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The Volatility of the German and Swiss Equity Markets

1. Introduction

With the development of active markets in derivative securities such as options, futures and options on futures, volatility estimation has become a key ingredient in the pricing of these instruments. Option traders find themselves in situations in which their portfolios may suffer large losses if the volatility of the underlying asset changes unexpectedly over time. Volatility also plays an important role in all market equilibrium models. In asset pricing models, such as the CAPM, the risk premium is determined by the conditional covariance between the expected asset return and a benchmark portfolio. Dynamic portfolio risk management is another example where the conditional future volatility plays an important role.

Finance academicians widely agree, see SCHWERT (1990), that the most appropriate measure of volatility is the standard deviation of the rate of return. The standard deviation measures the dispersion of returns and it is a useful volatility measure because it summarizes the probability of extreme return values.

Using daily stock prices, MANDELBROT (1963) and FAMA (1965) show that volatility changes over time and that, for short lags, time series of daily stock index returns exhibit positive autocorrelation. Furthermore, they show that daily stock index returns tend to have higher kurtosis than if they were normally distributed. Using a value-weighted and an equally-weighted index, AKGIRAY (1989) shows that daily US stock returns for the period 1963 to 1986 are not made up of serially independent realizations. Like FAMA (1965), AKGIRAY (1989) finds that large price changes are followed by large changes, and small price changes are followed by small changes. This finding not only holds for returns, but also for the squared return and the absolute return series. Note that, assuming a zero mean return over short intervals, the squared return is a proxy for the instantaneous variance of returns and the absolute return is a proxy for the instantaneous standard deviation of return. AKGIRAY (1989) finds that the autocorrelation is generally higher in the absolute return series, slightly lower in the squared return series and lowest in the return series, implying a persistence of volatility that is not present in the return series. Notice that serial correlation in volatility does not imply market inefficiency as is the case for serial correlation of returns.

Until recently there has been relatively little academic research explaining changes in the volatility of stock returns. SCHWERT (1990) presents a good overview of different explanations for changes in volatility. He distinguishes between short term volatility changes, which are related to the market microstructure of securities markets, and long term

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volatility, which depends on economic factors such as financial leverage, operating leverage, margin requirements and the condition of the economy.

BLACK (1976) argues that corporate leverage affects the long-term volatility of returns of common stocks. As stock prices increase, equity volatility should decrease as a result of the higher market value of the equity and, therefore, lower debt to equity ratios. The negative correlation between leverage of individual companies and the volatility of its stock price is now an accepted fact by both academics and practitioners. The corporate leverage argument, however, is unable to explain the variations in volatility for broad market indices. Aggregate leverage does not change quickly, and, at least for the US, has not changed that much over time.

FRANKS/SCHWARTZ (1990) show that the capital structure cannot be the sole determinant of changes in volatility. They find that inflation, real interest rates, and exchange rates are significant additional variables that explain changes in volatility. SCHWERT (1990) points out that there is strong correlation between the stock price volatility and the business cycle. Long-term volatility seems to increase during economic recessions.

Another explanation for the changes in market volatility, which is stressed mostly by regulators, is the level of margin requirements argument. Some regulators argue that personal debt used to finance stock purchases may cause crash situations. HSIEH/MILLER (1990) and ROLL (1989), however, have shown that there is no relation between stock return volatility and margin requirements.

While these factors are useful in describing long-term changes in volatility, they don't give an adequate explanation for short-term changes in volatility. The stock market crashes of October 1987 and 1989, which were accompanied by sharp volatility changes, are hard to reconcile with arguments which explain changes in the long-term volatility.

Usually changes in short-term volatility are explained by the structure of securities trading. KARPOFF (1987) finds evidence that increased trading activities are accompanied by increased stock return volatilities. However, it is difficult to determine the causes of the correlation between volatility and trading activity. The argument that trading volume directly causes volatility would only hold if all market participants wanted to trade simultaneously in the same direction.

The presence of changing volatilities of stock index returns may be modeled in two ways. On one hand there are models in which a linear dependence in daily stock index returns is assumed. On the other hand there are models in which a nonlinear dependence is assumed. AKGIRAY (1989) showed that models based on linear dependent daily stock index returns, even if they provide a good empirical fit to data, have severe shortcomings. The major criticism against linear dependent models, which does not apply to non-linear models, stems from the fact that they neglect information about the dependence on the squared values of returns, which is valuable for prediction purposes.

Unfortunately, as pointed out by PRIESTLEY (1981), statistical estimation of nonlinear models is very often intractable. Autoregressive Conditional Heteroscedasticity (ARCH) models are a good proxy for nonlinear processes. ARCH models, developed by ENGLE (1982), allow the first and second moments of the stock index return to depend on its past realizations. The return and the variance of return are modeled as linear functions, which facilitates the statistical estimation of the parameters. BOLLERSLEV (1986) extended the ARCH framework by allowing the variance to depend not only on lagged squared deviations from the mean return, but also on lagged variances. This modification is called Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model.

In this paper we present recent estimates of the volatility of the German and Swiss equity markets. We examine the time series properties of volatilities of the DAX and the SMI indices. We also focus on the correlation of the time series of returns and volatilities for these two markets. Since a major focus in the practical applications of volatility estimates is the forecasting of future volatilities, we investigate the use of ARCH and GARCH models as a tool for volatility forecasting. In section 2 of this paper we briefly describe the DAX and SMI indices and in section 3 we describe ARCH and GARCH models and analyze the time series properties of the data.

2. Description of Indices

2.1 Deutscher Aktienindex (DAX)

The main stock market index in Germany is the DAX, a capital weighted price index with some unique features which make it different from most other world indices. The DAX was created in 1987 by the Frankfurt Stock Exchange as Germany's first real time index. Cash dividends on the 30 stocks which make up the DAX are assumed to be reinvested in the dividend paying stocks. For this reason it is also called a performance index. CORDERO/DUBACHER/ZIMMERMANN (1988) argue that an investment performance index like the DAX is not useful as an underlying instrument for options and futures since it is harder to replicate in hedging strategies. The DAX as Germany's blue chip index contains only the 30 most liquid stocks which are continuously traded at the Frankfurt Stock Exchange.

The selection criteria for the DAX stocks has been:

1. High trading volume.
2. High market capitalization.
3. Early opening prices.

The DAX contains not only ordinary shares but also preferred shares. For Henkel Corporation only preferred non-voting shares are included in the index since all the ordinary shares are held by the Henkel family and are not exchange traded.

2.2 The Swiss Market Index (SMI)

The SMI is also a real time index. It is capital weighted and contains the 22 largest capitalization stocks of 19 different companies (i.e. some companies have more than one class of stock in the index). The SMI index was created in 1988 for the purpose of serving as the underlying asset for futures and option contracts traded on the Swiss Financial Futures and Options Exchange (SOFFEX).The selection criteria for the SMI index have been (see CORDERO/DUBACHER/ZIMMERMANN (1988)):

1. Arbitrage: The index should only contain stocks which allow arbitrage between the cash and the stock index futures market.
2. Information: The SMI index should contain liquid stocks to avoid the problem of outdated prices due to infrequent trading.
3. Representative index: The SMI should give an accurate description of the Swiss stock market.
4. The index should be easy to calculate.

As most other stock indices the SMI does not take into account the reinvestment of dividends.

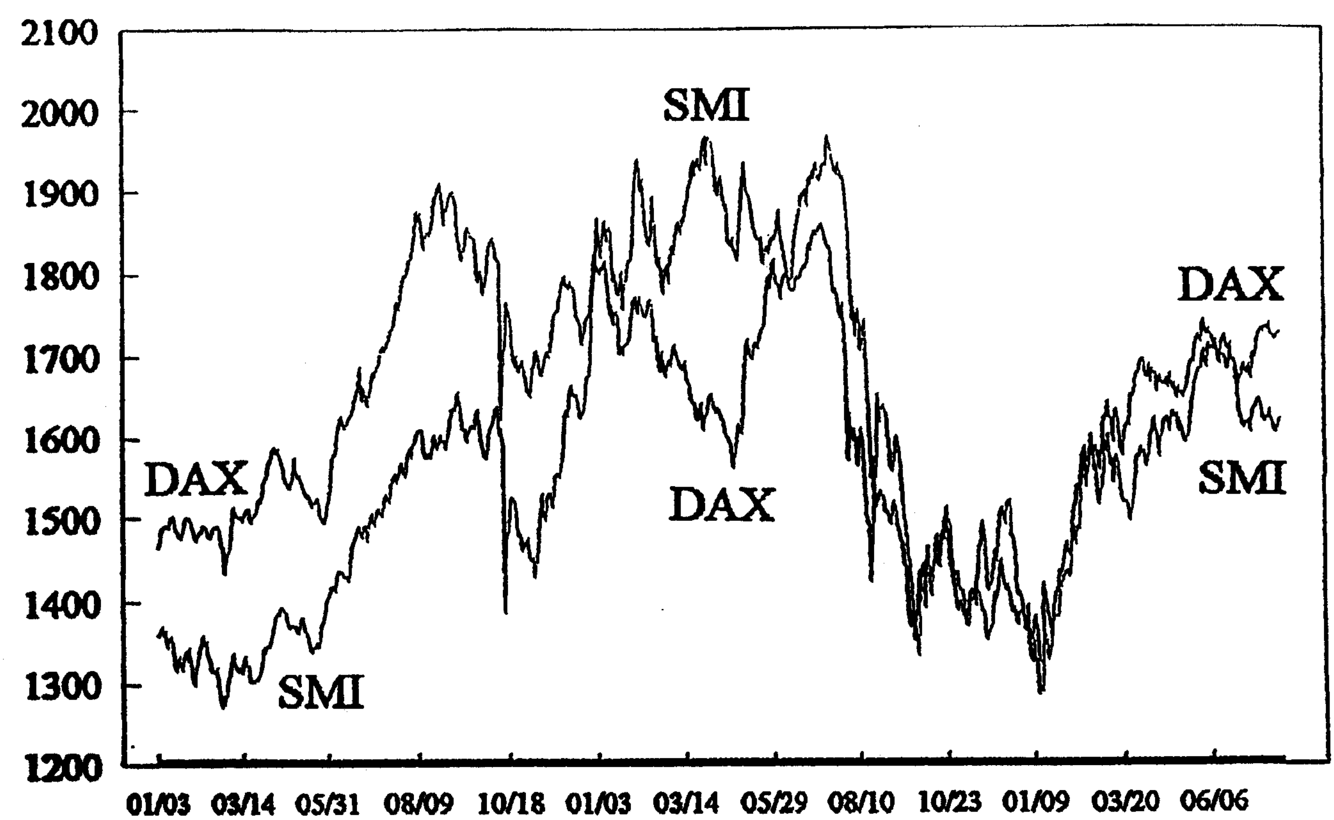
2.3 Volatility of stock market returns

We use daily values of the Deutscher Aktienindex (DAX) and of the Swiss Market Index (SMI) to estimate the return and the annualized standard deviation of stock market returns as a measure of volatility. The DAX data have been made available by the Frankfurter Wertpapierbörse AG for the period July 1988 to July 1991. Note that this period includes the German reunification. The Swiss Financial Futures and Options Exchange (SOFFEX) and the Association Tripartite Bourses (ATB) provided the Swiss data for the period January 1989 to October 1991. There were two minor changes in the composition of both the DAX and the SMI indices during the period covered.

Our volatility estimations are based on three different time series. Series 1 is the common period where data for both markets is available. This time series contains the period January 1989 to July 1991. Series 2 covers the German market for the period July 1988 to July 1991. Finally, series 3 covers the Swiss market for the period January 1989 to October 1991.

Initially we used two methods to estimate standard deviations. Every day during our observation period we used the returns of the last 21 trading days to compute the "rolling" standard deviations. This method gave us one volatility measure per day, but used in the computation overlapping observations. Once a month we used 21 non-overlapping daily returns for computing the "non-rolling" estimators for volatility, which gave us one volatility measure per month.
Figure 1 shows the level of the DAX index and the SMI index for the common period. From this figure we can see that the returns on both markets are highly correlated. The correlation coefficient for the common period is 0.71. These results are consistent with ROLL (1989b) who estimates a correlation of 0.68 between these markets for monthly returns from June 1981 to September 1987 based on the FT-Actuaries World Indices published by Goldman Sachs & Co. Keep in mind that the cumulative returns for both indices are not directly comparable since the DAX includes dividends and the SMI does not. Table 1 gives some descriptive statistics for the common period. For the period considered the German market with an average annualized standard deviation of 19.43% was more volatile than the Swiss market with an average annualized standard deviation of 15.77%. The results reported in Table 1 are computed using the rolling estimates of standard deviation. As mentioned earlier, the means are not directly comparable because dividends are included in the DAX, but not in the SMI.
As a proxy of the true ex-post volatility we use the absolute daily return for the specific day. Figure 2

Figure 1: DAX and SMI Index Price Level



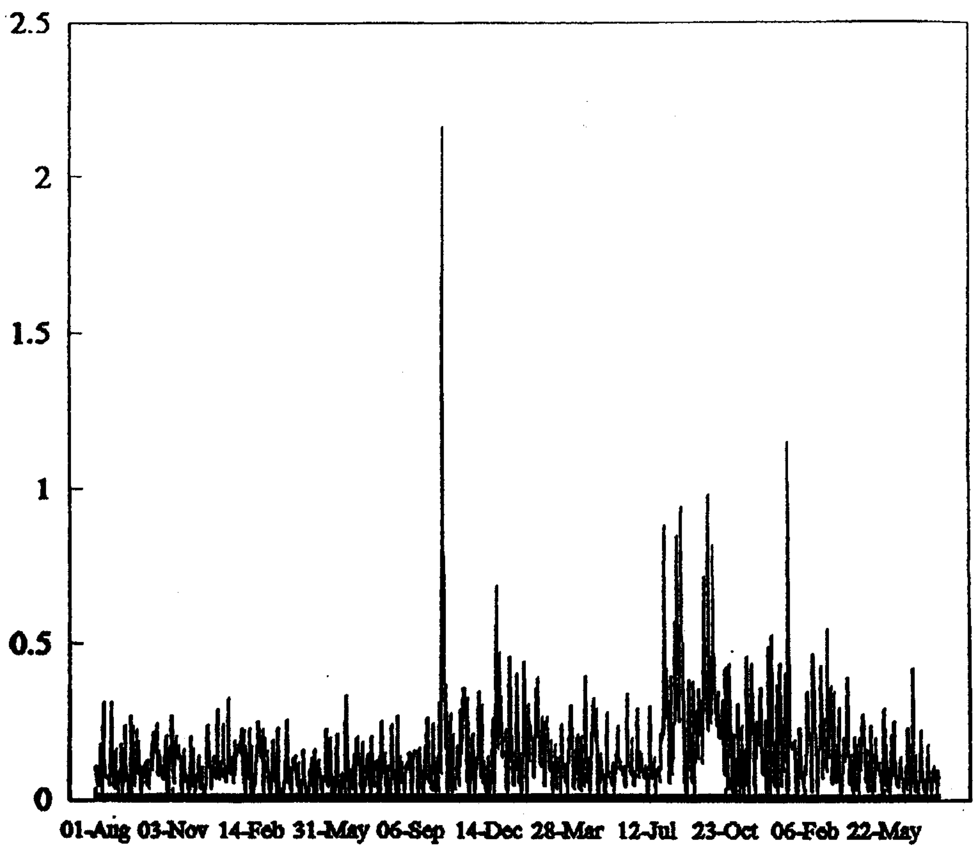
Domestic Currency.
Period: January 1989 to July 1991 (Daily Observations).
Sources: Frankfurter Wertpapierbörse AG, and ATB.

Table 1: Descriptive Statistics for the German and the Swiss Stock Markets (January 1989 to July 1991)

	Mean	Standard Deviation	Minimum	Maximum
DAX Return	0.00646	0.06758	- 0.19943	0.12663
SMI Return	0.00528	0.06015	- 0.1409	0.13079
DAX Volatility (Annualized)	0.1943	0.1062	0.0936	0.5673
SMI Volatility (Annualized)	0.1577	0.0947	0.0729	0.4721

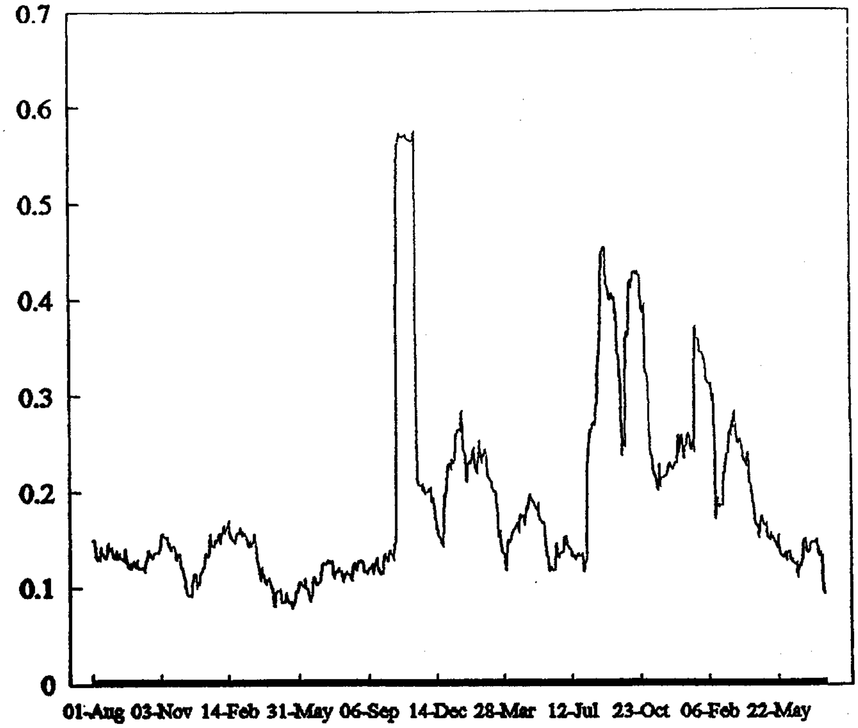
presents these annualized ex-post volatilities for the DAX index. Figure 3 reports the annualized rolling volatilities for the DAX index. The same information for the SMI index is provided in Figures 4, and 5. The correlation between the volatilities of the two markets is 0.93. The fact that the correlation between volatilities is substantially higher than the correlation between the stock index returns (0.71) is an intriguing result.
From the figures we also observe some extreme jumps in volatility. The first, common to both the DAX and SMI data corresponds to the mini-crash of October 16, 1989. The second, also common to both, in the summer of 1990 corresponds to the invasion of Kuwait. The third event is observable

Figure 2: Annualized ex-post volatility for the DAX index measured by the absolute daily return



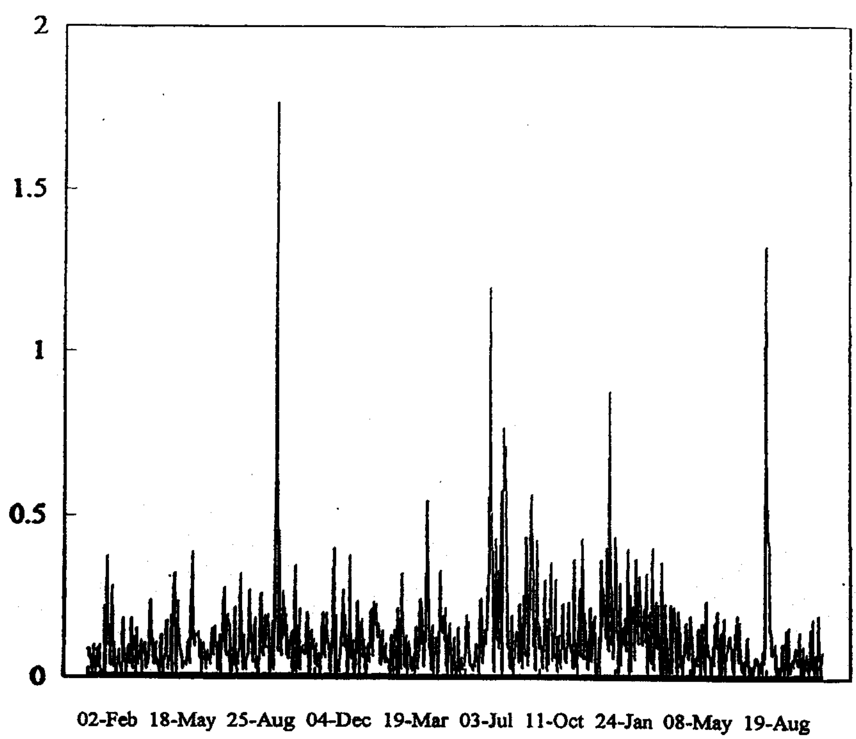
Period: July 1989 to July 1991.
Source: Frankfurter Wertpapierbörse AG.

Figure 3: Annualized historical rolling volatility for the DAX index estimated from daily returns



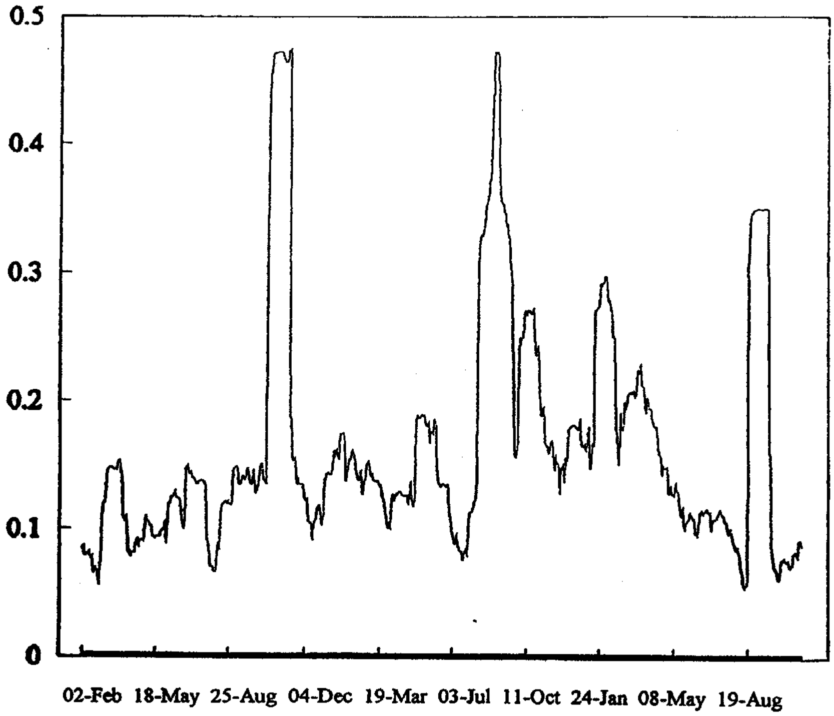
Period: July 1988 to July 1991. Source: Frankfurter Wertpapierbörse AG.

Figure 4: Annualized ex-post volatility for the SMI index measured by the absolute daily return



Period: January 1989 to October 1991. Source: ATB.

Figure 5: Annualized historical rolling volatility for the SMI index estimated from daily returns



Period: January 1989 to October 1991. Source: ATB.

only in the Swiss data because of data availability, and corresponds to the 'Gorbachev crash' in August 1991.

3. Time Series Properties of the Data

3.1 ARCH and GARCH Models

A statistical procedure designed especially to capture the changing nature of volatilities was recently developed by ENGLE (1982) and it is called autoregressive conditional heteroscedasticity (ARCH) model. The ARCH process allows the first and second moments of the stock index return to depend on its past values. The following is the simplest representation of the ARCH model:

$$r_t = \alpha + \epsilon_t$$

$$\epsilon_t \sim N(0,\sigma_t^2)$$

$$\sigma_t^2 = a + b * \epsilon_{t-1}^2 \quad .$$

It is assumed that the variance is a deterministic function of the lagged squared deviations from the mean return. The dependence of the first and second moment is formulated as a linear function. More general ARCH specifications allow for higher order lag square deviations in the variance equation. If a particular ARCH specification includes n lagged square deviations it is referred as ARCH(n). To estimate the parameters it is necessary to prespecify the conditional distribution function. In the above equation we assume a normal distribution. As AKGIRAY (1989) pointed out the model is flexible enough to admit other distributions. Within the model, return and variance processes are estimated jointly. Using daily returns we calculate maximum likelihood estimates for the ARCH model (using a numerical maximization procedure provided by the Econometrics Software Package Shazam). For large samples the choice of starting values for the numerical maximization procedure is not critical.
A more sophisticated variation of the ARCH framework developed by BOLLERSLEV (1986) is the

generalized autoregressive conditional heteroscedasticity (GARCH) model. A simple representation of the GARCH model is given by:

$$r_t = \alpha_t + \epsilon_t$$

$$\epsilon_t \sim N(0,\sigma_t^2)$$

$$\sigma_t^2 = a + b*\epsilon_{t-1}^2 + c*\sigma_{t-1}^2 \ .$$

Here it is assumed that the variance of the rate of return is a deterministic function of the lagged variance in addition to the lagged square deviation from the mean return. As pointed out by AKGIRAY (1989) GARCH processes can be seen as special cases of general random coefficient ARMA models. More general GARCH specifications allow for higher order lag square deviations and lag variances in the variance equation. If a particular GARCH specification includes n lagged square deviations and m lagged variances it is referred as GARCH(n,m). To estimate the parameters of the ARCH and GARCH processes the values of n and m in the equation have to be prespecified. The likelihood function can then be maximized for several combinations of the prespecified values. To obtain the optimal order of the process we compare the maximum values of the log-likelihood function. The ARCH model can be obtained from the GARCH model by setting c equal to zero. Lagrangean multiplier tests, developed by ENGLE (1982) and BOLLERSLEV (1986) can be used to test for ARCH and GARCH models. As an alternative χ^2 tests, based on likelihood ratios, can be used for statistical inference in ARCH and GARCH models.

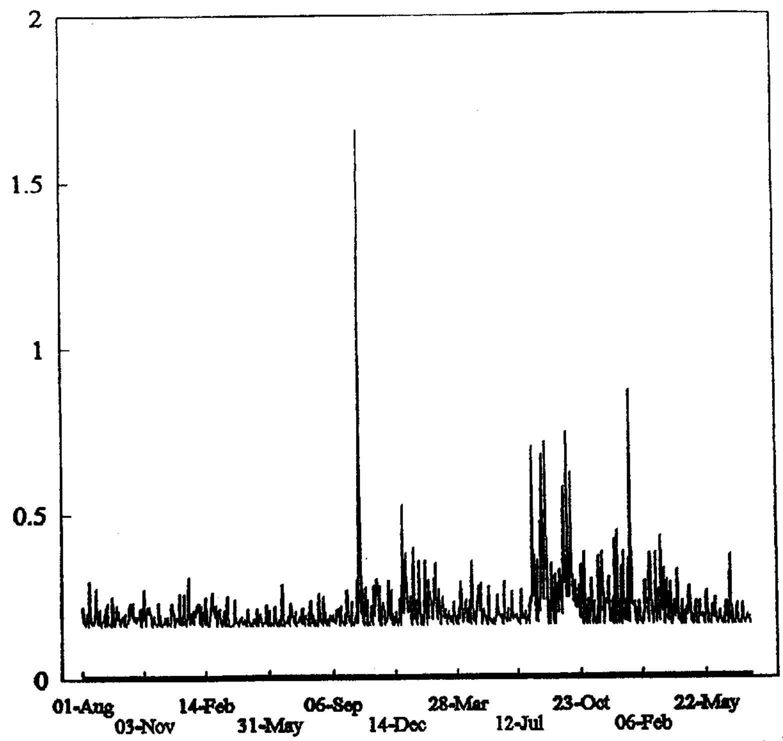
Table 2 contains estimates of the ARCH(1) and GARCH(1,1) models, both for the German and Swiss markets using all the data available for each market. The estimates of a in the ARCH(1) model for both markets are much smaller than the sample variances which were described in Table 1, implying that conditional variances change over time. The ARCH estimate of b is 0.57 with a standard error of 0.09 for the German market, and 0.18 with

Table 2: ARCH(1) and GARCH(1,1) Model Estimates for the German and Swiss Stock Markets

	Estimated Coefficient	Standard Error	t-statistics
DAX ARCH(1): Period July 1988 to July 1991			
Constant	0.001299	0.000404	3.22
a	0.000108	0.0000083	12.94
b	0.56792	0.090379	6.28
Log-likelihood = 2207.6			
DAX GARCH(1,1): Period July 1988 to July 1991			
Constant	0.00146	0.0003739	3.89
a	0.0000107	0.00000328	3.28
b	0.27505	0.0507	5.42
c	0.71986	0.0407	17.69
Log-likelihood = 2240.8			
SMI ARCH(1): Period January 1989 to October 1991			
Constant	0.000237	0.0004289	0.55
a	0.0001165	0.0000077	15.12
b	0.18508	0.06025	3.07
Log-likelihood = 2092.3			
SMI GARCH(1,1): Period January 1989 to October 1991			
Constant	0.0004348	0.0004134	1.05
a	0.0000419	0.0000109	3.84
b	0.17352	0.05479	3.17
c	0.54805	0.10322	5.31
Log-likelihood = 2103.3			

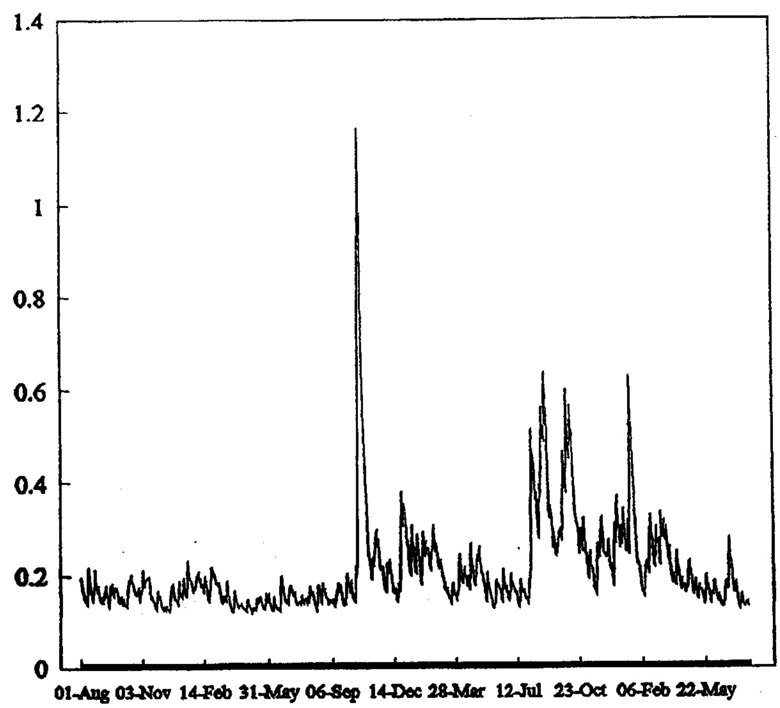
a standard error of 0.06 for the Swiss market. Thus, the relation between the lagged squared errors and variance is stronger for the German than for the Swiss market. By observing the GARCH estimate of c for both markets we conclude that, for the period considered, the persistence of volatility was also stronger for the German than for the Swiss market. The order of the ARCH and GARCH processes was found by applying Lagrangean multiplier tests. Within the group of GARCH processes, GARCH(1,1) shows the best result. By comparing for both markets the log-likelihood function values for the GARCH(1,1) process with those of the ARCH(1) process, we see that they are substantially greater. Therefore we can conclude that for both markets a GARCH(1,1) model fits the data much better than an ARCH(1) model. The estima-

Figure 6: Annualized ARCH(1) volatility forecast for the DAX index



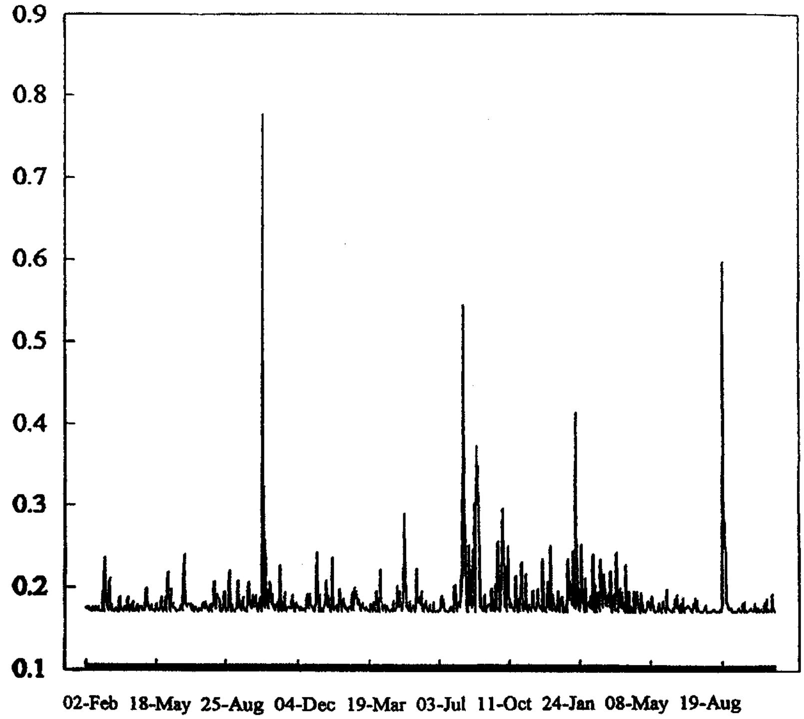
Period: July 1988 to July 1991. Source: Frankfurter Wertpapierbörse AG.

Figure 7: Annualized GARCH(1,1) volatility forecast for the DAX index



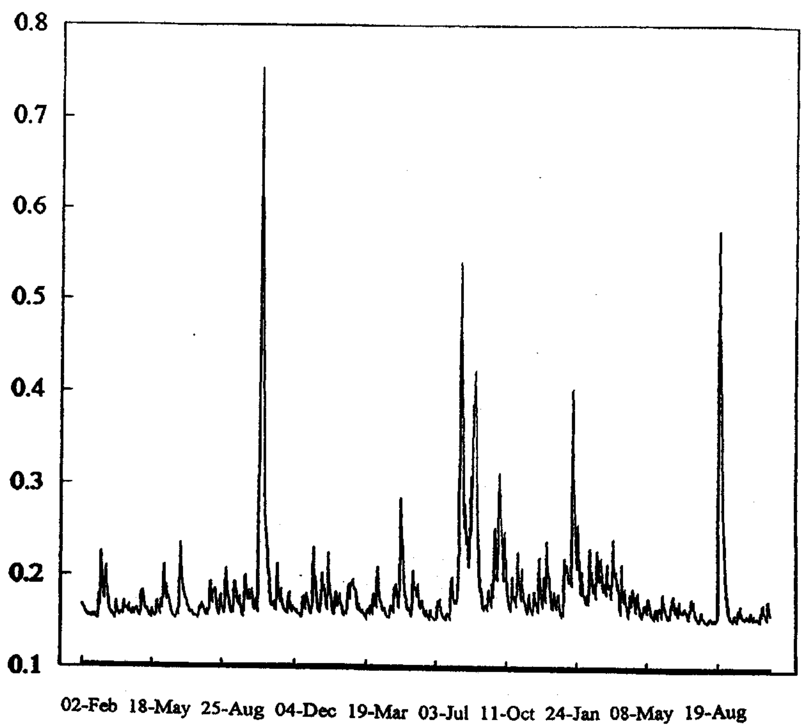
Period: July 1988 to July 1991. Source: Frankfurter Wertpapierbörse AG.

Figure 8: Annualized ARCH(1) volatility forecast for the SMI index



Period: January 1989 to October 1991. Source: ATB.

Figure 9: Annualized GARCH(1,1) volatility forecast for the SMI index



Period: January 1989 to October 1991. Source: ATB.

ted parameters of the GARCH(1,1) model are all statistically significant with the exception of the constant (with a t-value of 1.05) for the Swiss market.

The forecasted annualized volatilities corresponding to the ARCH(1) and the GARCH(1,1) models in both markets are shown in Figures 6 to 9. Note the similarity between these figures and ex-post volatilities presented in Figures 2 and 4. Only in very extreme market situations, such as the October 1989 crash, the ARCH and GARCH models underestimate the 'true' ex-post volatility. To evaluate the relative performance of the different volatility estimators, we computed the correlation between the ex-post volatility for both markets and the historical estimator, the ARCH estimator and the GARCH estimator for volatility. For both stock markets the GARCH estimator shows the highest correlation with the 'true' ex-post volatility (0.254 for the DAX and 0.202 for the SMI). The historical estimator performed better than the ARCH estimator for the German market (0.238 versus 0.162), but the ARCH estimator performed relatively better than the historical estimator for the Swiss market (0.178 versus 0.169).

4. Conclusions

Market volatility and its dynamics are an important ingredient in all index option models. In addition, market volatility plays a critical role in market equilibrium models of asset pricing. In this paper we study the volatility of the German and Swiss equity markets for a recent time period and look at its time series properties. It is significant that we observe a higher correlation between the volatilities in these two markets than between the returns. Applying to our data recent autoregressive conditionally heteroskedastic procedures, we find that a model in which the variance in a given period depends on lagged values of the variance and the lagged squared deviations shows the highest correlation with the 'true' ex-post volatility. As a proxy for the true ex-post volatility we use the absolute daily return for the specific day. As more data becomes available, a stronger test of the forecasting power of different volatility models would be to apply the forecasts to the pricing of index options.

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