Portfolio Insurance in the German Bond Market

1. Introduction

Remarkable developments have taken place on the national and international bond markets in the last two decades. The circulation of fixed interest rate securities from domestic issuers on the German bond market increased by 915 % from 143 bil. DM in January 1970 to 1,686 bil. DM in December 1991. In comparison, the nominal value of all shares quoted on the German stock exchanges only increased by 128 % from 28 to 64 bil. DM in the same period. The yields to maturity varied from 5.2% in March 1978 to 11.4% in September 1981, a range previously not considered. Together with this strong variation of yields a reduction of the average time to maturity from a little above 8 years to just under 4 years, with a tendency to rise again in recent years, can be observed. The price risks connected with interest rate changes will be shown by the following two examples:

- The price of the 6 % Bund with date of maturity 3/1/1993 fell from a 102.1 % on the 3/31/1978 to 75.15 % on the 4/4/1980 an investor would have lost 26% of his invested capital in this period.
- An increase of the current interest level on the German market for fixed securities by 1 % leads to approximately the same loss of value in DM as a price loss of 13% for all quoted shares.

Apart from the reductions in time to maturity, the increasing interest volatility had three further consequences: (1) New instruments were introduced on the primary and secondary markets for bonds to either limit the risks resulting from interest rate changes which creditors and/or debtors are subject to, or to redistribute these risks; (2) Some segments of the security markets changed from markets with long-term investors into trading markets. (3) New investment strategies were developed to control interest rate risks connected with bond portfolios. The following expositions are dedicated to this third sector.

2. Portfolio Insurance

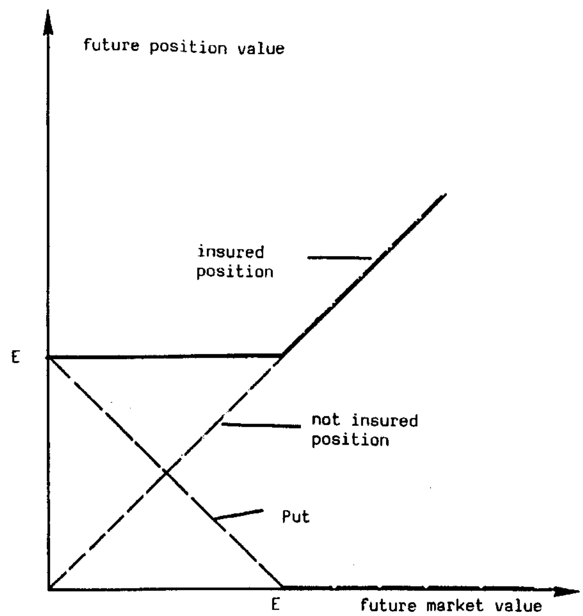
2.1 Protection of Asset Positions with Options

Principle

The owner of a European put has the right to sell an asset determined in an option contract at exactly one future point in time at a defined price (exercise price). He will make use of this right if the price is lower than the exercise price at the expiration date. So by buying a put - as shown in Figure 1 - it is possible to guarantee a lower bound for the future market value of an asset.

Protecting an asset position with a put is similar to a conventional insurance in as far as if the risk materializes the investor is indemnified, otherwise the insurance premium (the put price) is lost. Nevertheless there is a fundamental difference between the two types of insurance [1]: Insurance companies can diversify away the overall risk due to the immense number of small independent risks. This possibility is not available for the supplier of a market-value-dependent portfolio insurance. The equivalent protection with the help of a call shows up this difference more precisely. In this case a risk-free investment is supplemented by a high-risk option [2]:

Figure 1: Protecting the future market value of an asset with options.



Protection of Bond Portfolios
Transferring the principle of protecting a future asset position explained above to bond portfolios involves two central problems:
- In most cases there are no traded options, which would make it possible to control the price risk of a bond portfolio.
- Bond portfolios cause interest and redemption payments. The assumptions about how these values are to be reinvested influence the protection level and the achieved overall value of the portfolio at the end of the projected planning period.

The first group of problems is dealt with in section II. As far as the reinvestment of interest and redemption values is concerned there are two basically different possible procedures. Returns from the portfolio are either
- reinvested risk-free and separate from the portfolio to be protected or
- used to acquire further shares of the portfolio under consideration.

Insurance for the reinvested values against interest rate risks is possible under both assumptions, but requires a wide range of instruments. In the first case the reinvestment rate must be downside protected for every future coupon and redemption date. With the second alternative the price of the bonds to be bought additionally must be upside protected for the same points in time.
The first reinvestment alternative for interest and redemption payments was selected for the empirical part of the study.

2.2 Creation of Synthetic Options

Trade with options for bonds in the cash market has ceased completely in the Federal Republic of Germany. Options on bond portfolios or, alternatively, on bond indexes do not exist yet. The market for options on the Bund Future is not very liquid. Covering large bond portfolios by buying puts is therefore not possible at present.
One of the most important insights of Black and Scholes in their theory of pricing options on shares is the fact that, in an ideal situation, European options can be replicated synthetically from a suitably constructed portfolio of shares and a riskless asset, which must furthermore be continually rebalanced as the share price changes [3]. This methodology is transferred to bond options in the following and used as a portfolio insurance for bond portfolios [4].

Principle
The basic procedure for replicating an option will first be explained by an example. For this purpose the results of a portfolio consisting of a 10 %-bond and a loan at 6 % for a planning period of half a year

are presented in the following table. The first line contains the current and two possible future prices, the second line the accrued interest per unit which has to be paid by buying and selling the bond. The size of the short term credit is determined in such a way that it can be repaid by selling the bond in the event of an unfavourable bond price development.

Table 1: Replication of options.

	t	T	
Bond Prices	-100	95	105
Accrued Interest (10%)	-5	10	10
Credit (6%)	101.94	-105	-105
Portfolio	-3.06	0	10
Two Calls Long	-2·C	0	10

A comparison between the future value of the portfolio (line 4) and the future value of two calls, which permit the holder to buy two bonds at an exercise price of 100 in month t (line 5), shows that they coincide completely.

This simple example demonstrates that the future value of a European call at the expiration date can be created synthetically by buying bonds and taking a credit of suitable size. It is equally possible to represent the intrinsic value of a European put at the expiration date by selling a bond short and a risk-free investment.

The assumption that the bond can only assume two prices at the expiration date of the option is doubtlessly unrealistic. But this assumption is not necessary. If within a short period of time (e.g. 1 day) a small change of the bond price only leads to a small change in the value of the put, then one can prove that in short periods of time the stochastic change in the price of the bond and the resulting stochastic change in the value of the put are completely negatively correlated.

In other words: It is possible to construct a portfolio consisting of a suitable number of bonds and puts which is risk-free for a short period of time and which pays interest according to the current interest rate for a risk-free investment. This fundamental insight of the modern option pricing theory was used in another paper to derive a valuation relationship between options and bonds [5].

The application of the principle of replicating an option with two instruments traded on the market leads to two important results: the theoretical option price and the hedge ratio. The unrealistic example in table 1 produces an arbitrage-free call value of 1.53. The hedge ratio states how strongly the option price reacts in absolute terms if the bond price changes by 1 unit in a small period of time. For puts the following applies:

- The hedge ratio always has a value between 0 and 1.
- The higher the bond price, the smaller the hedge ratio.
- The hedge ratio is 0 (1) for bond prices above (below) the exercise price on the expiration day of the put.

If a portfolio made up from bonds and puts in such a way that the ratio between bonds and puts is equal to the hedge ratio then small changes in the bond prices are compensated by opposite changes in the put value. The portfolio is instantaneously risk-free. The qualitative relation

*"put + bond * hedge ratio = risk-free investment"* (1)

can, after "resolving" for the put, be written as

*"put = risk-free investment - bond * hedge ratio"* (2)

This is the basic equation for the synthetic generation of a European put, as was explained for a call by an example in table 1. At the same time it becomes obvious that a portfolio consisting of a risk-free investment and a bond sold short must be rebalanced continuously due to changes in the hedge ratio caused by the changes of the bond price. The synthetic creation of puts must therefore take place in the frame of a hedge ratio controlled *dynamic rebalancing strategy* [6].

Portfolio Insurance with Synthetic Options

Insuring the value of a portfolio consisting of bonds by means of synthetically created European puts requires the following steps:

1. *Determination of the point of time for which the portfolio should be insured:* The planning horizon can, for example, be defined by the date of a future large expense, the fund target, or the next performance-assessment of the portfolio manager.

2. *Determination of the insurance level:* The specification of a minimum value for the portfolio at the planning horizon leaves a considerable degree of freedom within a price range bounded by reasonable levels for interest rates. Selecting either the current price of the portfolio or the price which would arise at the planning horizon if interest rates do not change represent two obvious choices of the insurance level.

3. *Structure of the initial portfolio:* If puts for the instrument to be insured were available for the required expiration date, then the initial budget would be divided evenly - measured in nominal value units of DM 100 - between portfolio units to be insured and puts. But as, according to the assumptions, the puts must be created synthetically with the help of the equation (2), the volume of the resources invested in the instrument to be covered are reduced by the factor

1-hedge ratio.

The remaining amount

initial budget - amount invested risk-free

is used to buy bonds which, in relation to the planning horizon, are not subject to the interest rate risk. An example will explain the procedure.

Initial budget:	*10 million DM*
Price of the portfolio to be insured:	*102.5 %*
Accrued interest:	*1.3 %*
Theoretical put value:	*2.1 %*
Hedge ratio:	*0.55*

Allocation of the initial budget if puts are available:

Portfolio to be insured and puts:	*94,429 units with a nominal value of DM 100 (10 mill / (102.5 + 1.3 + 2.1)).*

Allocation of the initial budget with synthetic puts:

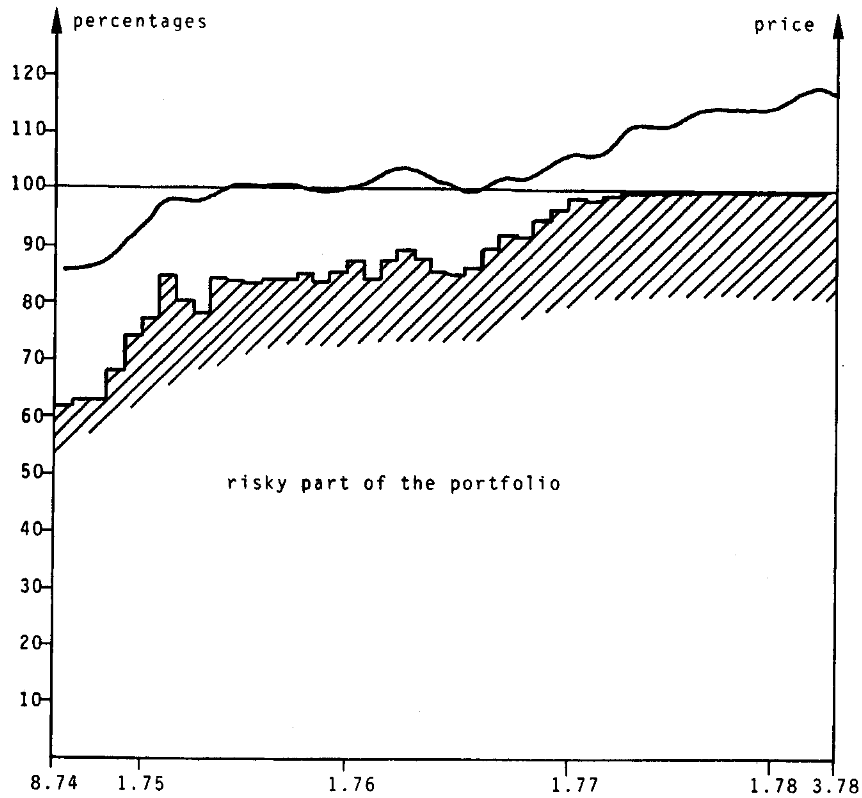
Portfolio to be insured:	*(1-0.55)*94,429 = 42,493 units ≈ 4,410,765 DM*
Risk-free investment:	*5,589,235 DM*

4. *Rebalancing of the portfolio:* Taking into account the current price of the instrument to be insured, the hedge ratio is calculated anew at every rebalancing point of time. The hedge ratio falls (rises) with rising (falling) prices and the portfolio is rebalanced in favour of the instrument to be insured (the risk-free investment). These rebalancements are carried out by means of a self-financing strategy, i.e. rebalancing does not result in any change of the portfolio's value. Figure 2 shows the portfolio development of the 8.5 % government bond which matures March 1985 between 8/1/1974 and 3/31/1978.

3. Empirical study

Although the insurance of *stock portfolios* by means of synthetic options was already carried out in grand style in the USA, it was not until 1987 - after a series of simulation studies with hypothetical prices [7] and a few casuistically orientated surveys [8] - that a major empirical study investigating the quality of this insurance strategy on the basis of market data supplied by the Standard & Poor 500 share index was published [9]. Comparable analyses for the North American or German *bond markets* are, to the knowledge of the author, not available yet.

Figure 2: Development of the portfolio structure with increasing prices.



3.1 Structure of the study

- The research period is based on the time interval ranging from January 1970 to December 1985. This period of time includes subperiods with both strongly increasing and decreasing yields to maturity as well as points of time with normal, flat and inverted term structures of interest rates.
- The periods of insurance are varied between one and six years. A new insurance strategy is initiated in quarterly intervals beginning with January 1970. This makes it possible to obtain between 40 (insurance period of 6 years) and 60 (insurance period of 1 year) simulation results. Due to the overlapping time intervals, though, these are not statistically independent of each other.
- The bond with the longest time to maturity is selected as the instrument to be insured from the spectrum of circulating bonds issued by the Government, the German National Railways and the National Post Office. The average time to maturity of these bonds is 11.2 years.
- The coupons collected in the periods of insurance are invested risk-free until the planning horizon. A control of the interest rate risk related to the income from these reinvestments requires, in analogy to the insurance of the price of a bond, the synthetic creation of interest rate futures or calls for every coupon date. This is not done due to the extensive insurance effort it would cause. This implies, however, that a complete elimination of the interest rate risk could no longer be achieved.

Due to the non-availability of zero bonds with a suitable time to maturity, the bond with the shortest remaining time to maturity expiring after the end of the planning horizon is chosen from all bonds issued by the Government, German National Railways or German National Post Office. This selec-

tion is based on the findings of an empirical study carried out at the same time [10].
- When selecting the exercise price of the synthetic option it must be taken into account that the bond price can change over time if the interest rate remains constant. Therefore the exercise price is set equal to that bond price which is quoted if the term structure of interest rates at the planning horizon is the same as at the beginning of the insurance period.
- The portfolio is rebalanced in monthly intervals. The distribution of funds between risky and risk-free portfolio parts is established by means of the current hedge ratio.
- Transaction costs of 0 % and 0.35 % considered when the portfolio position is bought, rebalanced and liquidated.
- Two questions referring to the evaluation of the portfolio come to mind: the question of the quality of the insurance and the question of the opportunity costs of this strategy. The quality of the insurance is, if, in retrospect, insurance was necessary, measured by the difference between the price of the portfolio and the insured value. Opportunity costs arise if, in retrospect, insurance turned out to be superfluous. They are quantified by the difference between the returns on investment achieved with and without insurance.

3.2 Results

Table 2 gives a first impression of the extent by which portfolio insurance reduces variability of returns in *percent* on investment over the various insurance periods.
First conclusions can be drawn from these results.
- Negative returns on investment can be avoided by portfolio insurance.
- With insurance intervals of up to 3 years the variability of the returns on investment is reduced by approximately 30 % in relation to the insured bond. Furthermore, the reduction of the maximum

Table 2: Returns on investment for the insured bond, the portfolio insurance and the risk-free investment (%).

Insurance period		1 year		3 years		6 years	
		ex TAC	inc TAC	ex TAC	inc TAC	ex TAC	inc TAC
Mean	bond to be insured	8.54	7.72	8.52	8.22	8.43	8.28
	portfolio insurance	8.40	7.14	8.44	7.93	7.93	7.63
	risk-free investment	7.29	6.68	7.89	7.67	7.85	7.72
Standard Deviation	bond to be insured	8.84	8.79	4.95	4.95	1.68	1.68
	portfolio insurance	5.68	5.61	3.42	3.41	1.47	1.46
	risk-free investment	2.24	2.18	1.63	1.61	1.05	1.05
Maximum	bond to be insured	28.19	27.13	18.35	18.05	11.12	10.96
	portfolio insurance	24.21	22.78	16.30	15.88	10.76	10.51
	risk-free investment	12.34	11.56	11.43	11.15	9.79	9.69
Minimum	bond to be insured	-10.16	-10.68	-1.58	-1.87	4.95	4.79
	portfolio insurance	1.11	0.11	3.18	2.90	4.80	4.62
	risk-free investment	3.47	2.75	4.76	4.63	5.64	5.56
Sample Size		60		52		40	

TAC: Transaction costs

Table 3: Difference between the price of the portfolio and the insured value relative to the initial budget (%).

Insurance period		1 year		2 years		3 years		4 years		5 years		6 years	
		ex TAC	inc TAC	ex TAC	inc TAC	ex TAC	inc TAC	ex TAC	inc TAC	ex TAC	inc TAC	ex TAC	inc TAC
Mean	[%] [BP]	-0.22 -22	-0.58 -58	-0.55 -27	-1.11 -54	-0.84 -28	-1.58 -49	-0.88 -21	-1.79 -40	-1.13 -21	-2.21 -37	-1.20 -16	-2.32 -31
Standard Deviation	[%]	0.93	1.06	1.21	1.28	1.20	1.09	1.67	1.50	1.94	1.76	1.59	1.27
Maximum	[%]	1.82	1.52	2.00	1.43	1.62	0.09	1.87	1.15	2.04	1.02	2.00	0.11
Minimum	[%] [BP]	-1.39 -139	-2.28 -228	-2.44 -119	-3.57 -174	-2.54 -79	-3.69 -115	-3.89 -86	-4.54 -102	-4.12 -68	-4.74 -84	-3.96 -52	-4.66 -61
Sample Size		24 (40%)		24 (43%)		24 (46%)		24 (50%)		20 (45%)		18 (45%)	

TAC: Transaction costs
BP: Basis points

gain potential is smaller than the insurance effect in the worst possible case.

- The negative influence of the transaction costs drops with increasing insurance periods.

The quality of the portfolio insurance is presented in table 3. The results are based on the evaluation of those simulations where the price of the bond is below the exercise price at the planning horizon.

The following statements about the insurance quality of the portfolio insurance can be deducted from the results presented in table 3:

- The number of cases in which insurance was justified retrospectively does not increase with the length of the planning horizon. This figure, relative to the sample size, first increases and then decreases.

- If the insurance quality, defined as the deviation of the price of the portfolio from the insured value, is related to the initial budget then it *drops* on average and in the worst possible case (with the exemption of the 6-year planning horizon) *with the length of the insurance period.* If, though, the insurance quality is measured as loss of return on investment in basis points per year then, with the inclusion of transaction costs, it improves on average and in the worst possible case with the length of the insurance period. Without transaction costs the results are not quite consistent, but show a tendency towards an increase in quality with the length of the planning horizon.

- Transaction costs lead to a doubling of the *average* deviations from the value to be insured. In the worst possible cases the influence of the transaction costs, measured in basis points, falls strongly with the planning horizon. In these cases the transaction costs, relative to the initial budget, reduce the value of the portfolio by a maximum of 1.15 %.

Table 4 presents the *average* advantages and disadvantages of the portfolio insurance in relation to both a strategy without insurance and the risk-free strategy. The *advantage (disadvantage)* compared to a strategy without insurance is defined as the

- *absolute difference* in the *terminal values* in DM of both strategies in those cases in which insurance proved to be *reasonable (not reasonable)* in retrospect.

Table 4: Advantages and disadvantages of portfolio insurance [DM].

Insurance period	1 year		2 years		3 years		4 years		5 years		6 years	
	ex TAC	inc TAC	ex TAC	inc TAC	ex TAC	inc TAC	ex TAC	inc TAC	ex TAC	inc TAC	ex TAC	inc TAC
Advantage	3.73	3.37	5.10	4.56	4.66	3.96	2.94	2.08	0.71	-0.31	-0.09	-1.14
Disadvantage	2.73	3.21	4.06	4.77	5.26	6.07	6.55	7.57	7.94	9.26	8.39	9.73
Comparison with strategy without insurance												
Advantage	3.70	3.00	6.16	5.13	7.64	6.45	7.80	6.43	5.15	3.50	3.87	2.17
Disadvantage	2.80	3.36	3.43	4.16	4.04	4.96	3.87	4.92	3.15	4.36	2.77	4.00

TAC: Transaction costs

In comparison with the risk-free investment the *advantage (disadvantage)* of the portfolio is measured by the
- *absolute difference* in the *terminal values* in DM of both strategies in those cases in which insurance *was not reasonable (reasonable)* in retrospect.

From Table 4 it is clear that, compared with the risk-free investment:
- without transaction costs the advantages of portfolio insurance outweigh its disadvantages,
- with transaction costs for the 1 year insurance period as well as for the 5 and 6 year planning horizons the disadvantages are greater than the advantages.

Compared with the strategy of no insurance, periods of 3 and more years are, even in an analysis based on average values as presented in Table 4, problematic.

3.3 Summary

A number of conclusions can be drawn from the results of the empirical study summarized in section 3.2 with the usual reservations with which the results of simulation studies should be interpreted:
- A perfect insurance of portfolios against the interest-rate risk is, as expected, not possible.
- Portfolio insurance for periods of more than 3 years must be considered problematic if it is performed as presented above.
- Portfolio insurance without transaction costs is attractive in relation to immunization strategies.
- Casuistic studies show that, without transaction costs, the quality of insurance can be considerably improved by weekly rebalancing.

Footnotes

[1] See the very clear introduction to portfolio insurance by SCHWARTZ (1986), pp. 9.
[2] This aspect is presented emphatically by Brignoli. See BRIGNOLI (1988), pp. 21.
[3] See BLACK/SCHOLES (1973), pp. 640.
[4] For the synthetic creation of stock options, an extensive example and about the practical problems see RUBINSTEIN/LELAND (1981), pp. 64 and pp. 70 as well as COX/RUBINSTEIN (1985), pp. 267.
[5] See BÜHLER (1988b).
[6] For other "dynamic asset allocation strategies" and their classification see RUBINSTEIN (1985), pp. 45ff., BOOKSTABER (1985), pp. 39, PEROLD/SHARPE (1988), pp. 16.
[7] See ASAY/EDELSBURG (1986), CLARK/ARNOTT (1987), ETZION (1986), ZHU/KAVEE (1988).
[8] See PLATT/LATAINER (1984), TILLEY/LATAINER (1985).
[9] See GARCIA/GOULD (1987).
[10] SEE BÜHLER (1988a), p. 36.

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