the results of tests concerning lower boundaries for option values and the put-call parity relationships. Section 4 deals with the validity of the BLACK and SCHOLES option pricing formula and analyzes the pricing errors. Lastly, in section 5, we conclude by providing tentative explanations for our findings and their implications for arbitrage pricing, for the learning process and for the accuracy of the BLACK and SCHOLES formula as applied to the Swiss market for standardized index option contracts.

2. SMI Options

2.1 The Swiss Market Index

The Swiss Market Index (SMI), underlying the index options traded on the SOFFEX, is representative of a concentrated market. Originally (December 1988), it included 24 stocks, representing 20 different firms. In 1991, these figures dropped to 22 stocks and 19 firms respectively. In July 1992, the composition was reduced again to 21 stocks and 18 firms [1]. However, this represents approximately 45% of the total Swiss stock market, and the correlation with comprehensive Swiss stock indexes, such as the SBC and the SPI [2], is very high: respectively 99% and 97% for the period 1989-1990 under review in this study. The high concentration is also reflected in the fact that five firms, representing 8 stocks, accounted for nearly 50% of the index in 1989 [3].

The SMI is a market weighted index, with a base figure retroactively set at 1500 on December 30, 1987. It is not adjusted for dividends, and is adjusted only imperfectly for changes in the capital structure of the component firms: changes take place every 6 months. The SMI value is calculated in continuous time: every change in the price of a constituent stock triggers a new index value. Therefore, the SMI composition has been carefully chosen to include the most actively traded stocks on the Swiss stock market.

Table 1: WILK-SHAPIRO test for the normality of SMI index returns

	Daily returns	Daily returns excluding 16,17/10/89 & 2,3/8/90	Weekly returns
Sample	495	491	100
Standard deviation (non annualized)	0.01	0.01	0.03
Skewness (1)	-1.61	-0.50	0.01
Kurtosis (2)	16.52	6.60	-0.31
W: Normal	0.89	0.95	0.99
Prob W < WN	0.0001	0.0001	0.823

(1) Skewness = E(X-Xmean)**2/standard error**3
(2) Excess Kurtosis=[E(X-Xmean)**4/standard error**4] -3

According to the assumptions underpinning the BLACK-SCHOLES model, the underlying asset's prices are lognormally distributed. To test for normality, we ran a WILK-SHAPIRO test (SHAPIRO and WILK, 1965) on the SMI index returns for the period under review: 1st January 1989 to 31st December 1990. We ran the test with and without the returns connected to the stock market crash of October 1989 and the invasion of Koweit in August 1990 (see Table 1). At a significance level of 1%, the results indicated that weekly fluctuations of the returns could be considered to have a normal distribution; daily fluctuations, however, could not [4]. The skewness and kurtosis statistics confirmed these results. Our finding for daily returns contrast with those of EVNINE and RUDD (1985) and of SMITH (1987), who could not reject normality for daily returns on the S&P 100 and Major Market indexes.

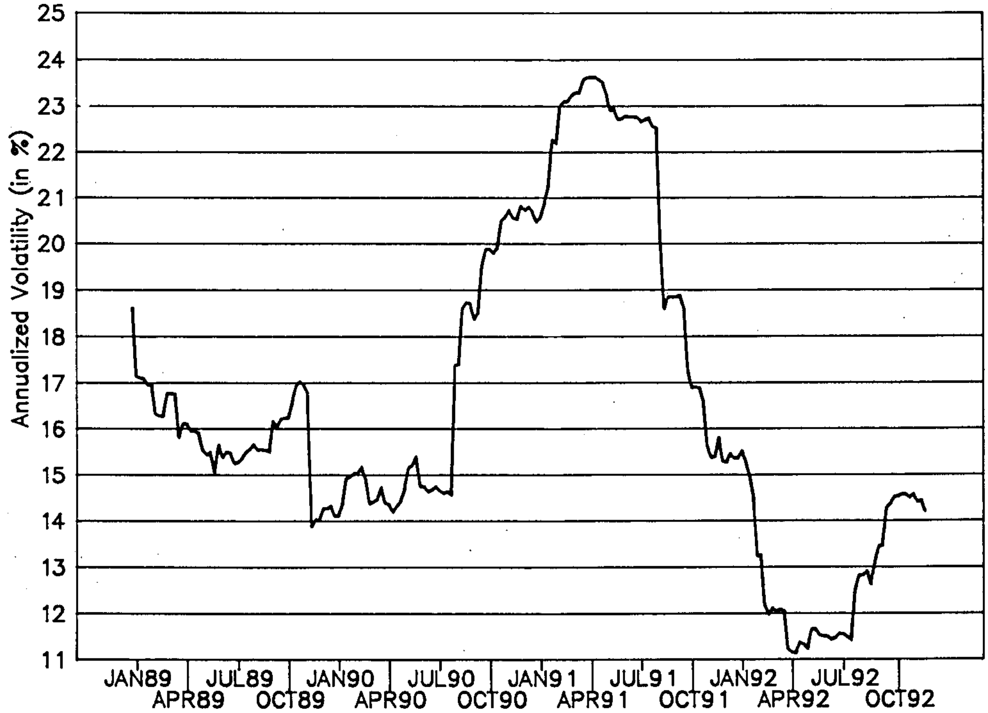
To test the conformity of SMI returns with BLACK and SCHOLES' assumptions, quarterly historical volatilities of the index returns were also computed. The observations indicate that the volatility could not be considered stationary (see Table 2). Even when volatilities connected to the stock market crash and the invasion of Koweit were excluded, the results were far from constant. As an illustration, Figures 1 and 2 plot the historical and implied volatility estimates of the SMI index return over our sample period and confirm the non-stationarity of this parameter, even if one can note from Figure 1 that the jump in volatility is mainly explained by the Koweit crisis, starting in August 1990[5].

It is also worthwile to mention here the real obstacles to efficient trading arising on the Swiss stock exchange. First of all, the Swiss stock market is very thin and illiquid for most medium to low capitalization traded shares. The thinness of the market, coupled with the restrictions and traditions precluding Swiss private and institutional investors from selling stocks short, explain the "apparent arbitrage opportunities" documented in the next section. This statement is reinforced in light of the late introduction (November 1990) of futures contracts on the SMI as a vehicle to circumvent the basket trading strategies and/or short selling restrictions faced by most investors. Finally, when we account for stocks indivisibility, the large nominal values of some shares (e.g. Roche), and the large transaction costs borne by investors active on the Swiss stock and options exchanges, we will acknowledge that even those strategies that could lead to a feasible arbitrage - by holding the compo-

Table 2: Volatility of SMI returns

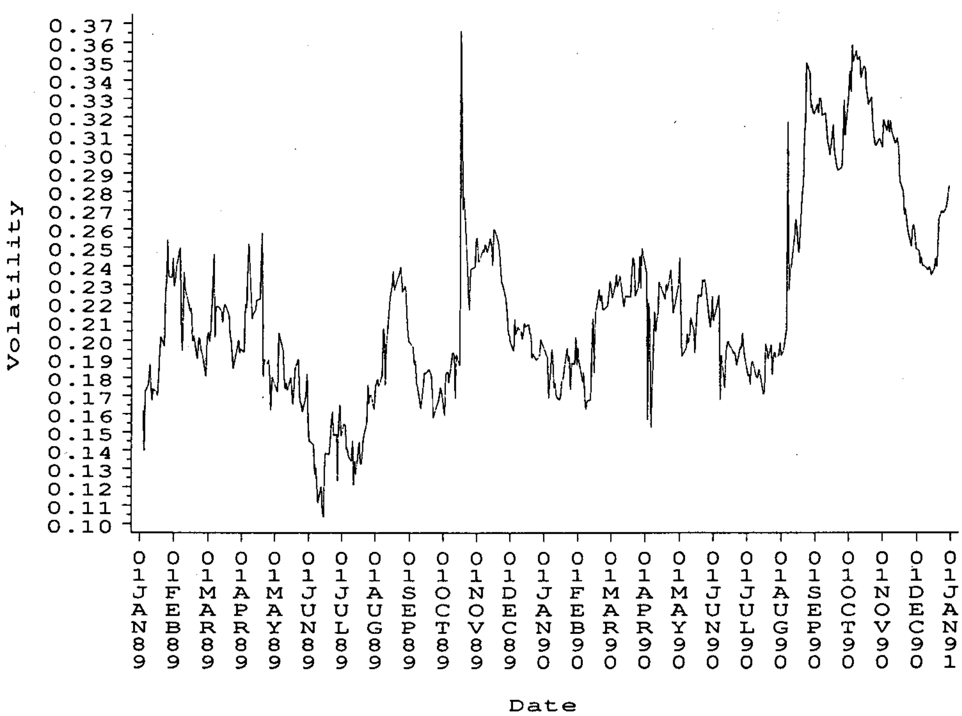
Quarter	Annualized quarterly volatility	
		excluding 16,17/10/89 & 2,3/8/90
1	10.44	10.44
2	12.26	12.26
3	12.40	12.40
4	27.44	14.18
5	14.06	14.06
6	14.81	14.81
7	28.58	28.72
8	18.62	18.62

Figure 1: Historical volatility SMI (weekly, 28/12/'88 to 25/11/'92)



Source: Union Bank of Switzerland

Figure 2: Evolution of the implied index volatility



nent stocks long - are actually unprofitable once these costs are added to the rational boundaries. For instance a single transaction on a Swiss stock amounts to an additional 0.825% of the stock's value, and similarly a buying or selling transaction of options supported in 1989 a fee of roughly 6% of the option's value for a customer (1% for a market maker), in the case of an option price of about Fr. 50.-. The thinness of the market further imposes large bid-ask spreads for stocks of small firms and deeply out-of-the-money options [6]. This contributes to maintain low liquidity and "apparent arbitrage opportunities" on that market.

Our observations of daily index return non-normality and changing index volatility, taken together with the market frictions arising in the framework of index option arbitrage trading, give us an idea of the extent to which the perfect market paradigm may be unappropriate in analyzing and pricing SOFFEX traded index options.

2.2 SMI options

SMI options were introduced by SOFFEX in December 1988, with a contract size equal to five times the value of the index. For maturity specifications, SOFFEX uses the three nearest months plus the nearest from the January/April/July/October cycle. American style options were first introduced and were traded until December 1990. During the second semester of 1990, European options expiring in 1991 were gradually introduced. Since January 1, 1991, only European options are traded.

The SMI option contract is cash-settled. The assigned option writer delivers in cash the difference between the settlement value of the index and the exercise price. At maturity, the settlement value is calculated over trades during a 30 min. interval of stock exchange trading: from 11.00 to 11.30 a.m.. For American options,.the daily settlement value was provided by the SMI closing price, computed as the average of thirty SMI values observed each minute during the last 30 minutes of stock exchange trading. This price was announced around 4.15 p.m., which corresponded approximately to closing of SOFFEX trading floor. However, this floor remained open for early exercise until 5 p.m.: during this lapse of time options could be exercised, but not traded.

It should be noted that most trading takes place in at-the-money options. For example, in 1989, it has been estimated by STUCKI (1992a) that options with striking prices in a 5% range around the index value represented 75% of all option trades.

2.3 Data

The tests reported in sections 3 and 4 below were conducted using two data bases. First of all, we used daily closing prices for the 2-year period from January 1, 1989, to December 31, 1990 [7]. As noted above, the SMI futures contract did not exist for most of this period: it only started trading in November 1990. Moreover, the options traded over the period 1989-1990 were American-style options, except for the last two quarters of the period, where SOFFEX began trading European options expiring in 1991. Consequently, in order to test theoretical relationships established for European options, we used only those options which were European, or could be assimilated to European options for not satisfying the rational early exercise conditions. We also noted in the preceding paragraph that the index closing value is an average taken over the last 30 mn. of stock exchange trading. This implies that there does not exist perfect simultaneity for our closing index and option prices. Still worse, the closing prices of SOFFEX options are not always given by transaction prices. When the options exchange is not sufficiently liquid - this is often the case with deep in-the-money or out-of-the money options - SOFFEX computes a "closing price". In some cases, this is the average of the bid and ask prices. In other cases, a member of the floor provides prices he would be prepared to quote. Finally, in rare cases, the missing prices may be computed using an option pricing model, such as the BLACK-SCHOLES model. Obviously, when this occurs, a bias is introduced into our tests of arbitrage rela-tionships. The data may suggest a violation of these relationships, but no real arbitrage opportunity has occured.

In order to ascertain the impact of these quirks in the data on the results of our tests, we used a second data base consisting of intradaily data. These data were compiled in continuous time over the period from March 31 to August 27, 1992 [8] and cover the following information: all values of the SMI index and of the SMI futures contract; all bid, ask and settlement prices of SMI options for different maturities and striking prices; all prices of the stocks underlying the index; and all Euro-Swiss franc interest rates for the relevant maturities.

The options traded during this latter period were all European. Thus, we were able to rerun our tests for truly European options, using intradaily market data, and for a period where the SMI futures contract traded and where the market had "learned" how to use and price these index options.

It was pointed out to us that we should limit our tests to liquid options series only, for the wide bid-ask spread for illiquid options introduces a bias in the results, even when transactions data are used. Given the above remark that most index options trade occurs for at-the-money options, this would have transformed our investigation into a test of whether *at-the-money index options* are priced in conformity with arbitrage considerations, or not. We believe that a valid theory of index options pricing should apply to all index options series. Thus, we did not exclude purposely in-the-money and out-of-the-money options from our data sets.

The next two sections present the results of the empirical tests performed with the two data sets.

3. Tests of Basic Arbitrage Relationships Using Daily and Intradaily Data.

Our first set of tests, based on the fundamental "no arbitrage opportunities" condition that has to be fulfilled in an efficient market, involved the lower boundaries verification for both puts and calls traded on the SOFFEX given that these options were first of the American type and later became European to facilitate agents' hedging needs. These tests were conducted on daily closing prices during the January 1989-December 1990 period as well as on intra-daily data on the European calls and puts traded over the subsequent March-August 1992 period. The purpose of the latter analysis is twofold: first, to check the learning experience of SOFFEX market participants; secondly, to verify the consistency of the results when settlement prices as well as intradaily transaction prices are taken into account simultaneously in order to reduce the impact of the artificial price setting mechanism of less liquidly traded options - essentially the out-of-the-money options - on the rational boundaries verification process.

3.1 Lower boundaries tests

This involved checking whether C_E and P_E, the prices of the European calls and puts, fulfilled the following conditions respectively:

$$C_E \geq I - Ke^{-r(T-t)} - D^* \qquad (1)$$

$$P_E \geq Ke^{-r(T-t)} - I + D^* \qquad (2)$$

where

I = The SMI index's value at day t.

K = The strike price of the option.

T-t = The annualized time to maturity of the option.

D* = The present value of all the dividends paid by the SMI component firms before the maturity date (T) of the option and weighted by the relative market capitalization of each firm.

r = The annualized short term interest rate prevailing at date t.

Further, we checked whether C_{US} and P_{US}, the prices of the American options, satisfied the following lower boundaries as summarized in SHEIKH's

(1991) study:

$$C_{US} \geq \text{Max}\Big\{0, I - K, \max_{j \leq J}\Big[I - \sum_{i=1}^{j-1} D_i e^{-r(T_i - t)} - K e^{-r(T_j - t)}\Big]; I - \sum_{i=1}^{J} D_i e^{-r(T_i - t)} - K e^{-r(T-t)}\Big\} \quad (3)$$

and

$$P_{US} \geq \text{Max}\Big\{0; K - I; \max_{j \leq J}\Big[K e^{-r(T_j - t)} - I + \sum_{i=1}^{j-1} D_i e^{-r(T_i - t)}\Big]; K e^{-r(T-t)} - I + \sum_{i=1}^{J} D_i\, e^{-r(T_i - t)}\Big\} \quad (4)$$

where:

$(T-t)$ = The annualized time to maturity of the American option.

(T_i-t) = The annualized time to payment of the ith dividend by one or more firms.

J = The total number of dividend payments until maturity date T of the option.

Table 3: Results for lower boundaries tests

	% of violations based on daily data		% of violations based on intra-daily data	
	%	N[a]	%	N[a]
1) Lower bound for European calls	0.3	1579	12.2	18168
2) Lower bound for European puts	1.0	1579	15.2	19201
3) Lower bound for US calls	8.7	9664		
4) Lower bound for US puts	5.1	9664		

[a] N denotes the total number of observations during the period.

The results are summarized in table 3.

By looking at this table, we can first, in light of the left hand side results based on daily data, state that the market had a positive learning experience since European options that traded during the last two quarters did effectively experience much less violations than their American predecessors. We also notice that American calls present more violations (8,7%) than American puts (5,1%), a result which can mainly be explained by a suboptimal early exercise policy for the latter calls (see below). However, when we look at intra-daily data over an even more recent period the results look much less encouraging since we observe 12.2% violations for the calls and 15.2% for the puts that traded over this period. Of course, part of these inefficiencies could arise from non synchronous data problems that are actually amplified during the day since the index's value will tend to lead or lag all its component stocks' prices except one. The latter problem disappears when we look at the closing prices since the stock market closes 10 to 15 mn before the SOFFEX thus "pooling" the option closing prices on this fixed index's closing value artificially. Also, the liquidity pattern of the SOFFEX being U-shaped, some of the intra-daily results can be attributable to liquidity effects. Moreover, we have been informed by traders that this U-shaped volume pattern will slightly decrease just before settlement which could explain, at least for the less liquid out-of-the-money options, an exogenous settlement price fixing mechanism that "improves" the results based on SMI closing prices (see section 2).

Finally, some of these violations have to be examined in light of the short selling restrictions - and costs - that made it difficult for most agents -before the SMI futures contract launching in November 1990 - to short sell the SMI component stocks in order to profit from this "apparent" calls' underpricing. The latter statement is of course reinforced by the non synchronicity problem between the cash and the options' markets that creates cash settlement related risk for that "apparent" arbitrage opportunity.

3.2 Early exercises of American options

Together with the short selling restrictions, the early exercise condition seems to be another factor explaining the greater number of violations observed among American calls, compared with puts. Indeed, by looking at the relation which states that the value of an American call or put should always exceed its intrinsic value, namely:

$$C_{US} \geq I - K \quad (5)$$

and

$$P_{US} \geq K - I, \quad (6)$$

we found 2,5% violations for the US calls [9] and 1,4% violations for the US puts. When talking about violations, we must take into consideration that these are ex-post tests. In reality, when the trader learns the SMI index's closing value at 4:15 pm, it is too late to buy or sell options. Although cash settlement of options exercised between 4:15 and 5:00 pm is based on the closing level which has just been announced, the options themselves can no longer be traded [10].
In addition to this timing issue, there seems to have been a misundertanding of the rational exercise policy followed by holders of American SMI call options. Early exercise was often triggered by the linkages between the Swiss and US stock markets. After the Swiss market is closed, Swiss option traders keep an eye on Wall Street and trade on their anticipations of US market induced expected-changes on the SMI at the opening the next day. In some cases, bad news would thus trigger them to exercise their calls fearing that the index would drop, although we know that this policy overlooks the rational optimal early exercise policy of American calls - based on dividend and interest rate flows analysis.

3.3 Tests of the Put-Call Parity relationships

Tests of the Put-Call Parity were conducted both with American and European options as well as with daily and intra-daily prices in order to assess the efficiency of the relative pricing mechanism of the SOFFEX. We tested the American put-call boundaries and the European put-call parity in absolute terms, respectively based on the following bounds:

a) For the American options

If $|\text{Diff}\,1| \leq \alpha$ and $|\text{Diff}\,2| \leq \alpha$,

the put-call boundaries for the American options are respected at level α, where

$$\text{Diff 1} = C_{US} + Ke^{-r(T-t)} - I - P_{US} \quad (7a)$$

$$\text{Diff 2} = P_{US} - K + I - D^{*} - C_{US} \quad (7b)$$

α = tolerance level for the absolute error term.

b) For the European options

The put-call parity relationship holds when

$|\text{Diff}| \leq \alpha$

where

$$\text{Diff} = P_E - C_E - Ke^{-r(T-t)} + I - D^{*}. \quad (8)$$

First of all, we provide summary statistics on the bias, namely its magnitude, mean and standard deviation as they were detected on American and European options both from daily as well as intra-daily data: see table 4.
By looking at this table, one realizes that there has been a slight improvement of the parity relationships between 1989 and 1990 (Panels A and B). It is essentially reflected in the transition to European style traded options which have been gradually introduced since the last quarter of 1990 and which have an exercise policy more easily understood by market participants. Another explanation is that the

Table 4: Statistics on Put-Call Parity bias

Panel A: American options		
	1989	*1990*
Mean Diff 1	-16.79	-17.41
Std. deviation Diff 1	14.50	15.76
Mean ¦Diff1¦	17.14	17.77
Mean Diff 2	-2.08	-3.04
Std. deviation Diff 2	11.38	13.27
Mean ¦Diff2¦	8.44	9.73
Number of observations	5078	4586
Panel B: European options		
Mean Diff		-11.76
Std. deviation Diff		12.52
Mean ¦Diff¦		12.23
Number of observations		1579
*Panel C: European options & intra-daily data**		
		March/August 92
Mean Diff		9.80
Std. deviation Diff		15.52
RMSE of Diff		18.36
Number of observations		2585

*For intra-daily data tests, we only took the prices of calls and puts based on the same value of the SMI index and for which the interval of price quotation did not exceed one minute.

market gradually learned to price and to arbitrage these new instruments. An interesting result is that the magnitude and sign of the significant biases [11] for American options seem to indicate that the violation is most often on the short hedge restriction side and would thus involve, to be exploited profitably, the short selling of the component stocks of the SMI. The latter activity was severely precluded for most agents trading on the SOFFEX by the regulations and transaction costs incurred on securities lending facilities.

The intra-daily 1992 data based results tend to show, after the launching of the SMI futures, persistent arbitrage opportunities and noise. This phenomenon had to be expected given the synchronicity related problems that closing prices tend to hide. Furthermore, these apparent arbitrage opportunities need to be redefined and analyzed in light of the transaction costs incurred by market participants trading on the SOFFEX.

For that purpose, we look at the put-call parity relationships after accounting for the minimal and maximal fees an agent would have to support in order to make the arbitrage. We shall define α, the tolerance level, as representing the aggregate transaction costs incurred by buying (selling) the index's component stocks and simultaneously buying (selling) the puts (calls) on the SOFFEX [12] for different market participants. We shall therefore retain 3 scenarios.

Scenario 1:

Low cost scenario applying to a market maker on the SOFFEX:

- index's component stocks [13] trading fee of 0.2%;
- fee per option contract: 0.25 frs;
- exercise fee per option contract: 2.00 frs;
- value of α: 6.00 frs.

Scenario 2:

High cost scenario applying to a market maker:

- index's component stocks' trading fee: 0.5% on the SOFFEX;
- fee per option contract: 0.50 frs;
- exercise fee per option contract: 2.00 frs;
- average bid and ask spread: 2.08 frs;
- α = 14 frs.

Scenario 3:

Transaction costs applying to an agent trading on the SOFFEX:

- index's component stocks' trading fee: 0.70%;
- fee for trading one option contract (average): 1.24 frs;
- exercise fee per option contract: 4.00 frs;
- average bid and ask spread: 2.08 frs;
- α= 20.50 frs.

The results [14] for these three values of , respectively 6 frs, 14 frs and 20.50 frs are provided in Table 5 for all types of options and for daily as well as

Table 5: Results of Put-Call Parity tests

	α = 6.00	α = 14.00	α = 20.50
Panel A: American options			
% of violations in 1989 (N = 5078)	20%	7.1%	2.5%
% of violations in 1990 (N = 4586)	22.5%	6.6%	1.6%
In violating sample			
Mean Diff 1	-28.49	-36.94	-43.34
Standard Deviation Diff 1	15.79	14.35	17.12
RMSE Diff 1	32.58	39.64	46.59
Mean Diff 2	11.79	18.86	24.15
Standard Deviation Diff 2	8.44	8.81	11.70
RMSE Diff 2	14.50	20.81	26.83
Number of violations	2049	666	201
Panel B: European options			
% of violations (N = 1579)	18.3%	6.3%	2.5%
In violating sample			
Diff	-28.23	-39.79	-47.16
Standard Deviation Diff	14.82	9.89	8.07
RMSE	31.87	28.70	47.85
Number of violations	289	100	39
*Panel C: European options and intra-daily data**			
% of violations (N = 2585)	32%	14.5%	12.5%
In violating sample			
Mean Diff	25.60	44.73	49.36
Standard Deviation Diff	19.50	12.73	5.77
RMSE	32.19	46.51	49.69
Number of violations	828	376	324

* In this panel all pairs of call and put prices are based on the same spot value of the SMI and are quoted within less than one minute time intervals.

intra-daily data. By looking at this table, we see that there is very little room for arbitrage opportunities on the SOFFEX once we take into account a realistic transaction fees schedule for a SOFFEX investor who has to face the reality of a wide bid and ask spread, as well as of its variability [15], in addition to the transaction costs.

The intra-daily data based results show that, when correcting for the non synchronous data problems, some arbitrage opportunities remain although one would have to check whether these results apply to real trading opportunities given the thinness of the market and the fact that for most investors, a tolerance level of 20.50 frs is still a too "reasonable" measure of the effective costs of trading SOFFEX options. However, during the 1992 observation period, the futures SMI contract was trading and this should have reduced the inefficiencies in the put-call relationship. To summarize, the set of results on the SMI options' rational boundaries do suggest that the market participants took time to learn, letting inefficiencies gradually decrease. However, even after taking into account synchronicity problems and transaction costs schedules, one must acknowledge that some traders could have done profitable riskless arbitrage strategies using SMI index options, provided the latter did not have the effect of creating negative liquidity effects and thus did not involve large trades' executions.

4. The Performance of the BLACK and SCHOLES Pricing Model

In this section, we present the results of testing the hypothesis that the BLACK and SCHOLES model prices SMI index options adequately. Although we have seen in the two preceding sections that some basic assumptions of the model are not verified, the importance of this model for option pricing theory and option trading justify to investigate its pricing performance. The model will be tested using our two data bases. Before presenting the results and analyzing the pricing errors, we first give some indications about the formulas and the parameter values used in this empirical study.

4.1 The formulas

Because of the generally recognized importance in option pricing of dividend payouts on the stocks included in the underlying index, we have adjusted the BLACK and SCHOLES (1973) formula to take the dividend factor into account. There has been research done to determine what the magnitude of the fall-off implicit in option prices is, on ex-dividend day. In our tests, we shall assume that the fall-off is equal to the dividend, although this may lead to underpricing the options (KAPLANIS, 1986), and we shall also assume that the dividend amounts and payment dates are known in advance. This assumption is reasonable, given Swiss firms' highly predictable dividend policy. After adjustment for the actual dividend payouts, we get the following expressions:

$$C_E = I^* N(d_1) - Ke^{-r(T-t)} N(d_2) \qquad (9)$$

$$P_E = Ke^{-r(T-t)} N(-d_2) - I^* N(-d_1) \qquad (10)$$

where $d_1 = \dfrac{\ln(I^* / K) + (r + \sigma^2 / 2)(T-t)}{\sigma\sqrt{T-t}}$

$d_2 = d_1 - \sigma\sqrt{T-t}$

$I^* = I - D^*$ represents the value of the index adjusted for future dividend payments,

$N(\bullet)$: cumulative standard normal distribution,

σ : annualized index return volatility,

and the other symbols as defined in section 3.

The formulas adjusted for a constant dividend yield can be expressed as follows:

$$C_E = Ie^{-\delta(T-t)} N(d_1) - Ke^{-r(T-t)} N(d_2) \qquad (11)$$

$$P_E = Ke^{-r(T-t)} N(-d_2) - Ie^{-\delta(T-t)} N(-d_1) \qquad (12)$$

where

$$d_1 = \frac{\ln(I / K) + (r - \delta + \sigma^2 / 2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

δ = instantaneous dividend yield, assumed constant,and the other symbols as above.

4.2 The parameters

Of the seven parameters included in these formulas (I, K, T-t, r, D*, σ and δ), the first five can be observed easily. The index return volatility and the instantaneous dividend yield, however, must be estimated.

We have used three methods to estimate σ : a moving (30 day) historical volatility, a constant historical volatility and an implied volatility. For the tests with daily data, the constant historical volatility was given each year by the annualized standard deviation of all weekly index returns during that year. The results are 15.4% for 1989 and 19.76% for 1990. For the tests with intradaily data, a constant volatility is calculated daily as the annualized standard deviation of all tick-to-tick index returns during that day. Using 48 days over the period from March 31 to July 16, 1992, we obtain an average daily volatility parameter of 6.22% with an estimated standard deviation of 1.64%. This somewhat low level of the volatility is not due to the fact that we abstract from the close-to-open volatility since it is well-known that the volatility is largely caused by trading (see FRENCH and ROLL, 1986). As HULL (1989, p.122) notes, the empirical results suggest that non-trading periods may be ignored for the measurement of historical volatility.

The implied volatility method uses the information provided by option values at t to obtain a volatility estimate subsequently plugged into the option pricing formula at t+1. With intradaily data, the volatility plugged into each call (put) is the implied volatility calculated from the last call (put) traded [16]. With daily data, we use a least squares implied volatility. This method estimates the volatility plugged into the formula on day t by taking into account option values for the different strike prices and expiration dates on day t-1. We constructed a function F which is equal to the square of the sum of the deviations between the model option prices and their market prices, divided by two:

$$F(\sigma) = \frac{\Sigma_i [w_i - w_i(\sigma)]^2}{2}$$

where

w_i : market option premium for a given strike price and expiration date,

$w_i(\sigma)$: model price for the option generated by the BLACK and SCHOLES model, as a function of σ.

We then minimized this second-degree polynominal with respect to σ. The solution is the implied volatility. Average implied volatility over the two-year period was 22%.

In equations (9) and (10), the dividends paid out by a particular firm are weighted according to the firm's relative market capitalization share in the Swiss Market Index. The weights attributed to the various firms included in the index during the periods our study is concerned with were not avail able on a daily basis. As we did have half-yearly data on the weighting, we conducted the daily tests under two extreme hypotheses: the undated changes in capital all took place at the beginning of the half-year period, or, on the contrary, at the end. There was no difference observed in the pricing errors expressed relative to the market premium, under the two hypotheses. In absolute terms, the difference in pricing error was negligible. The results presented below are based on the assumption that changes in capital took place at the beginning of the half-year periods.

The instantaneous dividend yield, δ, was constructed by dividing the sum of total annual weighted dividends by the average level of the SMI index that same year. The dividend yield was 4.6% in 1989, and 5,3% in 1990. The interest rates were chosen in the same way that they were for the put-call parity tests. The time remaining to expiration was annualized as were all the other parameters.

4.3 The results based on daily data

We tested formulas (9) and (10) three times, using the three different methods of estimating the volatility described above: the constant annual volatility, the moving (30 day) historical volatility, and the least squares implied volatility. Then, we tested formulas (11) and (12) also three times, using the same volatility estimates. Thus, we get six models, differentiated by their volatility estimates and by the procedure to take dividends into account. A ranking of the six models is provided in Table 6, respectively for calls and puts. The ranking is based on the root mean squared error (RMSE) of the pricing error expressed in relative terms: (Model

Table 6: Model ranking according to performance
(Smallest RMSE of the relative pricing error)

CALLS		PUTS	
1. Implied volatility and actual dividend payouts	0.43	1. Implied volatility and constand dividend yield	0.10
2. Constant volatility and actual dividend payouts	0.43	2. Implied volatility and actual dividend payouts	0.10
3. Implied volatility and constant dividend yield	0.47	3. Constant volatility and constant dividend yield	0.25
4. Constant volatility and constant dividend yield	0.49	4. Historical volatility and constant dividend yield	0.28
5. Historical volatility and constant dividend yield	0.63	5. Constant volatility and actual dividend payouts	0.32
6. Historical volatility and actual dividend payouts	0.67	6. Historical volatility and actual dividend payouts	0.35

Table 7: Average relative put pricing errors
(t statistic in parentheses)

	Actual Dividend payouts	Constant Divident yield
Constant volatility	-0.25 (-47.04)	-0.16 (-31.56)
Historical (moving) volatility	-0.24 (-39.13)	-0.16 (-26.50)
Implied volatility	-0.06 (-26.19)	0.03 (12.79)

price - Market price) / Market price. When the RMSEs are equal, the models are ranked according to their average relative pricing errors. The table shows that the two models based on an implied volatility dominate, especially for puts. The two models based on a moving historical volatility perform poorly. In terms of dividends, the models using actual dividend payouts fare better for calls, but the ranking is reversed for puts: a constant dividend yield provides lower RMSEs.

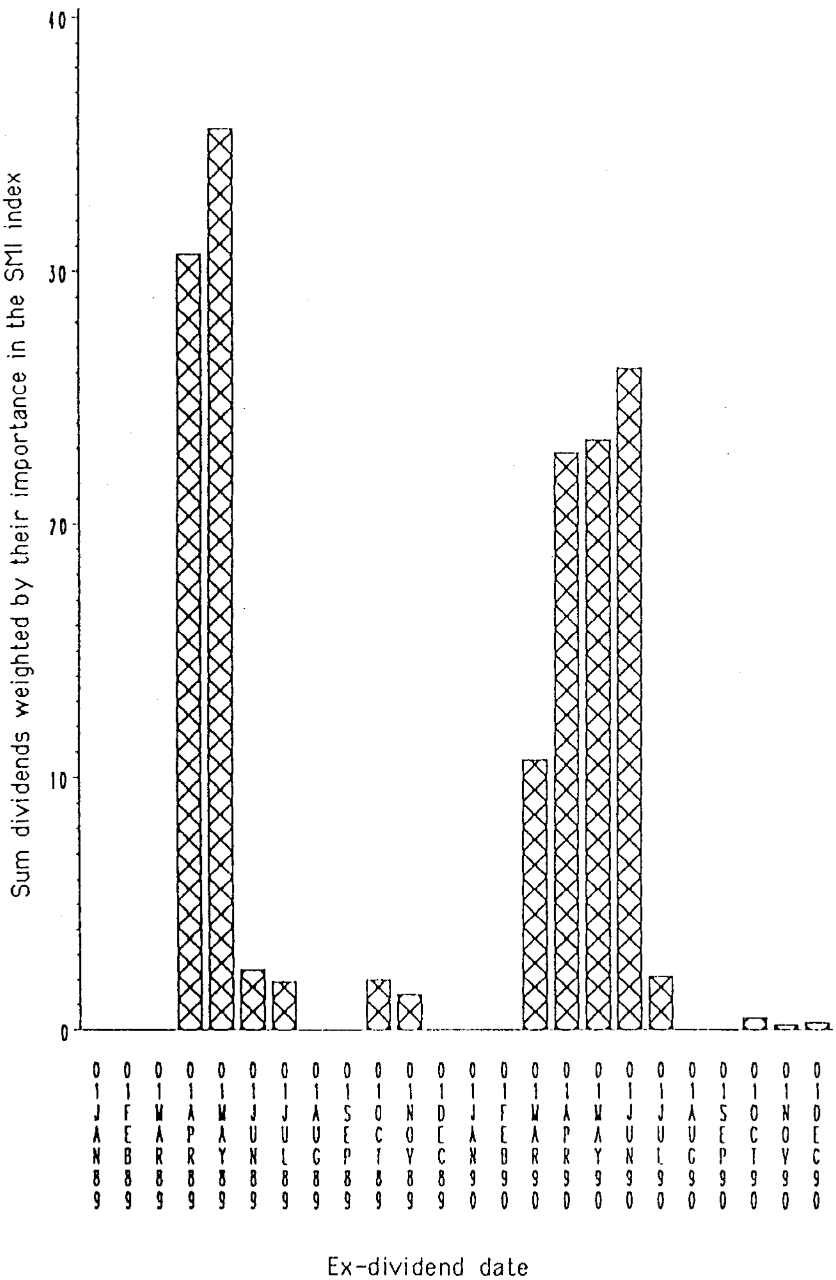
The fact that we get better results when using implied volatilities is not surprising, since option traders themselves use implied volatility to price options, thus turning the model into a self-fulfilling prophecy. It seems strange, however, that using a constant dividend yield produces better results for puts, given the discrete nature of the dividends paid on the stocks underlying the SMI index: Figure 3 shows that dividend payments are concentrated in the second quarter each year. This enigma can be solved by recalling that we only have a six-month sample of European puts. Our put data cover only the third and fourth quarters of 1990, corresponding to those options expiring in 1991. Table 7 displays the average relative pricing errors for puts. It shows that our models tend to underprice puts on average, but that underpricing is reduced - or even turned into overpricing - when a constant dividend yield is used. This is easily explained by comparing formulas (10) and (12): the latter formula tends to overprice options, relative to the former, when there are, in fact, no forthcoming dividend payouts before the options expiration date; and, indeed, during the second semester of 1990, forthcoming dividends were very limited. Thus, using a constant dividend yield produces better results for puts, because it limits the propensity of the BLACK and SCHOLES model to underprice the options.

In the remainder, we will focus our analysis on three models only: the two models using an implied volatility, and the model using a constant (annual) volatility and actual (discrete) dividend payments. For each of these models, the average pricing errors in absolute and in relative terms can be found in Tables 8-10. We have subdivided the results according to two parameters: the time to expiration and the relationship between the strike and the spot prices of the index. The upper panel is for absolute errors, the lower panel for relative errors. In each table, the second and fifth columns give estimations of the average pricing error, along with Student t-statistic for the null hypothesis. The third and sixth columns provide RMSE values.

The t-statistics indicate that, in almost all cases, the pricing errors were significantly different from zero. Thus, overall, the BLACK and SCHOLES model did not price European SMI index options accurately during the period under review. Using an implied volatility and actual dividend payouts (Table 9), the model overpriced the calls by 2% on average, and underpriced the puts by 6% on average. Our results for calls confirm those of BEINER (1991), who used the 1989 SMI calls which could be assimilated to European calls. FIGLEWSKI (1988), who tested the BLACK and SCHOLES model using NYSE index options, an implied volatility and discrete dividend payouts, found that the model overpriced the call premiums by 4% on average. In contrast, SHEIKH (1991), who used transaction data on S&P100 calls, found that the model underpriced calls on average [17].

The results presented in the lower panel of Table 9 show that the RMSEs for puts are lower than for calls, although the average relative pricing errors are lower for calls. This is an indication that

Figure 3: Monthly distribution of SMI-weighted dividends in 1989 and 1990



underpricing of puts by this model is steadier than overpricing of calls. The observed mispricing pattern may be due to our use of the same implied volatility for both option categories. Recent empirical evidence documents different average volatility estimates for calls and puts on an underlying index: see HARVEY and WHALEY (1992). The average implied volatility of calls is less than the average implied volatility of puts. Using the same implied volatility tends to overprice calls and underprice puts. The model using implied volatility and constant dividend yield (Table 10) genera-

Table 8: Model using constant volatility and actual dividend payouts

Absolute pricing error: model price - market price

	CALLS			PUTS		
	N	m [t]	RMSE	N	m [t]	RMSE
Total	9380	-7.08 [-63.04]	12.98	1539	-27.43 [-63.85]	38.18
In-the-money	2359	-3.82 [-20.17]	9.97	916	-25.72 [-48.01]	30.40
At-the-money	4300	-8.63 [-47.32]	14.75	514	-30.39 [-39.11]	35.12
Out-of-the-money	2721	-7.44 [-39.86]	12.26	109	-27.76 [-17.45]	32.30
Expiration < 1 month	2666	-2.22 [-18.48]	6.58	52	-13.69 [-14.59]	15.24
Expiration 1 to 3 months	4526	-7.98 [-53.32]	12.84	659	-17.95 [-40.16]	21.29
Expiration 3 to 5 months	1695	-9.57 [-29.13]	16.56	610	-36.65 [-56.01]	40.04
Expiration > 5 months	493	-16.54 [-23.84]	22.60	218	-33.57 [-29.87]	37.42

Relative pricing error: (model price - market price) / market price

	CALLS			PUTS		
	N	m [t]	RMSE	N	m [t]	RMSE
Total	9380	-0.24 [-65.37]	0.43	1539	-0.25 [-47.04]	0.32
In-the-money	2359	-0.02 [-19.31]	0.06	916	-0.12 [-38.24]	0.15
At-the-money	4300	-0.13 [-39.35]	0.25	514	-0.39 [-59.04]	0.41
Out-of-the-money	2721	-0.61 [-76.00]	0.73	109	-0.70 [-58.74]	0.70
Expiration < 1 month	2666	-0.30 [-34.33]	0.54	52	-0.18 [-5.89]	0.28
Expiration 1 to 3 months	4526	-0.24 [-47.61]	0.41	659	-0.21 [-25.76]	0.29
Expiration 3 to 5 months	1695	-0.19 [-28.24]	0.33	610	-0.30 [-33.92]	0.36
Expiration > 5 months	493	-0.16 [-22.03]	0.23	218	-0.27 [-25.20]	0.31

Table 9: Model using implied volatility and actual dividend payouts

Absolute pricing error: model price - market price

	CALLS			PUTS		
	N	m [t stat]	RMSE	N	m [t stat]	RMSE
Total	9380	1.34 [19.26]	6.88	1539	-8.00 [-32.13]	12.62
In-the-money	2359	-0.08 [-0.65]	6.35	916	-9.26 [-26.61]	14.01
At-the-money	4300	1.47 [14.35]	6.91	514	-6.11 [-16.44]	10.40
Out-of-the-money	2721	2.36 [17.98]	7.26	109	-6.40 [-9.45]	9.50
Expiration < 1 month	2666	0.19 [2.15]	4.75	52	-10.68 [-11.25]	12.65
Expiration 1 to 3 months	4526	0.42 [5.16]	5.50	659	-7.51 [-21.41]	11.72
Expiration 3 to 5 months	1695	4.17 [19.52]	9.72	610	-8.51 [-24.35]	12.11
Expiration > 5 months	493	6.28 [11.94]	13.26	218	-7.43 [-7.62]	16.15

Relative pricing error: (model price - market price) / market price

	CALLS			PUTS		
	N	m [t stat]	RMSE	N	m [t stat]	RMSE
Total	9380	0.02 [5.45]	0.43	1539	-0.06 [-26.19]	0.10
In-the-money	2359	-0.00 [-0.77]	0.04	916	-0.04 [-24.43]	0.06
At-the-money	4300	0.04 [12.89]	0.21	514	-0.07 [-16.58]	0.12
Out-of-the-money	2721	0.02 [1.34]	0.75	109	-0.15 [-9.74]	0.22
Expiration < 1 month	2666	-0.08 [-7.25]	0.58	52	-0.09 [-7.14]	0.13
Expiration 1 to 3 months	4526	0.04 [7.50]	0.37	659	-0.05 [-16.01]	0.09
Expiration 3 to 5 months	1695	0.12 [15.50]	0.33	610	-0.07 [-22.26]	0.10
Expiration > 5 months	493	0.11 [9.99]	0.27	218	-0.05 [-5.52]	0.13

Table 10: Model using implied volatility and constant dividend yield

Absolute pricing error: model price - market price

	CALLS			PUTS		
	N	m [t stat]	RMSE	N	m [t stat]	RMSE
Total	9380	2.67 [-27.64]	9.73	1539	4.23 [15.34]	11.61
In-the-money	2359	-7.51 [-41.86]	11.51	916	5.51 [14.02]	13.09
At-the-money	4300	-1.88 [-12.21]	10.30	514	3.25 [8.35]	9.37
Out-of-the-money	2721	0.29 [2.28]	6.62	109	-1.88 [-2.98]	6.82
Expiration < 1 month	2666	-0.65 [-8.40]	4.05	52	-6.49 [-7.30]	9.07
Expiration 1 to 3 months	4526	-2.64 [-20.18]	9.21	659	0.80 [2.66]	7.72
Expiration 3 to 5 months	1695	-5.09 [-15.25]	14.67	610	6.26 [14.54]	12.32
Expiration > 5 months	493	-5.46 [-9.48]	13.90	218	11.48 [12.13]	18.07

Relative pricing error: (model price - market price) / market price

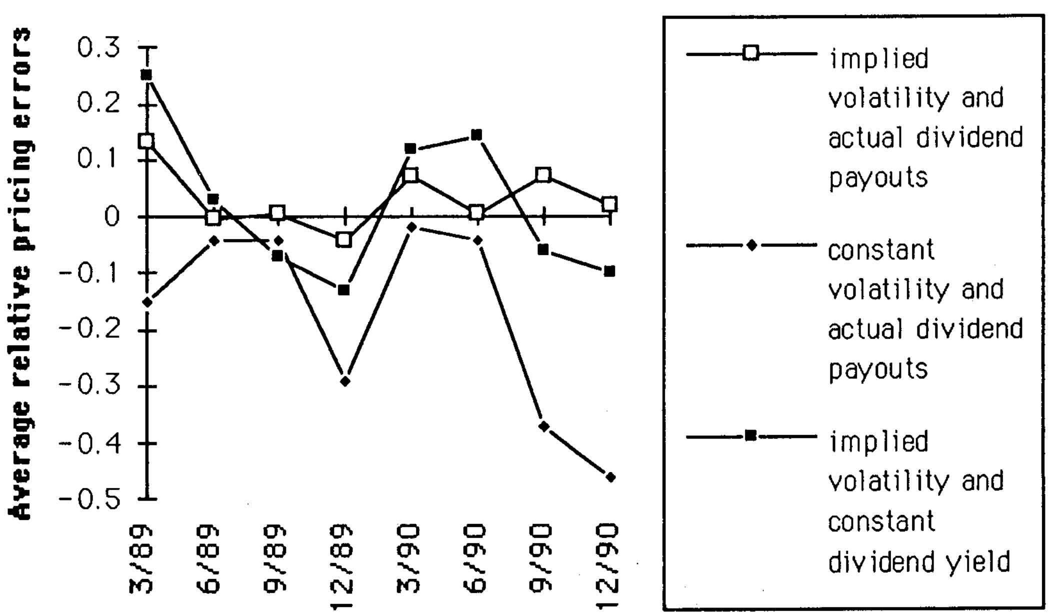
	CALLS			PUTS		
	N	m [t stat]	RMSE	N	m [t stat]	RMSE
Total	9380	-0.01 [-3.12]	0.47	1539	0.03 [12.79]	0.10
In-the-money	2359	-0.04 [-36.46]	0.06	916	0.03 [15.75]	0.06
At-the-money	4300	0.02 [4.86]	0.28	514	0.05 [9.91]	0.12
Out-of-the-money	2721	-0.05 [-3.48]	0.79	109	-0.03 [-1.85]	0.19
Expiration < 1 month	2666	-0.10 [-10.05]	0.55	52	-0.06 [-5.56]	0.10
Expiration 1 to 3 months	4526	0.02 [2.84]	0.48	659	0.02 [5.74]	0.09
Expiration 3 to 5 months	1695	0.03 [4.10]	0.31	610	0.03 [9.03]	0.08
Expiration > 5 months	493	-0.01 [-1.40]	0.19	218	0.10 [10.92]	0.17

tes the smallest average relative pricing errors. It underprices calls and overprices puts on average, although some categories of calls (puts) are overpriced (underpriced). Overpricing of puts by this model has been explained above. Underpricing of calls may also be explained by the constant dividend bias. Looking at Figure 4, plotting the results for calls over the whole period, we see that this model performs relatively well only on average. The constant dividend yield generates underpricing when there are, in fact, no forthcoming dividend payouts before the option expiration date (two last quarters of each year), and errs in the opposite direction when numerous ex-dividend dates come before expiration, and sizable payouts have been smoothed by the constant rate (two first quarters of each year). In contrast, the more realistic discrete dividend model generates much more regular results.

Table 8 shows that the model using a constant volatility systematically underprices the calls and puts of each category. This is due to the fact that the constant volatility is lower than the implied volatility: we noted previously that the latter was 22% on average, while the former reached only 15.4% in 1989 and 19.76% in 1990. The graph of Figure 4 shows that underpricing is particularly large in the fall of 1989 and 1990, two periods characterized by a jump in market volatility (see Section 2, and more particularly Table 2). For calls and puts, average relative mispricing is largest for out-of-the-money options, which benefit most from an increase in volatility.

Considering the relationship between the strike price and the index value across the different models, we see that in-the-money options are priced best, in terms of RMSE. Out-of-the-money options generate the poorest performance [18]. As opposed to the results of FIGLEWSKI (1988), HULL and WHITE (1987), BEINER (1991) and SHEIKH (1991), the out-of-the-money call options are not underpriced in the model with implied volatility and discrete dividend payouts: they are overpriced on average, although not significantly (in relative terms).

Figure 4: Average relative call pricing errors in 1989-1990



Within the various time to expiration subsets, the model using implied volatility and discrete dividend payouts generates the smallest average relative pricing errors for options with expiration dates 1 to 3 months away. These results differ from those of FIGLEWSKI and BEINER. FIGLEWSKI's best percentage results were for calls with the longest time to expiration, while BEINER's best percentage results were for those closest to expiration. FIGLEWSKI found that the bias was less clear in absolute terms. He concluded that the use of the percentage measure, which includes the option premium in the divisor, is responsible, since prices increase with time to expiration. Indeed, we find that calls closest to expiration generate the lowest mispricing in absolute terms, for the three models. However, FIGLEWSKI's remark does not hold for puts. For instance, in Table 10, the best results are obtained for puts with the expiration date 1 to 3 months away, both in absolute and in relative terms. If we look at RMSEs across the three tables, we find that calls with the longest time to expiration are priced best in relative terms, but not in absolute terms: in this case, calls with the shortest time to expiration generate the lowest RMSEs. For puts, no clear pattern arises, both in absolute and in relative terms, using average mispricing or RMSE.

4.4 Analyzing the pricing errors based on daily data

We have regressed the relative pricing error on the degree of moneyness ratio (I-K)/I, the time to expiration (T-t), the volatility estimate (σ), and the present value of SMI-weighted actual dividend payouts (Divi), in order to test whether the tendencies observed in the preceding paragraph were significant. The regressions were conducted for our six models; however, we present in Table 11 only the results for the three models on which our analysis has been focused [19],[20]. These results inspire the following remarks:

- All the intercept terms are significantly negative, except for puts in the constant dividend/implied volatility model. This suggests that some unknown factors tend to generate underpricing of index options by the BLACK and SCHOLES model.
- The two models based on an implied volatility generate lower adjusted R^2, maybe reflecting non linearities in the pricing error structure of these models. Moreover, the adjusted R^2 are generally higher for puts than for calls. This may be due to the reduced period of observation for European puts: 6 months instead of 2 years for calls.
- The coefficient for the (I-K)/I ratio is always significantly positive for calls, always significantly negative for puts. Thus, in-the-moneyness reduces underpricing, with a larger impact for the constant volatility (and historical volatility) models. Deep out-of the money options are underpriced by the model to the greatest extent, thus reflecting the mispricing structure found by FIGLEWSKI (1988) and SHEIKH (1991). FIGLEWSKI points out that "this result is consistent with the hypothesis that some investors are willing to pay a premium over fair value for options that offer a lottery ticket type of payoff structure: a low probability of a very high percentage profit." An alternative would be that our implied volatility estimate does not include deep out-of-the money options. However, the bias exists across all models, and our tests of the implied volatility models using a sample which included all option premiums for volatility estimation did not change the results.
- The time to expiration has a very low impact, but significantly positive in most cases. Thus, the model underprices options which are close to expiration and overprices those for which maturity is far in the future. SMITH (1987), HULL and WHITE (1987) and SHEIKH (1991) also found that the BLACK and SCHOLES model underpriced shorter maturity index calls. An explanation could be that we did not include options which were

Table 11: Regression of relative pricing error
(t-statistics in parentheses)

CALLS	Constant volatility, actual dividends	Implied volatility, actual dividends	Implied volatility, constant dividends
Intercept	-0.58	-0.25	-0.29
	(-25.65)	(-12.49)	(-13.84)
(I-K)/I	2.68	0.30	0.48
	(98.73)	(6.18)	(9.50)
T - t	0.00	0.00	-0.00*
	(19.25)	(15.66)	(-0.05)
σ	1.71	0.79	0.90
	(13.65)	(9.59)	(10.59)
Divi	0.00	-0.00*	0.01
	(4.16)	(-0.33)	(36.18)
Adj. R^2	0.53	0.04	0.14
PUTS	Constant volatility, actual dividends	Implied volatility, actual dividends	Implied volatility, constant dividends
Intercept	-0.39	-0.12	0.03*
	(-46.56)	(-6.26)	(1.65)
(I-K)/I	-1.46	-0.24	-0.02*
	(-58.57)	(-13.46)	(-0.91)
T - t	0.00	0.00	0.00
	(4.87)	(2.99)	(12.03)
σ	...	0.09*	- 0.27
	...	(1.45)	(-3.70)
Divi	-0.16	-0.03	-0.04
	(-12.63)	(-2.95)	(-2.91)
Adj. R^2	0.70	0.10	0.11

* : An asterisk denotes lack of significance at 5% level.

under two weeks from expiration in our implied volatility estimate. However, testing the implied volatility models using a complete sample of options did not change the result.

- As expected, an increase in the volatility estimate tends to increase the model price (reduce underpricing). However the coefficient is not significant, or negative, in the implied volatility models for puts.
- Lastly, actual SMI-weighted dividend payouts have a very low impact on call mispricing, although significantly positive in most cases. They always have a significant and negative impact on put mispricing. This result shows that the influence of dividends is underestimated by the model - even the model with discrete dividend payouts. Payments of dividends do not decrease the theoretical call price (increase the theoretical put price) as much as they should to reflect behavior in the market.

4.5 The results based on intradaily data

The tests presented in the previous paragraphs were performed again using our intradaily data base for 1992. We tested four models which differ according to the volatility estimate (daily historical volatility, or implied volatility of the last call/put traded), and the underlying instrument (SMI index itself, or futures on the SMI index). The four models used actual dividends. Our tests using the futures as the underlying instrument were motivated by several comments pointing to the fact that SOFFEX participants trade SMI index options as if these options were options on the futures - not options on the index. For tests using the futures, we modified formulas (9) and (10) appropriately, and we replaced the volatility estimates of the index by those of the futures. The overall results for the four models are presented in Table 12 in terms of RMSEs computed for relative pricing errors. The table shows that taking the futures into account does not improve the results when the historical volatility

Table 12: RMSEs for tests using intradaily data

	Historical volatility	Implied volatility
Index	Calls : 0.49 Puts : 0.58	Calls : 0.03 Puts : 0.03
Futures	Calls : 0.54 Puts : 0.53	Calls : 0.16 Puts : 0.23

Table 13: Results with intradaily data. Model using historical volatility and actual dividend payouts.

Absolute pricing error: model price - market price

	CALLS			PUTS		
	N	m [t stat]	RMSE	N	m [t stat]	RMSE
Total	6118	-11.70 [-70.25]	17.51	7652	-7.74 [-176.45]	8.63
In-the-money	368	-42.04 [-42.19]	46.19	--	--	--
At-the-money	5750	-9.76 [-76.13]	13.77	7242	-7.98 [-180.62]	8.81
Out-of-the-money	--	--	--	410	-3.43 [-30.13]	4.14
Expiration < 2 month	4227	-6.43 [-107.85]	7.51	5554	-6.64 [-160.18]	7.32
Expiration 2 to 4 months	1740	-21.77 [-52.63]	27.78	2055	-10.74 [-120.53]	11.48
Expiration > 4 months	151	-42.99 [-53.35]	44.12	43	-5.33 [-10.10]	6.35

Relative pricing error: (model price - market price) / market price

	CALLS			PUTS		
	N	m [t stat]	RMSE	N	m [t stat]	RMSE
Total	6118	-0.42 [-121.96]	0.49	7652	-0.50 [-142.85]	0.58
In-the-money	368	-0.33 [-35.28]	0.38	--	--	--
At-the-money	5750	-0.42 [-118.08]	0.50	7242	-0.48 [-138.20]	0.56
Out-of-the-money	--	--	--	410	-0.86 [-66.40]	0.90
Expiration < 2 month	4227	-0.39 [-89.02]	0.48	5554	-0.46 [-111.08]	0.55
Expiration 2 to 4 months	1740	-0.45 [-92.40]	0.50	2055	-0.61 [-102.89]	0.67
Expiration > 4 months	151	-0.65 [-56.63]	0.66	43	-0.31 [-10.03]	0.37

is used, and deteriorates them considerably when using implied volatility. Table 13 and 14 detail the results for the two models based on the spot index [21]. Comparing Table 13 with Table 8 (or with results from the historical volatility model using daily data), we can see that the performance of the model has not improved over time, or by switching to intradaily transaction data. All the pricing errors are significantly negative and are quite large: an average of 42% for calls and 50% for puts. Comparing Table 14 with Table 9, we see that the implied volatility model, which dominated already, has still

Table 14: Results with intradaily data. Model using implied volatility and actual dividend payouts

Absolute pricing error: model price - market price

	CALLS			PUTS		
	N	m [t stat]	RMSE	N	m [t stat]	RMSE
Total	6118	0.065 [6.49]	0.79	7652	0.01 [2.30]	0.48
In-the-money	368	0.033* [0.43]	1.47	--	--	--
At-the-money	5750	0.067 [7.06]	0.73	7242	0.014 [2.42]	0.49
Out-of-the-money	--	--	--	410	-0.009* [-0.58]	0.32
Expiration < 2 month	4227	0.096 [8.24]	0.76	5554	0.015 [2.29]	0.48
Expiration 2 to 4 months	1740	-0.007* [-0.36]	0.81	2055	0.009* [0.86]	0.47
Expiration > 4 months	151	-0.052* [0.51]	1.26	43	-0.096* [-1.17]	0.55

Relative pricing error: (model price - market price) / market price

	CALLS			PUTS		
	N	m [t stat]	RMSE	N	m [t stat]	RMSE
Total	6118	0.002 [6.25]	0.03	7652	0.001 [2.11]	0.03
In-the-money	368	0.001* [1.01]	0.01	--	--	--
At-the-money	5750	0.002 [6.17]	0.03	7242	0.001 [2.25]	0.02
Out-of-the-money	--	--	--	410	0.002* [0.50]	0.07
Expiration < 2 month	4227	0.003 [6.50]	0.03	5554	0.001* [1.82]	0.03
Expiration 2 to 4 months	1740	0.000* [0.45]	0.02	2055	0.001* [1.26]	0.03
Expiration > 4 months	151	0.001* [0.53]	0.02	43	-0.006* [-1.21]	0.03

* : An asterisk denotes lack of significance at 5% level.

Table 15: Regression of relative pricing error (intradaily data) (t statistics in parentheses)

CALLS	Historical volatility	Implied volatility
Intercept	-0.78 (-62.76)	0.03 (16.66)
(I-K)/I	10.92 (112.25)	0.24 (11.77)
T - t	-0.00 (-9.21)	-0.00 (-3.89)
σ	7.36 (45.25)	-0.22 (-15.04)
Divi	-0.01 (-29.06)	0.00 (7.12)
Adj. R^2	0.70	0.04
PUTS	**Historical volatility**	**Implied volatility**
Intercept	-0.88 (-121.37)	-0.03 (-7.14)
(I-K)/I	-13.51 (-264.20)	-0.09 (-4.43)
T - t	-0.00 (-10.91)	0.00* (1.69)
σ	5.75 (68.05)	0.19 (7.73)
Divi	0.01 (71.99)	0.00* (1.16)
Adj. R^2	0.91	0.01

* : An asterisk denotes lack of significance at 5% level.

improved its performance. The pricing errors are quite small, and although they remain significant on average, they have become unsignificant for several option categories: 7 out of 10 in relative terms. The model still overprices calls, but underpricing of puts has been replaced by almost exact pricing: average relative error of 7 in 10'000 (rounded to 1 in 1'000 in Table 14). Only at-the-money puts are significantly overpriced at the 5% level. As with daily data, in-the-money calls and calls in the intermediate maturity range (2 to 4 months) are priced most accurately. This may be explained by more active trading, and thus higher liquidity in this segment of the SMI calls market.

4.6 Analyzing the pricing errors based on intradaily data

The results of the regression analysis are presented in Table 15. Most of the regression coefficients are significant, indicating that the BLACK-SCHOLES model is characterized by serious parameter biases, more particularly when the historical volatility is used. Comparing Table 15 with Table 11, it is startling to see that model heterogeneity in terms of regression fit has sharpened: We get adjusted R^2 of 70% and 91% for the historical volatility model, respectively for calls and puts, compared with values of 4% and less than 1% for the implied volatility model. Using the latter suppresses most of the parameter biases. It is also interesting to note that the signs of the coefficients are not stable across both data sets. For instance, using the implied volatility model, only the strike price bias is confirmed. It is significantly positive for calls, and negative for puts, in both Tables 11 and 15. All the other coefficients either have opposite signs, or have become unsignificant. Thus, our regression analysis across both data sets confirms that in-the-moneyness reduces underpricing (increase overpricing) of index options by the BLACK and SCHOLES model. Finally, one notes that the volatility bias remains quite significant, even in the case of the implied volatility model.

5. Conclusion

The purpose of this research was twofold: on one hand, test some basic arbitrage relations on the Swiss index options market; on the other hand, verify that the BLACK and SCHOLES formula prices European options accurately on this market. We used two data set for our tests: daily data over the two-year period 1989-1990, and intradaily data over a period of four months in 1992.
Results in Section 3 point out that basic arbitrage

relationships seem to be frequently violated on the SMI options market, even when intradaily transaction data are used, and market participants have had time to adjust to trading in this new instrument. Profitable arbitrage opportunities are significantly reduced when transaction costs are taken into account, but do not vanish altogether. A thinly traded market, asynchronous data problems for the daily data tests, and the fact that our tests are conducted ex post may explain the remaining violations.

Results in Section 2 and 4 indicate that the BLACK and SCHOLES pricing formula is not quite satisfactory for the purpose of valuating European SMI options. Firstly, basic assumptions of the model - lognormality of changes in the index value and constancy of the index return volatility - are not verified [22]. Secondly, the model misprices calls and puts significantly on average, even when the model performance is (somewhat artificially) improved by using an implied volatility estimate. In this latter case, the model overprices calls and underprices puts on average, when daily data are used for the tests. The results from intradaily data show that overpricing of calls is reduced, and that underpricing of puts has been turned into slight overpricing. Comparison between a model with actual (discrete) dividend payouts and a model with constant (continuous) dividend flow has demonstrated the importance of using the former when analyzing the Swiss stock market derivative instruments.

Mispricing by the BLACK and SCHOLES model has also been submitted to regression analysis. We have found several parameter biases, especially when we used a constant or historical volatility estimate. However, most of these biases are not stable across observation periods. Only the strike price bias seems to be confirmed: out-of-the-money options are underpriced by the model, whereas in-the-money options tend to be overpriced. This could possibly be explained by liquidity considerations and/or by the fact that some market participants are willing to pay a premium for the leverage provided by out-of the-money options. Taken together, these results do not lead to an outright rejection of the BLACK and SCHOLES formula applied to SMI index options, but they point to the fact that some of the model's simplifying assumptions matter when the model is used in an environment characterized by non stationary volatility and infrequent trading.

Footnotes

[1] In 1993, the Swiss Market Index is composed of 23 stocks, representing 15 firms only.

[2] SBC = Swiss Bank Corporation; SPI = Swiss Performance Index.

[3] In 1993, the same five firms account for 77.67% of the SMI. These firms are : Ciba Geiby, Nestlé, Roche, Sandoz and Union Bank of Switzerland.

[4] If the probability that the WILK-SHAPIRO W statistic being less than the WN value (corresponding to the normal distribution assumption) is sufficiently small, then the null hypothesis (normality) is rejected.

[5] The characteristics of the implied volatility of SOFFEX stock and index options are analyzed by STUCKI (1992b). His results, based on data for 1988 and 1989, confirm that the implied volatilities fluctuate widely from day to day.

[6] For deep out-of-the-money options, the spread may account for 40% of the bid price.

[7] These data were provided by SOFFEX and Telekurs. SOFFEX provided SMI call and put premiums, for the various expiration dates and strike prices, volumes of options exercised at each date, as well as SMI values. Telekurs provided the daily EuroSwiss franc interest rates for various maturities, and the daily closing prices of the stocks included in the index. SOFFEX also supplied the halfyearly weights assigned to these stocks, and information about changes in the capital structure of the component firms. Finally, dividends paid on the stocks included in the index were provided by Telekurs, through the intermediary of the Union Bank of Switzerland.

[8] The data were compiled with the help of the Computer Center at the University of Geneva and the data flow was provided by Reuters.

[9] EVNINE and RUDD (1985) who tested the same relationship using S&P and MMI calls also found violations in approximately 2% of the cases tested.

[10] LEFOLL, ORMOND and VELASQUEZ (1990) tested

boundary conditions using intra-daily common stock option prices on the SOFFEX. They took transaction costs into account and found that there were no violations of the lower boundary conditions.

[11] If one looks at the mean and standard deviation of Diff 1 and Diff 2 in panel (A). Furthermore, in a discussion of our paper, STUCKI (1992a) reports that from August to November 1989, one could identify 6,3% of violations of the long hedge condition against 56,8% for the short hedge condition.

[12] We assume zero transaction costs when trading in the default free bonds market.

[13] All the computations apply to an average value of the SMI of 1650.

[14] These are illustrative examples that have ignored the privileges or absence of them some market makers may have when trading the component stocks of the SMI or when trading combined option positions.

[15] The standard deviation of the absolute bid-ask spread was equal to 2.8 frs. in 1992.

[16] We exclude values of options which are at least 10% out-of-the-money and values of options with expiration date under 2 weeks away.

[17] FIGLEWSKI (1988) and SHEIKH (1991) use data on American options to test the BLACK-SCHOLES model.

[18] This may be explained by the fact that out-of-the-money options were excluded from the computation of the implied volatility estimates.

[19] The results from the other three models are very similar to those we present. They are available from the authors upon request.

[20] We do not report the DW statistic as our data sample is a mixture of cross-section and time-series data.

[21] Unfortunately, we do not have data for in-the-money puts and out-of-the-money calls, due to a programming failure. The screening of data flow from Reuters was not adapted as the index moved up and down during the first semester of 1992.

[22] Additional research by the same authors has shown that a stochastic volatility model does not improve pricing accuracy, compared with the implied volatility BLACK-SCHOLES model.

References

BEINER, N. (1991): "Evaluation et conditions d'arbitrage sur le marché des options sur l'indice SMI : quelques résultats préliminaires", Finanzmarkt und Portfolio Management 5, pp.270-279.

BLACK, F. and M. SCHOLES (1973) : "The pricing of options and corporate liabilities", Journal of Political Economy 81, pp. 637-654.

DAY, T. and C. LEWIS (1988): "The behavior of the volatility implicit in the prices of index options", Journal of Financial Economics 22, pp. 103-122.

EVNINE, J. AND A. RUDD (1985): "Index options : The early evidence", Journal of Finance 40, pp. 743-756.

FIGLEWSKI, S. (1988): "Arbitrage based pricing of stock index options", Review of Futures Markets 7, pp. 250-270.

FRANKS, J. and E. SCHWARTZ (1991): "The stochastic behavior of market variance implied in the prices of index options", Economic Journal 101, pp. 1460-1475.

FRENCH, K. and R. ROLL (1986): "Stock return variances: The arrival of information and the reaction of traders", Journal of Financial Economics 17, pp. 5-26.

HARVEY, C. and R. WHALEY (1991): "S&P 100 index option volatility", Journal of Finance 46, pp. 1551-1561.

HARVEY, C. and R. WHALEY (1992): "Market volatility prediction and the efficiency of the S&P 100 index option market", Journal of Financial Economics 31, pp. 43-73.

HULL, J. (1989) : "Options, Futures and Other Derivative Securities, Englewood Cliffs: Prentice Hall.

HULL, J. and A. WHITE (1987): "The pricing of options on assets with stochastic volatilities", Journal of Finance 42, pp. 281-300.

KAPLANIS, C. (1986): "Options, taxes and ex-dividend day behavior" Journal of Finance 61, pp. 411-424.

LEFOLL, J. (1994): "Cours de transaction et parité des options sur actions à la SOFFEX", Journal de la Societé de Statistique de Paris 135, No.2.

LEFOLL, J., ORMOND, R. and M. VELASQUEZ (1990): "Arbitrage conditions for option pricing on the SOFFEX", Finanzmarkt und Portfolio Management 4, pp. 129-143.

PIROTTE, H. and P. TAMBURINI (1994): "Evaluation et contenu informationnel du futures sur l'indice SMI", Working paper, HEC Lausanne.

SHAPIRO, S. and M.B. WILK (1965): "An analysis of variance test for normality (complete samples)", Biometrika 52, pp. 591-611.

SHEIKH, A. (1991): "Transaction data tests of S&P 100 call option pricing", Journal of Financial and Quantitative Analysis 26, pp. 459-475.

SMITH, P. (1987) : "Stock index options : Analysis, valuation and empirical tests", Doctoral dissertation, New York University.

STUCKI, T. (1992a) : "Comments to 'Market efficiency and index option pricing", Meeting of the European Finance Association, Lisbon, August.

STUCKI, T. (1992b): "Eigenschaften der impliziten Volatilitäten der SOFFEX-Optionen", Finanzmarkt und Portfolio Management 6, pp. 396-413.