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The Valuation of Options for Constant Elasticity of Variance Processes

A Review of the Theory and Empirical Evidence for Options Written on Swiss Stocks

1. Introduction

In this study, we use daily prices to estimate the elasticity factors of a sample of Swiss stocks following BECKERS'(1980) lines for the estimation of the characteristic exponent in COX's (1975) and COX/ROSS' (1976) Constant Elasticity of Variance (CEV) processes. We investigate the cross firm, as well as the time properties of the elasticity factors and assess the relative pricing performance of two option pricing models, namely the Black/Scholes model and the square root model. In contrast to BECKERS (1980), the pricing performance test is performed using actual call option contracts traded on the Swiss Options and Financial Futures Exchange (SOFFEX), instead of simulations. The motivation for this paper rests on two considerations. First, the stochastic volatility problem has received considerable attention in the financial literature during the past two decades. Such attention is justified by the fact that the volatility is one of the inputs required in determining the equilibrium prices of many financial assets and a correct specification of the process governing the second order moment is hence critical (BOYLE/ ANANATHANARAYANAN (1977), MERTON (1976a and b), RENDLEMAN/O'BRIEN (1990)). In the framework of option pricing, the most popular model (BLACK/SCHOLES (1973) model) relies on the assumption of a constant variance through time. Empirical evidence however suggests that the assumption of a constant variance is inconsistent and one then easily understands the tremendous effort to provide an alternative option valuation model that explicitly takes account of the non stationarity of the stock price returns (HOFMANN/PLATEN/ SCHWEIZER (1992), SCHWERT/SEGUIN (1990), SCOTT (1987)). Second, and to focus on Switzerland, one must reckon that, little has been done on the characterization of the stock price distribution in relation to the pricing of options written on Swiss stocks. CHESNEY/CHRISTOPHI/GIBSON/LOUBERGE/SCHLAFFER (1992) showed that the returns to the Swiss Market Index were leptokurtic and that a stochastic volatility option pricing model may be more suited for the valuation of the SMI option contracts. STUCKI (1992) uses the implied volatility to show that, intradaily, the implied volatilities are more or less constant while they change heavily from day to day. More recently, CHESNEY/GIBSON/LOUBERGE (1993) conducted tests for the CEV and stochastic volatility models for the pricing of SMI options.

This paper is similar in spirit to that of BECKERS (1980) but its major contribution lies in the exten-

* I am very grateful to Professor R. Gibson and Mr. D. Mirlesse for inspiration and helpful comments, as well as to an anonymous referee for his detailed comments and critics which led to a complete revision of the first version of the paper. I am also indebted to N. Tuchschmid for his suggestions at the early stages of this paper and to L. Gardiol an D. Isakov for computer assistance.

sion of Beckers' work to test the behaviour of the elasticity factors through time and to perform a comparative pricing performance test on the Black-Scholes model and the square root model. Further more, it supplements the work on SMI option contracts with a survey of option contracts written on individual Swiss stocks.

The paper is structured as follows. Section 2 presents an overview of the Constant Elasticity of Variance processes, as well as a review of some past empirical results on the subject. Section 3 covers the empirical validation of the CEV processes in the Swiss context, with a description of the data and methodology. Section 4 discusses the pricing performance of the Black and Scholes and the square root models. Section 5 summarizes and concludes the paper.

2. The Constant Elasticity of Variance Processes

2.1 The Theory

The theory of the Constant Elasticity of Variance processes (CEV) stems from the work of COX (1975) and COX/ROSS (1976). The idea for this class of models originated from the search of an option pricing formula that explicitly takes into account the heteroskedasticity of stock price returns evidenced by casual empirism (BLACK (1976), FAMA (1970), BLATTBERG/GONEDES (1974)). The concern for the properties of the stock price distribution is justified by the potentially high sensitivity of the expected option value at maturity to the characteristics of the underlying distribution.

The basis of the CEV processes is the assumption that the volatility of equity returns is inversely linked to the stock price, that is, when the stock price rises, its volatility has the tendency to fall and vice versa. The economic foundations of such a relationship are established by using arguments drawn from the theory of the firm. In fact, we know from the work of MILLER/MODIGLIANI (1958) that the value of the firm is independent of its capital structure in a world without taxes and bankruptcy costs; only the investment policy and the operating risk inherent to it may determine the value of the firm. For strategic reasons, firms may internally change their common stock return distribution through technological innovations and/or mergers and acquisitions. If the risk taken on new projects is high relative to the promised cash inflows, this will cause the firm's value to fall while its volatility expressed in terms of the standard deviation will increase. FAMA (1970) supplements this explanation by arguments drawn from multiperiod consumption-investment theory. Departing from state dependent utility functions (which imply that tastes for given bundles of consumption goods can be state dependent) and the fact that the investment opportunities available in any given future period may depend on events occurring in preceding periods, FAMA (1970) shows that, such a universe does not lend itself to securities' prices whose volatilities are constant. The constant volatility hypothesis goes with the restrictive assumption that the consumer's tastes for given bundles of consumption goods and services are independent of the state of the world. Thus, if in each period aggregate consumer-investors plan their consumption and investment over multiple future periods, then the variances for securities may change over time as new information arises and new individuals (preferences) bid for risky assets in the capital markets.

It is worth to mention that several other directions have been taken by researchers in modelling the heteroskedasticity of the stock price returns. ARCH-type (Autoregressive Conditional Heteroskedasticity) and GARCH-type (Generalized ARCH) models establish a conditional functional relationship between the current volatility and the past volatilities. In the case of ARCH models introduced by ENGEL (1982) if Y is the state variable, the value of Y at time t writes as follows:

$$Y_t = a_0 + \sum_{i=1}^{j} a_i Y_{t-i} + \varepsilon_t \qquad (1)$$

where ε_t is the white noise of mean zero and variance

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \tag{2}$$

While equation (1) is a classical ARMA relation, the variance of the white noise as given by equation (2) is set to be dependent on past realizations of ε (thus removing the assumption of homoscedasticity). With financial data, this model captures the tendency for volatility clustering, i.e., for large (small) price changes to be followed by other large (small) price changes, but of unpredictable sign.
STEIN/STEIN (1991) analytically studied the case where the volatility of the stock price returns is driven by an arithmetic Ornstein-Uhlenbeck process such that:

$$dP = \mu P dt + \sigma P dZ_1 \tag{3}$$

and

$$d\sigma = -\delta(\sigma - \theta)dt + kdZ_2 \tag{4}$$

where P is the stock price, σ is the "volatility" of the stock, k, μ, δ, and θ are fixed constants, and dZ_1 and dZ_2 are two independent Wiener processes. As is evident from (4), the volatility is supposed to be mean reverting, the long run average level being θ.
HULL/WHITE (1987) concentrate on the case where the variance σ^2 of the stock returns is assumed to satisfy the following stochastic process:

$$d\sigma^2 = \alpha\sigma^2 dt + \xi\sigma^2 dw \tag{5}$$

with α and ξ being two constants independent of the stock price, and dw a standard Brownian motion. Similar models have been studied by JOHNSON/SHANNO (1987), SCOTT (1987), and WIGGINS (1987), among others.
The Constant Elasticity of Variance class of models which are the concern of this paper establish a deterministic inverse relationship between the stock price and its volatility. In a standard log-normal diffusion process, the stock price (S) is supposed to satisfy the stochastic differential equation:

$$dS = \mu S dt + \sigma S dZ \tag{6}$$

But when the volatility is inversely linked to the stock price, with characteristic exponent θ, the variance (υ^2) is set to be equal to:

$$\vartheta^2 = \sigma^2 S^{\theta-2} \tag{7}$$

and the stochastic differential equation (6) becomes:

$$dS = \mu S dt + \sigma S^{\theta/2} dZ \tag{8}$$

The contingent claim valuation formula that takes account of the inverse relationship is given below. Let r denote the riskless interest rate, τ , the time remaining till the expiration of a European call denoted C with exercise price E. The set of assumptions necessary for the CEV processes are the same as for the Black/Scholes model, except that, the variance of the stock price returns follows the process described in equation (7). Note that, it follows from (7) that the elasticity of the variance (υ^2) with respect to the stock price (S) is equal to:

$$\eta = \frac{\partial\vartheta^2}{\partial S} X \frac{S}{\vartheta^2} = \theta - 2 \tag{9}$$

Since θ is supposed to be a constant, the elasticity of the variance as given by relation (9) is constant (hence the name of Constant Elasticity of Variance). Within the framework of BLACK/SCHOLES (1973) or the fairly general framework of HARRISSON/KREPS (1979), the partial differential equation that the option price must satisfy when σ^2 is constant is known:

$$\tfrac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 + \frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} - rC = 0 \tag{10}$$

But when the variance is set to be dependent of the stock price level as specified by relationship (7), the differential equation (10) becomes:

$$\frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^\theta + \frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} - rC = 0 \tag{11}$$

COX (1975) solves equation (11) by using the neutral hedge argument and under rational option price boundaries constraint [1]. The manipulation of Cox's formula for empirical purpose is not as easy as the Black/Scholes' formula. Only MAC BETH/ MERVILLE (1980) have been able to test it after imposing some restrictions on the constants of the model.

Because of the difficulties in implementing Cox's formula, researchers have concentrated on three special cases of CEV processes. When $\theta = 2$, the elasticity of the variance as given by equation (9) is equal to zero and relationship (11) reduces to equation (10) and the partial equilibrium price of the call option is given by the Black and Scholes' formula in the absence of dividend payout:

$$C_{bs} = S \cdot N\left[\frac{\ln(\frac{S}{E}) + (r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}\right] - e^{-r\tau} \cdot E \cdot N\left[\frac{\ln(\frac{S}{E}) + (r - \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}\right] \tag{12}$$

N[.] stands for the cumulative normal distribution function.

When $\theta = 1$, the elasticity of the variance equals (-1) and

$$\vartheta = \sqrt{\vartheta^2} = \frac{\sigma}{\sqrt{S}}$$

that is, the volatility is inversely proportional to the square root of the equity price. Cox gives the valuation formula (square root model) in this special case, as reported in BECKERS (1980). Let:

$$y = \frac{4rS}{\vartheta^2(1 - e^{-r\tau})}$$

$$z = \frac{4rE}{\vartheta^2(e^{r\tau} - 1)}$$

and w a parameter which takes on values 0 and 4 such that:

$$h(w) = 1 - \frac{2}{3}(w+y)(w+3y)(w+2y)^{-2}$$

$$q(w) = \frac{[1 + a - b - c]}{\left[2h^2 \frac{(w+2y)}{(w+y)^2}(1 - (1-h)(1-3h))\frac{(w+2y)}{(w+y)^2}\right]^{1/2}}$$

with

$$a = h(h-1)(w+2y)(w+y)^2$$

$$b = \frac{h(h-1)(2-h)(1-3h)(w+2y)^2}{2(w+y)^4}$$

$$c = \left(\frac{z}{w+y}\right)^h$$

and the value of the call in the square root process writes as follows:

$$C_{sm} = S \cdot N(q(4)) - Ee^{-r\tau} \cdot N(q(0)) \tag{13}$$

N(.) still stands for the cumulative normal distribution function.

When $\theta = 0$, the elasticity of the variance relative to the stock price is equal to (-2) and the variance is inversely proportional to the stock price. COX/ ROSS (1976) give the proper call valuation formula known as the absolute model:

$$C_{ab} = (S - Ee^{-r\tau})N(y_1) + (S + Ee^{-r\tau})N(y_2) + v[n(y_1) - n(y_2)] \quad (14)$$

where N(.) and n(.) stand for the cumulative normal distribution and the unit normal density function, respectively and

$$v = \vartheta\sqrt{\frac{1-e^{-2r\tau}}{2r}}$$

$$y_1 = (S - Ee^{-r\tau})/v$$

$$y_2 = (-S - Ee^{-r\tau})/v$$

2.2 Past Empirical Work on the CEV Processes

On the empirical side, a major contribution to the CEV processes is that of MAC BETH/MERVILLE (1980). Based on the closing prices of all call options written on six US stocks, the two authors assessed the relative performance of the Black/ Scholes model and Cox's original CEV model. The main result of their study is that Cox's formula tends to fit market prices better than the Black/ Scholes formula and may thus be a prime alternative to circumvent the systematic bias often observed with the Black and Scholes' formula. However, as MANASTER (1980) comments, this finding is not surprising given the procedure followed by Mac Beth/Merville for the estimation of the constants of the model. In fact, MacBeth/Merville use a three-step procedure to estimate the parameters θ and σ. First, they use the Cox's model and search for values of θ for which the implied values of σ are most nearly constant for each stock. Second, they use the Black/Scholes model and their previous result to obtain the implied values of υ. The third step involves the estimation of σ on a daily basis. This procedure therefore ensures that the best information available from the Black/ Scholes model is extracted and the reported outperformance of the CEV models comes at no surprise.

BECKERS (1980) for his part suggested an original procedure - to be detailed in Section 3 - for the estimation of θ and based on the results of a simulation, he also concludes to the superiority of the CEV models. FERRI/KREMER/OBERHELMAN (1986) followed Beckers' methodology to analyze the price characteristics of a sample of warrants written on 50 stocks and under the constant elasticity of variance hypothesis; their finding does not support the inverse relationship, but due to special issues encountered in the pricing of corporate warrants, care must be taken in generalizing their conclusions [2].

3. Empirical Tests of the CEV Models in the Swiss Context

3.1 Data and Methodology

The purpose of this paper is to estimate the vector of elasticity factors of a sample of Swiss stocks following the basic lines of BECKERS (1980). Moreover, the time properties, as well as cross firm properties of these elasticity factors will be explored and a comparative performance test will be performed on the Black/Scholes and the Square Root option pricing models. Our sample is limited to the basket of Swiss stocks continuously traded. We collect daily prices for each stock over a four year time horizon (from November 1, 1988 to October 30, 1992) and call option prices for six stocks over a full month (November 1992). The stock and call prices are obtained from the data bases, respectively of the Finance Group and of CEDIF at the University of Lausanne.

Since the stock prices from the data base are adjusted for stock splits, we just incorporated the dividends to the ex-dividend day stock price, assuming a 100% price decline after the dividend payout [3].

3.2 Estimation and Tests of the Elasticity Factors

From the work of BECKERS (1980), the elasticity factor (θ) can be obtained from the regression coefficient of the following equation:

$$\mathrm{Ln}\left|\mathrm{Ln}\frac{S_{t+1}}{S_t}\right| = a + b\,\mathrm{Ln}\,S_t \tag{15}$$

In fact, we know that, in the CEV processes

$\upsilon = \sigma S^{(\theta-2)/2}$

and

$$\mathrm{Ln}(\vartheta) = \mathrm{Ln}\sigma + \frac{(\theta-2)}{2}.\mathrm{Ln}S.$$

Beckers argues that, since the CEV class of distributions differs from the lognormal in scale only , the ratio of

$$E\left|Ln\frac{S_{t+1}}{S}\right|$$

to the standard deviation is approximately constant for the entire class. We can therefore approximate

$$\frac{(\theta-2)}{2}$$

by the regression coefficient (b) of equation (15). The justification for such a procedure lies in the fact that Beckers did not have intra-daily prices for his analysis. He uses daily closing prices and this is certainly the major limitation of his study.
Note however that, in order for

$$\mathrm{Ln}\left|\mathrm{Ln}\frac{S_{t+1}}{S_t}\right|$$

to be defined, S_{t+1} must be different from S_t. In cases where $S_{t+1} = S_t$, Beckers adjusts S_t by adding 0.0625 (the tic at that time). While this methodology may be appropriate for US stocks which have unitary market prices hardly above \$100, it cannot be

Table 1: Grid of tics

Stock Price Range	Tic
$1 \leq S \leq 50$	0.05
$50.25 \leq S \leq 200$	0.25
$200.50 \leq S \leq 500$	0.50
$501 \leq S \leq 2000$	1.00
$2005 \leq S \leq 10\,000$	5.00
$S > 10\,000$	25.00

applied uniformly to Swiss stocks which have unitary prices ranging from more than Sfr 100 to more than Sfr 5000. We therefore decided to adjust S_t using the grid of tics obtained from the Geneva Stock Exchange, as shown in Table 1.
Since our sample consists of the continuously traded stocks, there were very few cases where S_t needed adjustment. We run the simple regression as specified in equation (15) by considering three periods: a four-year time horizon, a 12-month time horizon (four regressions for each stock) and a 6-month time horizon (eight regressions for each stock). The justification for such a subdivision comes from our aim to test the behaviour of the elasticity factors across time. Table 2 shows a synthesis of the estimations for the period running from November 1, 1991 to October 30, 1992. The estimations for the four-year period are relegated in exhibit 1 because our analysis is going to be concentrated on the one-year estimations. The last column of table 2 shows the F statistic which has been used to perform a Chow test for the stability of the regression coefficients (b). We do not report the results of the estimations for the-month time horizon because the hypothesis that the six-month elasticity factors may be different from the one-year characteristic exponents was rejected. We thus keep the one-year estimates for the remainder of the study.

Some interesting results appear in table 2 (and in exhibit 1):

- the first observation one can make is that the R^2 are very low; that has almost always been a

Table 2: Estimations of parameters θ for the period running from 11/01/1991 to 10/30/1992

	Security	b	a	F-stat	R^2	t-stat for $b = 0$	t-stat for $b = -1/2$	t-stat for $b = -1$	t-stat for $a = 0$	$\theta = 2b + 2$	F-stat for Chow test
1	Adia I	-0.344	-2.212	2.15	0.009	-1.47	0.66	2.81	-1.63	1.312	5.94*
2	Alusuisse-Lonza I	-1.660	5.309	7.77*	0.030	-2.78	-1.94@	-1.10	1.45	-1.320	3.30*
3	Alusuisse-Lonza N	-1.288	2.936	3.38*	0.014	-1.84@	-1.12	-0.41	0.69	-0.576	1.89
4	Ascom I	-1.398	5.807	11.51*	0.045	-3.40	-2.18	-0.96@	1.85	-0.796	1.61
5	BBC I	-1.083	4.060	2.53	0.010	-1.59	-0.85	-0.12	0.72	-0.166	8.58*
6	BBC N	-1.532	5.252	3.69*	0.016	-1.92@	-1.29	-0.66	0.99	-1.064	5.52*
7	BBC PS	-0.768	0.193	1.53	0.006	-1.23	-0.43	0.37	0.04	0.460	6.63*
8	Ciba-Geigy I	-2.433	10.705	3.75*	0.015	-1.93@	-1.53	-1.14	1.31	-2.886	1.99
9	Ciba-Geigy N	-1.820	6.754	4.71*	0.019	-2.17	-1.57@	-0.97	1.24	-1.640	2.56
10	Ciba-Geigy PS	-1.330	3.558	2.80	0.011	-1.67	-1.04	-0.41	0.69	-0.660	1.83
11	CS Holding I	-1.550	6.488	1.50	0.006	-1.22	-0.82	-0.43	0.68	-1.100	1.53
12	CS Holding N	-1.870	5.922	2.51	0.010	-1.58	-1.16	-0.73	0.85	-1.740	9.49*
13	Elektrowatt I	-1.560	6.533	2.22	0.009	-1.49	-1.01	-0.53	0.80	-1.120	1.65
14	Ems I	-4.040	24.522	7.05*	0.028	-2.65	-2.32	-2.00	2.16	-6.080	1.96
15	Fischer I	-1.110	3.014	2.19	0.009	-1.48	-0.81	-0.15	0.58	-0.220	3.03*
16	Holderbank Fin. I	0.310	-6.987	0.09	0.000	0.31	0.81	1.31	-1.13	2.620	3.83*
17	Merkur N	-2.640	9.979	10.00*	0.042	-3.16	-2.56	-1.96@	2.11	-3.280	4.26*
18	Nestlé I	1.760	-17.668	1.87	0.008	1.37	1.75	2.14	-2.00	5.520	4.84*
19	Nestlé N	1.710	-17.282	2.11	0.009	1.45	1.88	2.30	-2.15	5.420	6.72*
20	Nestlé PS	-1.400	4.919	0.91	0.004	-0.95	-0.61	-0.27	0.44	-0.800	6.14*
21	Roche I	-0.200	-3.683	0.05	0.000	-0.22	0.33	0.90	-0.49	1.600	10.10*
22	Roche GS	-0.290	-2.995	0.12	0.000	-0.35	0.24	0.83	-0.44	1.420	7.73*
23	Sandoz PS	0.550	-9.774	0.19	0.000	0.43	0.82	1.22	-0.97	3.100	4.06*
24	SBG I	-0.440	-2.294	0.14	0.001	-0.37	0.04	0.47	-0.29	1.120	2.61
25	SBG N	13.13	-70.490	8.25*	0.031	2.87	2.98	3.09	-3.13	28.260	0.31
26	SBV I	-2.800	10.888	15.60*	0.059	-3.95	-3.24	-2.54	2.72	-3.600	3.84*
27	SBV N	-1.680	4.420	4.36*	0.017	-2.09	-1.46@	-0.84	0.98	-1.360	3.29*
28	SBV PS	-2.990	11.658	7.77*	0.031	-2.79	-2.32	-1.85@	1.95	-3.980	1.96
29	SMH N	0.900	-11.079	7.15*	0.028	2.67	4.15	5.63	-4.74	3.800	3.73*
30	SMH PS	0.950	-11.375	8.59*	0.034	2.93	4.46	6.00	-5.07	3.900	5.57*
31	Sulzer N	-1.225	2.663	5.85*	0.024	-2.42	-1.43@	-0.44	0.83	-0.450	6.25*
32	SGS I	-3.180	17.886	6.59*	0.027	-2.56	-2.16	-1.76@	2.02	-4.360	5.62*
33	SVB I	-1.300	3.665	2.96	0.011	-1.72	-1.06	-0.40	0.71	-0.600	1.68
34	Winterthur I	-1.070	3.524	1.83	0.007	-1.35	-0.72	-0.09	0.54	-0.140	0.77
35	Winterthur PS	-1.710	6.063	9.47*	0.037	-3.08	-2.18	-1.28@	1.69	-1.420	3.42*
36	Zürich I	-4.160	26.424	7.02*	0.028	-2.65	-2.33	-2.01	2.21	-6.320	5.06*
37	Zürich N	-1.330	4.102	1.68	0.007	-1.29	-0.80	-0.31	0.58	-0.660	1.55

* indicates that F-stat is higher than the critical F (3.00 for the 1-year estimations at 95% confidence level and 2.80 for the Chow test at 99% confidence level).

@ indicates that the regression is valid (F-stat is higher than the critical F) but the null hypothesis based on a t test could not be rejected at the 95% confidence level.

characteristic in Beckers' regression, indicating that the specification may be imperfect;

- 30 regression coefficients for the 1-year estimations (and 31 regression coefficients for the 4-year estimations) are negative, giving a first indication about the inverse relationship; but in the following, we will concentrate only on valid regressions, that is, regressions for which the F statistic is greater than the critical F (18 valid regressions);
- when we consider the 1-year estimations and take into account the statistical significance of the regressions, the evidence for the CEV processes is supported only in 15 cases (15 coefficients are negative or equal to zero for valid regressions), at the level of confidence 95%;
- only three out of the 18 valid coefficients are positive (SBG N, SMH N, and SMH PS); the case for positive relationship between the stock price and its volatility can be investigated in the framework of RUBINSTEIN's (1983) displaced diffusion model;
- to check if some of the securities can be characterized as conforming to the Black/Scholes model (that is, $\theta = 2$, or $b = 0$), the t statistic for $b = 0$ has been computed; only three stocks have failed to pass the test (Alusuisse-Lonza N, BBC N and Ciba-Geigy I). These stocks may thus be assumed to conform the Black / Scholes model;
- similarly, tests were performed for $\theta = 1$ ($b = -1/2$) and for $\theta = 0$ ($b = -1$). For four stocks, the hypothesis that $b = -1/2$ ($\theta = 1$) could not be rejected (square root model) whereas five stocks failed to pass the test for $b = -1$ (or $\theta = 0$) and may thus conform to the absolute process;
- in all, six stocks could not be classified in either group;
- strictly speaking, there is no justification for the use of the Black/Scholes option pricing model for the valuation of SOFFEX options since only three characteristic exponents may equal to 2;

Table 3: Extreme values of the estimations

	4-year estimation	1-year estimation
Minimum of b	-2.35	-4.16
Minimum of a	-70.16	-70.49
Minimum of θ	-2.70	-6.32
Maximum of b	13.03	13.13
Maximum of a	8.03	26.42
Maximum of θ	28.06	28.26

- the reasons for the variation of the elasticity factor across securities can only be evidenced by the study of the financial and operating leverage of the firms comprised in the sample. Table 3 below contains a short summary of our analysis.

In order to test the cross firm and cross year behaviour of the elasticity factors, a Chow test was performed. The cross firm stability test was achieved by running a single regression of the following form:

$$\mathrm{Ln}\left|\mathrm{Ln}\frac{S_i(t+1)}{S_i(t)}\right| = a + b\,\mathrm{Ln}\,S_i(t)$$

with $i = 1, 37$ (number of securities) and $t = 1$, number of observations for security i during year 1991/1992 [4]. The idea is to test the hypothesis that b1 = b2 = b3 = ...= b37 = b following Beckers' methodology. We obtain $b = 0.08$ in the single regression and an extremely high value of F statistic(27.428) using the sum of the squared residuals from the single regression, the number of restrictions and the sum of the squared residuals from the 37 individual regressions [5]. We can therefore reject the null hypothesis that one charateristic exponent could be used for all stocks. Similarly, a Chow test was used to see if the 4-year θ (b) can be assumed equal to the one-year θ (b). This step involved running five regressions for each stock in the sample (the 4-year regression and four 1-year regressions). The last column of Table 2 shows the computed F values. Considering a significance level of 99% (critical F = 2.80) the null hypothesis is rejected in 22 cases, indicating that the elasticity factor is changing from year to year. We repeat this procedure by comparing the six-month coefficients to the one-year coefficients and we were unable to reject the null hypothesis in all the cases. Since the six-month estimations are assumed equal to the 1-year estimations, we kept the 1991/1992 estimations for the remainder of the paper. These results are consistent with long term or low speed shifts in θ.

4. Tests of the Black/Scholes model and the square root model

The estimation of the elasticity factors showed that none of the CEV models (including the Black/Scholes model) could be used to price all SOFFEX options. Only COX's (1975) original model may be considered if one wants to apply the CEV models. In practice, the use of the Black/Scholes model is equivalent to imposing the same elasticity factor for all the stocks ($\theta = 2$). The purpose of this section is to compare the ability of the Black/Scholes model (which assumes a perfect inelasticity) and the square root model (which assumes a perfect unitary elasticity) to fit market prices of SOFFEX call options' prices. This procedure is equivalent to imposing $\theta = 2$ and $\theta = 1$ for all the stocks.
Before we go any further in our analysis, it may be interesting for the reader to get a brief insight on the SOFFEX stock options. SOFFEX options (call and put options) are of the american type, that is, they give their holder the right to buy (sell) five stocks (size of the contract) until the expiration date and at the specified exercise price. The range of strike prices is set so that there are always options trading at-the-money, in-the-money and out-of-the-money. The expiration date is the saturday following the third friday of the expiration month. Under the current market structure, during any month, there are options expiring in the three upcoming months (plus options expiring in one of the following four months: January, April, July or October). The transaction fee is 5% of the value of the contract, meaning that any seller of an option will receive 5% less than the quoted price, while a buyer of an option will pay 5% above the quoted price.
We assume that the SOFFEX call prices are efficiently determined and we compare the model prices to the market prices in order to apprehend the importance of the deviations from market prices for each model. We consider 49 call option contracts written on six stocks (see table 4 below): 14 contracts were maturing in January 1993, 18 in February 1993, 17 in April 1993 and three strike prices for each stock and each expiration date. The methodology followed is to compute for each model [6] the daily theoretical prices for the full month of November 1992. We assume no dividend payout, no premature exercise and use a riskless interest rate of 6.20%, which was the average Euro Swiss franc short term interest rate prevailing in November 1992.The historical volatility (1991/1992) is used in the Black/Scholes model whereas υ (see table 4 below) is the relevant parameter in the square root model.

υ is computed using the following formula:

$$\vartheta^2 = \left[\frac{\sigma^2}{\sum_1^n (1/S_t)}\right].n$$

where n stands for the number of observations and σ for the standard deviation in the traditional Black/Scholes model. A complete demonstration of how to obtain υ can be found in FERRI/KREMER/OBERHELMAN (1986). This demonstration is

Table 4: Sample of underlying stocks

	σ	υ	θ
Alusuisse I	30.54%	6.415	-1.320
BBC I	27.94%	17.036	-0.166
SBV I	26.83%	4.476	-3.600
SMH N	35.90%	11.087	3.800
SBG I	30.65%	8.300	1.120
Zürich I	23.28%	10.410	-6.320

based on BECKERS (1980), COX/RUBINSTEIN (1985), and MAC BETH/MERVILLE (1980).
As can be seen from table 4, the underlying stocks are selected to reflect the distribution of the elasticity factors observed above (negative and positive characteristic exponents). Table 5 presents a synthesis of the results stemming from the comparison of both models' pricing performance. The analysis focuses on the pricing bias calculated as follows:

$$\text{Bias} = \frac{[\text{Model Price} - \text{Market Price}]}{\text{Market Price}} X100.$$

For each option contract and for each model, the average bias and the standard deviation are calculated as well as the t ratio. Table 5 is only a short summary of exhibit 2, which contains all the statistics.
The biases are analysed with respect to the time to maturity and the degree of moneyness. The following conclusions can be drawn from table 5 (and from exhibit 2):

- for all options and for the two models, the average bias is positive, although the bias with the Black/Scholes model is smaller than the one with the square root model (13.32% <23.5%);
- the t ratios of the biases for the two models are far above the critical t value (2 with 20 degrees of freedom and at the confidence level 95%) and the two models have similar extreme values of t.
- for all in-the-money options and for short term in-the-money options (expiring in January 1993), the Black/Scholes model systematically yields negative biases (market price > model price) while the sign of the biases is opposite for the square root model;
- for at-the-money options and out-of-the-money options and for all maturity dates, the Black/Scholes model systematically yields positive biases (market price < model price) but the intensity of the bias seems to have no correlation with the expiration date;
- the square root model produces positive biases (market price < model price) for any exercise

Table 5: Comparative pricing performance of two option pricing models

	BLACK/SCHOLES MODEL					SQUARE ROOT MODEL				
	Average bias (%)	Minimum stand. dev.	Maximum stand. dev.	Minimum |t| stat	Maximum |t| stat	Average bias (%)	Minimum stand. dev.	Maximum stand. dev.	Minimum |t| stat	Maximum |t| stat
All options	13.32	1.66	65.11	0.02	18.22	23.50	0.75	52.33	0.20	18.05
In-the-money options	-0.03	1.66	20.59	0.17	9.28	10.12	0.75	19.14	0.20	8.29
At-the-money options	23.60	3.09	18.92	0.76	18.22	27.35	2.87	35.81	1.24	18.05
Out-of-the money options	20.93	5.96	65.11	0.02	8.22	38.11	3.69	52.33	0.69	15.03
January in-the-money options	-13.26	1.71	20.50	0.17	9.28	4.13	0.75	19.14	0.32	1.37
February in-the-money options	6.35	2.04	5.65	0.45	4.32	13.51	2.15	5.49	1.00	8.29
April in-the-money options	6.82	1.66	10.62	0.24	5.87	12.73	1.70	11.10	0.20	7.91
January at-the-money options	12.38	4.52	18.92	0.87	1.72	22.90	4.54	35.81	1.24	3.52
February at-the-money options	28.04	3.09	11.71	0.76	18.22	29.28	2.87	11.87	1.61	18.05
April at-the-money options	26.63	7.10	10.73	0.79	5.36	29.14	6.17	12.63	2.05	5.22
January out-of-the money options	25.43	28.44	53.39	0.36	0.75	41.23	41.10	51.69	0.75	1.01
February out-of-the money opt.	13.61	9.62	19.03	0.33	2.53	38.38	3.69	13.37	1.71	15.03
April out-of-the money options	26.00	5.96	65.11	0.02	8.22	35.44	10.63	52.33	0.69	3.09

price and expiration date;

- the observed biases of the square root model also seem to have no correlation with the time to maturity but they tend to intensify with the extend to which the option is out-of-the-money; this may however be a simple statistical issue;
- the standard deviations of the biases for the two models are unstable across strike prices and expiration dates;
- even for the stock whose θ is closest to 1 (SBG I), the performance of the square root model is not improved;
- we should mention that the intensification of the biases for out-of-the-money options expiring in April may be due to the omission of dividends in our estimations (SBG, SBV and Alusuisse traditionally pay dividends in April).

Obviously, it is difficult on the basis of our findings to conclude to the superiority of one model over the other. Only when the analysis is limited to deep in-the-money options expiring in January, can the square root model be thought to outperform the Black/Scholes model. But taking the results together, one must agree that the solution to the changing volatility problem in option pricing models does not lie in the square root model, at least within the Swiss context. This result is in line with the observed distribution of the elasticity factors, where only 15 stocks (out of 37) could be characterized as following a CEV process stricto sensu.

5. Summary and Conclusions

This paper has investigated Cox's and Cox/Ross' Constant Elasticity of Variance processes in the framework of option valuation in the Swiss context. The methodology used is that of Beckers, but we went two steps further to investigate the time properties of the elasticity factors and to assess the comparative performance of the Black/Scholes model and the square root model. In contrast to Beckers, we use actual option prices to test the two option pricing models and not simulations.The observed distribution of the elasticity factors does not suggest that the CEV processes are generalized in Switzerland. But the elasticity factors can be assumed to remain constant over a reasonable time horizon (one year in our case). The comparative option pricing performance test leads to the conclusion that even if the CEV models have very sound theoretical foundations, their superiority over the Black/Scholes model cannot be presumed. This conclusion is in line with CHESNEY/GIBSON/LOUBERGE (1993) who provide results which do not support the CEV and stochastic volatility models for the pricing of SMI options.

Exhibit 1: Four-year estimations of the CEV model (from 11/01/1988 to 10/30/1992)

	Security	b	a	F-stat	R^2	t-stat for b	t-stat for a	$\theta = 2b + 2$
1	Adia I	-0.431	-1.710	61.21	0.058	-7.81	-4.51	1.138
2	Alusuisse-Lonza I	-0.230	-3.464	1.46	0.001	-1.21	-2.89	1.540
3	Alusuisse-Lonza N	0.058	-5.205	0.06	0.000	0.26	-3.79	2.116
4	Ascom I	-0.883	2.051	36.98	0.036	-6.08	1.78	0.234
5	BBC I	-0.162	-3.602	1.06	0.001	-1.03	-2.75	1.676
6	BBC N	0.160	-6.070	0.60	0.000	0.77	-4.36	2.320
7	BBC PS	0.167	-5.967	1.30	0.001	1.13	-6.17	2.330
8	Ciba-Geigy I	-0.896	0.813	16.35	0.016	-4.05	0.56	0.208
9	Ciba-Geigy N	-0.813	0.171	18.85	0.018	-4.34	0.14	0.374
10	Ciba-Geigy PS	-0.680	-0.606	12.09	0.012	-3.49	-0.49	0.640
11	CS Holding I	-1.389	5.414	19.45	0.019	4.42	2.26	-0.778
12	CS Holding N	-0.893	0.141	12.24	0.012	-3.50	0.09	0.214
13	Elektrowatt I	-0.439	-2.071	1.19	0.001	-1.09	-0.64	1.122
14	Ems I	-1.370	4.707	25.14	0.024	-5.01	2.35	-0.740
15	Fischer I	-0.577	-0.734	10.27	0.010	-3.20	-0.56	0.846
16	Holderbank Fin. I	-0.860	0.249	7.48	0.007	-2.73	0.12	0.280
17	Merkur N	-1.600	4.213	8.87	0.009	-2.98	1.36	-1.200
18	Nestlé I	-0.704	-0.703	3.89	0.003	-1.97	-0.29	0.592
19	Nestlé N	-0.226	-3.882	0.75	0.000	-0.86	-2.22	1.548
20	Nestlé PS	-1.299	4.340	15.02	0.014	-3.87	1.75	-0.598
21	Roche I	-0.255	-3.075	1.54	0.001	-1.23	-1.81	1.490
22	Roche GS	-0.119	-4.377	0.56	0.000	-0.75	-3.60	1.760
23	Sandoz PS	-0.705	0.202	5.37	0.005	-2.31	0.08	0.590
24	SBG I	-1.080	2.022	10.86	0.010	-3.29	0.94	-0.160
25	SBG N	13.03	-70.169	32.29	0.030	5.68	-6.20	28.06
26	SBV I	-0.703	4.467	28.17	0.027	-5.32	2.48	0.594
27	SBV N	-0.834	-0.449	5.28	0.005	-2.29	-0.21	0.330
28	SBV PS	-2.350	8.031	35.79	0.034	-5.99	3.64	-2.700
29	SMH N	0.383	-7.517	9.86	0.009	3.16	-9.61	2.760
30	SMH PS	0.150	-5.848	1.23	0.001	1.11	-6.74	2.300
31	Sulzer N	-0.529	-2.004	2.17	0.002	-1.47	-0.89	0.942
32	SGS I	-0.832	1.006	7.29	0.007	-2.70	0.46	0.336
33	SVB I	-1.660	-0.281	16.80	0.016	-4.11	-0.22	-1.320
34	Winterthur I	-0.723	0.784	3.83	0.003	1.95	0.25	0.554
35	Winterthur PS	-1.160	2.519	13.13	0.013	-3.62	1.20	-0.320
36	Zürich I	-0.857	1.450	5.89	0.005	-2.42	0.53	0.286
37	Zürich N	-1.455	-0.404	6.09	0.006	-2.46	-0.20	-0.910

Exhibit 2: Comparative Pricing Performance of the Black/Scholes and Square Root Models

	BLACK/SCHOLES MODEL			SQUARE ROOT MODEL		
	Average Bias (%)	Stand. Dev. of Bias (%)	t-stat of Bias	Average Bias (%)	Stand. Dev. of Bias (%)	t-stat of Bias
Options January deep in the money						
ZURJAN1700	-15.88	1.71	-9.28	-0.36	0.75	-0.48
SMHJAN1100	-16.97	10.48	-1.62	-6.60	12.13	-0.54
SBGJAN650	-22.21	6.64	-3.34	-4.89	6.99	-0.69
ALUJAN380	-6.54	20.50	-0.32	26.37	19.14	1.37
BBCJAN3200	-1.21	7.09	-0.17	8.49	11.76	0.72
SBVJAN220	-16.78	4.61	-3.64	1.80	5.47	0.32
Options January at the money						
ZURJAN1900	19.91	18.92	1.05	27.00	19.71	1.36
SMHJAN1200	11.78	8.84	1.33	10.21	7.08	1.44
SBGJAN750	7.80	4.52	1.72	16.01	4.54	3.52
ALUJAN420	NA	NA	--	44.45	35.81	1.24
SBVJAN260	10.05	11.52	0.87	16.86	9.86	1.70
Options January out of the money						
ZURJAN2000	10.41	28.44	0.36	52.52	51.69	1.01
SMHJAN1400	40.46	53.39	0.75	31.67	41.86	0.75
SBVJAN300	NA	NA	--	39.51	41.10	0.96
Options February deep in the money						
ZURFEB1800	10.36	2.42	4.28	17.84	2.15	8.29
SMHFEB1200	9.62	5.65	1.70	7.82	3.93	1.99
SBGFEB750	15.28	3.53	4.32	20.95	4.41	4.75
ALUFEB340	-0.93	2.04	0.45	11.20	2.46	4.55
BBCFEB3200	8.44	3.40	2.48	17.76	4.62	3.84
SBVFEB240	-4.67	4.79	0.97	5.50	5.49	1.00
Options February at the money						
ZURFEB1900	31.56	3.28	9.62	36.69	3.60	10.19
SMHFEB1300	39.96	7.58	5.27	20.52	5.82	3.52
SBGFEB800	56.33	3.09	18.22	51.80	2.87	18.05
ALUFEB360	4.26	5.58	0.76	19.13	5.71	3.35
BBCFEB3400	26.90	11.71	2.29	34.02	11.87	2.86
SBVFEB260	9.27	8.16	1.13	13.54	8.37	1.61
Options February out of the money						
ZURFEB2000	25.30	10.00	2.53	55.47	3.69	15.03
SMHFEB1400	NA	NA	--	40.60	9.80	4.14
SBGFEB850	NA	NA	--	NA	NA	--
ALUFEB380	-9.69	9.62	-1.00	20.64	10.60	1.94
BBCFEB3600	33.62	19.03	1.76	54.94	13.37	4.10
SBVFEB280	5.21	15.44	0.33	20.26	11.78	1.71

	BLACK/SCHOLES MODEL			SQUARE ROOT MODEL		
	Average Bias (%)	Stand. Dev. of Bias (%)	t-stat of Bias	Average Bias (%)	Stand. Dev. of Bias (%)	t-stat of Bias
Options April deep in the money						
ZURAPR1800	2.54	10.44	0.24	8.21	10.95	0.74
SMHAPR1100	-3.45	9.47	-0.36	-2.06	9.90	0.20
SBGAPR750	22.74	3.87	5.87	25.07	4.05	6.19
ALUAPR340	6.59	10.62	0.62	17.68	11.10	1.59
BBCAPR3200	10.74	2.26	4.75	18.60	2.35	7.91
SBVAPR240	1.80	1.66	1.08	8.89	1.70	5.22
Options April at the money						
ZURAPR1900	19.31	10.73	1.79	23.52	11.46	2.05
SMHAPR1300	34.51	7.10	4.86	16.83	6.17	2.72
SBGAPR800	49.36	9.20	5.36	45.80	8.76	5.22
ALUAPR360	5.69	7.17	0.79	17.11	7.16	2.38
BBCAPR3400	20.81	8.82	2.35	33.30	12.63	2.63
SBVAPR280	30.10	9.94	3.02	38.32	9.46	4.02
Options April out of the money						
SMHAPR1400	NA	NA	--	32.93	10.63	3.09
SBGAPR850	49.05	5.96	8.22	NA	NA	--
ALUAPR400	10.98	33.15	0.33	35.79	32.47	1.10
BBCAPR3800	42.53	31.65	1.34	36.43	52.33	0.69
SBVAPR300	1.44	65.11	0.02	36.63	39.56	0.92

Footnotes

[1] Cox's general formula is not reproduced here for the sake of clarity; only three special cases of CEV models corresponding to $\theta = 0$, 1 and 2 will be discussed. Similarly, the dividend versions of the CEV models will be overlooked since the tests are conducted under the assumption that there's no dividend payout before the expiration of the call option contracts considered.

[2] One standard approach in the pricing of warrants is to value them as otherwise identical options and adjust the obtained prices by the dilution factor. The discussion around the valuation of warrants is about whether the state variable must be the underlying stock or the total value of the issuing company.

[3] Such a procedure may be imperfect because some studies tend to show that the stock price does not fall by the full amount of the dividend.

[4] Because some stocks did not trade on particular days, we do not have the same number of observations for all the stocks in the sample; the total number of observations for the 37 stocks is 9 157.

[5] The reader interested in the way to perform a Chow test for more than two subgroups is refered to the original paper of Beckers or to PINDYCK/RUBINFELD (1987), pp. 123-126.

[6] We thus had to compute 49 x 21 = 1 029 theoretical prices for the Black and Scholes model and the same number for the square root model (a total of 2 058).

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