

The Estimation of Multiple Factor Models and their Applications: The Swiss Equity Market

1. Introduction

Since the seminal work of MARKOWITZ (1959) and SHARPE (1964), the Capital Asset Pricing Model has dominated the academic literature on portfolio management. Central to this model is the portfolio or stock beta which should be the only relevant determinant of return and therefore also the crucial risk measure. In other words the Capital Asset Pricing Model uses beta as the single explanatory factor.

Recently however the Capital Asset Pricing Model has been challenged by the Arbitrage Pricing Theory of ROSS (1976). Based on a set of less stringent assumptions, the APT shows that the stock return may be determined by a limited set of factors. Although beta could be one of these factors, APT itself does not provide any insight into the nature, magnitude and number of relevant factors.

Recent empirical work by FAMA/FRENCH (1992) casts further doubt on the prevalence of beta as the sole or even major determinant of stock returns: studying US stock returns over the period 1962 through to 1989, they find that the company's market capitalisation as well as its book to price ratio (book value divided by market value) are more significant explanatory variables of return than beta (which in a purely statistical sense was insignificant).

Arguably it could therefore be said that academic research has recently moved closer to investment

practice: investment analysts and portfolio managers have never accepted beta as the only (or even as an important) decision variable. Indeed, investment managers have always used a multitude of criteria in building portfolios : interest rate sensitivity, price/earnings ratio, earnings growth and its stability, sensitivity to general economic activity, exchange rate sensitivity, price to book ratio are only a few of the characteristics which a portfolio manager would associate with a given company before thinking of its beta.

Hence, multiple factor models now dominate in both the academic and the real world as the leading paradigm describing the behaviour of stock returns. Unfortunately, so far academic research has not succeeded in theoretically deriving which factors influence stock returns.

In addition, there is no consensus on the empirical methods which could be used to identify these multiple factors. In this article we will compare two methodologies to estimate multiple factor models as applied to the Swiss stock market.

The article is structured as follows. In section 2 we give a general description of an equity multiple factor model. We concentrate upon different empirical methods which can be used to estimate these types of models and select two methodologies which will be applied to the Swiss equity market. Section 3 discusses the data used and section 4 details the different steps involved in the estimation of a fundamental factor model for Switzerland. In section 5

we develop a statistical factor model, which in section 6 is compared to the fundamental model. In section 7 this comparison is elaborated upon through an empirical test of both the statistical and the fundamental model. The major conclusions of this comparison are summarized in section 8. The last three sections of the paper illustrate the use of the fundamental factor model in a portfolio management context. In section 9 a static portfolio risk analysis is described whereas section 10 details the central role of the risk model for portfolio construction purposes. Finally, section 11 emphasises the importance of the multiple factor model for performance attribution and analysis, enabling us to differentiate between luck and skill in portfolio management. Section 12 summarizes the major findings of the paper.

2. Multiple Factor Models

A multiple factor model in its most general form posits that asset returns are generated by the returns to a set of common factors and the exposures of the assets to those factors:

$$R_i(t) = \sum_{j=1}^k X_{ij}(t) f_j(t) + \epsilon_i(t) \quad (1.1)$$

where

- $R_i(t)$ = excess return to security i in period t (i.e. return to security i in excess of the short term risk free rate)
- k = number of common factors
- $X_{ij}(t)$ = loading of security i on factor j (also referred to as the exposure of security i to factor j)
- $f_j(t)$ = return on factor j in period t
- $\epsilon_i(t)$ = specific return to security i in period t (i.e. the fraction of the return which cannot be explained by or attributed to the common factors)

For example, a stock may have an excess return of 10% in a particular month and the factor model may attribute this return as 5% to factor one (e.g. economic growth) and 3% to factor 2 (e.g. change in inflation) and the remaining 2% to the specific or non-factor performance of the stock. In matrix notation equation (1.1) can be rewritten as

$$R_t = X_t f_t + \epsilon_t \quad (1.2)$$

Equation (1) simply states that there are good reasons why Winterthur, Swissair, Nestle and Holderbank have different returns over any holding period. However, there is probably no consensus on these reasons: some investors may explain the differences in terms of Swissair and Nestle being more exchange rate sensitive than Winterthur, and Holderbank; others may emphasise the differential behaviour of the industrial versus the insurance sector, still others may concentrate on a larger inflation sensitivity of some companies etc.. Multiple factor modelling tries to fill in equation (1) by identifying the number of factors needed - as well as their nature and relevance - to fully explain these cross sectional return differences.

Equation (1.2) has a straightforward equivalent in a risk sense:

$$V_t = X_t F_t X_t^T + \Omega_t \quad (2)$$

where

- V_t = covariance matrix of asset risks
- F_t = factor covariance matrix (i.e. covariance matrix of factor returns)
- X_t = matrix of asset exposures
- Ω_t = (diagonal) matrix of specific variances

Equation (2) is important because it captures the risk dimension of the multiple factor model. Naively one could think that there are factors in the stock market which - month by month - give us a steady or constant return: in this naive world food stocks could for instance have a return of 1% each and

every month. In this case we would have a characteristic (being a food stock) which has a constant return without any risk (variability). It is one of the basic tenets of the financial world that there is no reward without risk. As a result, the above naive hypothetical case is not realistic: factors which drive returns will also have some risk associated with them.

Inevitably therefore, the impact of the factors which drive stock returns (in equation (1)) will fluctuate through time (i.e. the factor returns will have some variance and -arguably - some covariance amongst each other). In other words the variability of stock returns through time (and their correlations) can be partially attributed to the (co-) variability of the factor returns : airline stocks are more volatile because of the variability of exchange rates, food stocks are less volatile because they are less sensitive to the general economic cycle etc.. This differential volatility of stock returns is explained in equation (2) using the same factors which drive the returns in equation (1).

In sum, multiple factor models describe stock returns and their (co-) variances as a function of a limited set of risky attributes. The identification of these attributes and their riskiness is an empirical issue which has been addressed using 3 different approaches:

1. predetermine X (sensitivity to the individual factors) and estimate f (factor returns) using cross-sectional regression techniques for equation (1). Typically the X will be company or stock characteristics such as capitalisation, P/E ratio, leverage, earnings growth, dividend yield etc., whereby a list of predetermined company attributes is posited to have an impact on stock returns. As a result of the multiple regression one can determine which variables have discriminatory power (i.e. have significant factor returns).

This methodology was pioneered by Barr Rosenberg (ROSENBERG (1974), ROSENBERG/MARATHE (1976)) who was amongst the first to recognise the importance of factors other than beta in the return generating process (see also KING (1966)).

2. predetermine f and estimate X using time series regression techniques for equation (1) Predetermined factor returns can be the excess long term bond return, return on commodities such as oil and gold, changes in inflation, exchange rate movements etc. (See for instance BERRY/BURMEISTER/MC ELROY (1988) and CUENOT/REYES (1992)). The regression will yield the sensitivity of each stock to the individual factors (i.e. X_{ij})

3. simultaneous estimation of X and f using a statistical procedure called factor analysis of matrix V . Note that in this case X and f are purely statistical constructs which in themselves have no direct economic meaning. However in subsequent analyses the factor return series may be correlated with economic series (bond returns, inflation, oil prices etc.). Alternatively the factor exposure matrix X can be rotated onto a predetermined target matrix, possibly consisting of groups of assets with different properties (such as high yielding, low exchange rate sensitivity etc.). However the interpretation of these factors remains more of an art than a science.

The factor analysis method was used most frequently in the initial tests of the APT (see for instance ROLL/ROSS (1980), although a significant controversy rages about the robustness of the methodologies used (SHANKEN (1982), CHO (1984), CONNOR/KORAJCZYK (1988)).

In our study we will apply methods 1 and 3 to the Swiss equity market and will refer to them as the fundamental model and the statistical model respectively.

3. Data

The estimation universe for this study consists of the 250 largest Swiss stocks (as measured by market capitalisation), irrespective of their type (i.e. the universe includes bearer and registered shares as well as participation and dividend right certificates). All data used for this study was supplied by UBS in Zurich. Total returns were calculated on a monthly basis and are fully adjusted for dividends, splits, rights issues etc. All analyses are

done on the basis of excess returns which are obtained by subtracting off the 3 month risk free rate (3 month Euro-Swiss Franc rate). Balance sheet and income statement based ratios were calculated from the published data as provided by UBS. Whenever a financial ratio is calculated (such as for instance leverage), it is assumed to be fully in the public domain 6 months after the end of the fiscal year (to avoid relating balance sheet or income statement data to returns at a time when this information was not fully public and therefore could not have any impact on the returns). [1]

Data is available since January 1980 but some of the company characteristics are based on a 5 year history (such as earnings growth, variability in earnings, average dividend yield etc.). Therefore the estimation period for both models uses the data from January 1985 through December 1989.

4. The Fundamental Multiple Factor Model

The estimation of equation (1) using a predetermined set of factor exposures starts from the identification of a list of variables which investment analysts and portfolio managers typically use in their day-to-day decision making. The Appendix summarises all relevant descriptors used in the development of the Swiss fundamental factor model. Using such a large list of explanatory variables in a regression analysis may obviously lead to collinearity problems (i.e. a number of these ratios such as book leverage and debt to assets measure similar characteristics which makes it virtually impossible to distinguish between them in terms of their impact on the stock returns). Therefore a cluster analysis technique was used to combine those attributes which capture similar characteristics. The cluster procedure reduced the number of explanatory variables to the 8 risk indices summarised in Table 1a. We would also expect that the industrial activity of the company has a strong discriminating impact on its returns (food stocks can be expected to perform differently from airline companies, banks from building companies etc.). Therefore, the industry

affiliation of the company was also used in addition to the balance sheet and market information based attributes as an important stock characteristic (each company is 100% assigned to one industry although ideally we would have liked to take into account the diverse nature of some company activities).

For each month t a stock can be characterised along each of the above 20 dimensions: it will have a certain market capitalisation, dividend yield, leverage etc. and will belong to one of the 12 industries. of Table 1b [2].

A generalised least squares cross-sectional regression relates the excess return of the stocks to their respective attributes, resulting in a factor return associated with each of these 20 factor exposures. The factor return measures that part of the excess return which - *ceteris paribus* - would be earned by a stock which has a unit exposure to this attribute (i.e. which is positioned at the 83rd percentile or one standard deviation above the mean) on this risk index. For example, the cross-sectional regression (1) for the month of August 1992 would have resulted in the estimates summarised in Table 2.

For example, in August 1992 a portfolio which would have consisted 100% of Utility companies (and assuming that each company would have had an average size, yield, leverage etc.), would have a negative excess return of -11.1%. Conversely a (hypothetical) portfolio which would have had unit exposure to leverage (i.e. a portfolio which would have been positioned at the 83rd percentile on the leverage dimension) would have had a positive return of 2.5% [3].

In the context of our study, this process is repeated every month for the period between January 1985 and December 1989, resulting in a 60 month series of factor returns. Figures 1 through 3 summarise the (cumulative) time series behaviour of some of these factor returns. (i.e. the cumulative return to a hypothetical portfolio which month after month has unit exposure to this attribute and is neutral with respect to all other characteristics).

Note for instance the positive slope of the yield risk index factor return, implying that - on average and *ceteris paribus* - high yielding stocks commanded a

Table 1a: Risk Indices Fundamental Swiss Multiple Factor Model.

-
1. SIZE values total assets and market capitalisation to differentiate between large and small stocks.
Components:
 - + Log of Capitalisation
 - + Log of Total Assets
 2. SUCCESS identifies recently successful stocks using price behaviour (measured by historical alpha and relative strength), and to a lesser degree, earnings growth information.
Components:
 - + Relative Strength
 - + Historical Alpha
 - + Earnings Growth, 5 years
 3. YIELD measures the company's current dividend yield and yield over previous years.
Components:
 - + Current Yield
 - + Average Yield, 5 years
 4. VOLATILITY predicts the company's volatility based on its historical behaviour. Unlike beta, which measures only the market responsiveness of a stock, this risk index measures a stock's overall variability, including its response to the market.
Components:
 - + Historical beta
 - + Cumulative Range, 1 year
 - + Historical Beta * Historical Sigma
 5. VALUE captures the extent to which a company's ongoing business is priced inexpensively in the marketplace by looking at cash flow to price, earnings to price, and sales to price.
Components:
 - + Cash Flow to Price
 - + Earnings to Price
 - + Sales to Price
 6. EARNINGS VARIABILITY measures a company's historical earnings variability and cash flow fluctuations over the past five years.
Components:
 - + Variance of Earnings
 - + Variance of Cash Flow
 7. GROWTH is primarily an indicator of a company's historical growth.
Components:
 - + Earnings Growth, 5 years
 - + Asset Growth, 5 years
 - + Sales Growth
 - + Recent Earnings Change
 - + Change in Assets
 8. FINANCIAL LEVERAGE captures the financial structure of the firm.
Components:
 - + Debt to Assets
 - + Book Leverage
-

Table 1b: Industry Classification Fundamental Swiss Factor Model.

Firms are assigned to industries according to the Swiss Performance Index classification.

The industry classifications are:

1. Banking	7. Utilities
2. Insurance	8. Chemicals and Pharmaceuticals
3. Transportation	9. Food and Beverage
4. Department Stores	10. Electronics
5. Other Services	11. Building and Construction
6. Machinery	12. Miscellaneous

positive risk premium. In fact, a pure factor portfolio positioning itself at the 83rd percentile on the yield dimension, would have cumulatively outperformed the market (i.e. the SPI index) by 13.2% over the period January 1985 through December 1989. Similarly large companies tended to have a negative risk premium until the last quarter of 1988 i.e. small companies tended to outperform large companies.

The adjusted R squared for the cross-sectional regressions averaged .46 over the entire 5 year period. The model has a tendency to explain cross-sectional differences well in months when excess returns are large (the R squared was .93 in October 1987), whereas the R-Squared tends to be low when the stock market on average doesn't move much in a given month [4]. All 20 explanatory variables on average were statistically relevant over the 60 month period, implying that we are capturing effects which between 1985 and 1989 had a noticeable impact on stock returns. Table 3 summarises the F- stats for the 8 fundamental factor returns (over the 60 month sample period).

Each cross-sectional regression (1) also yields an estimate of each stock's specific return (i.e. the error term $\varepsilon_i(t)$). The variance of this specific return over the 60 month period provides an estimate of the stock specific variance and captures the extent to which the stock is influenced by idiosyncratic events. For example Infranor has a stock specific standard deviation of 37% per year.

Table 2: Factor Return Regression Swiss Fundamental Model; August 1992.

Factor	Regression Coefficient	T-Stat
Size	-0.003	-1.06
Success	0.007	1.51
Yield	0.002	0.47
Volatility	-0.005	-0.94
Earnings Variability	0.011	1.73
Growth	-0.002	-0.32
Leverage	0.025	4.20
Value	-0.029	-3.93
Banks	-0.042	-4.40
Insurance	-0.055	-5.19
Transport	-0.083	-3.37
Department Stores	-0.043	-2.39
Other Services	-0.049	-4.58
Machinery	-0.055	-3.93
Utilities	-0.111	-5.73
Chemicals	-0.025	-2.76
Food & Beverages	-0.036	-3.29
Electronics	-0.076	-5.45
Building & Construction	-0.055	-3.20
Miscellaneous	-0.040	-3.08

Adjusted R-Squared: 0.36

The 60 month history of factor returns can be used to compute the factor covariance matrix F . Given the matrix of company exposures X as well as the stock specific variances (arranged in a diagonal matrix Ω), all ingredients of equation (2) have been identified.

Table 3: Significance of Fundamental Factor returns (critical F - value at 90% confidence is 2.06).

Factor	F-Stat	Percent of Time Abs(T-Stat) > 2
Size	7.30	45
Success	4.34	29
Earnings Variability	3.79	29
Volatility	3.28	26
Value	2.47	24
Growth	2.31	17
Yield	2.30	16
Leverage	2.07	18

Figure 1: Cumulative Time Series of Size Factor Return.

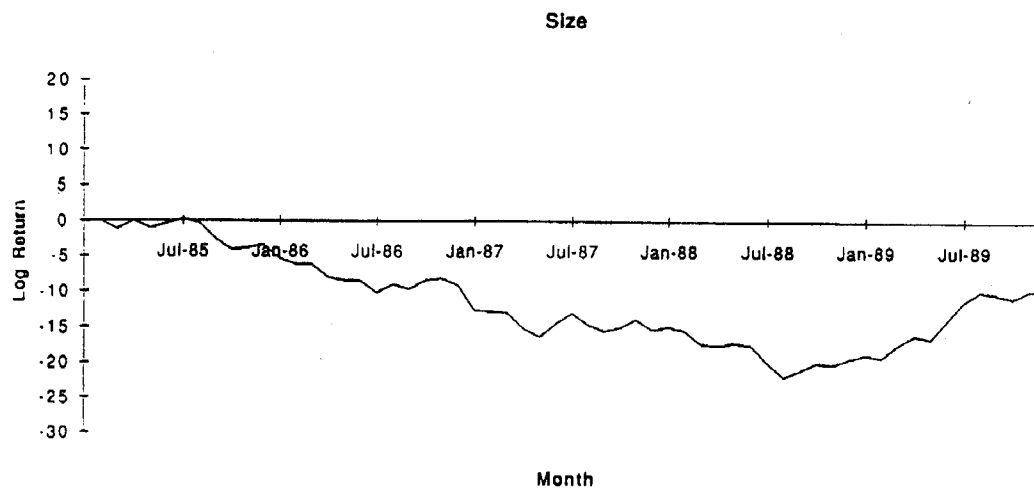


Figure 2: Cumulative Time Series of Yield Factor Return.

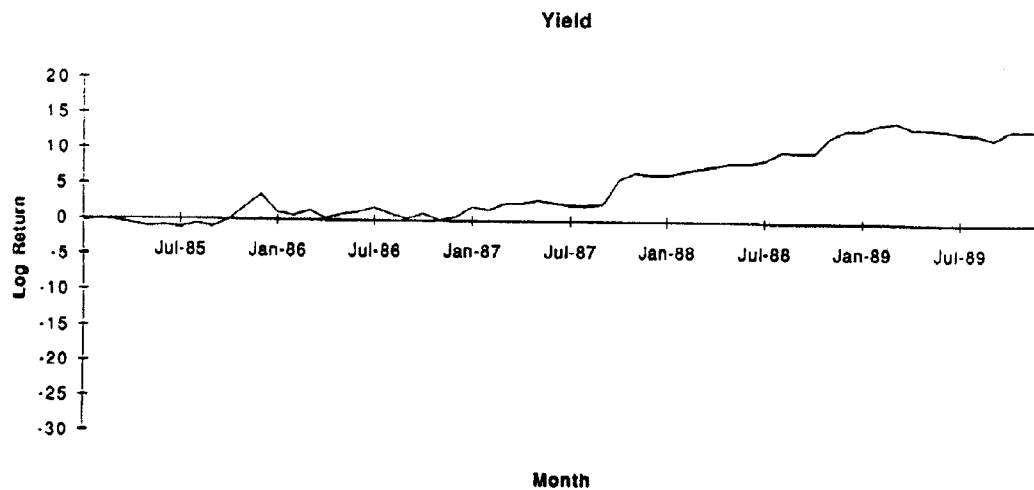
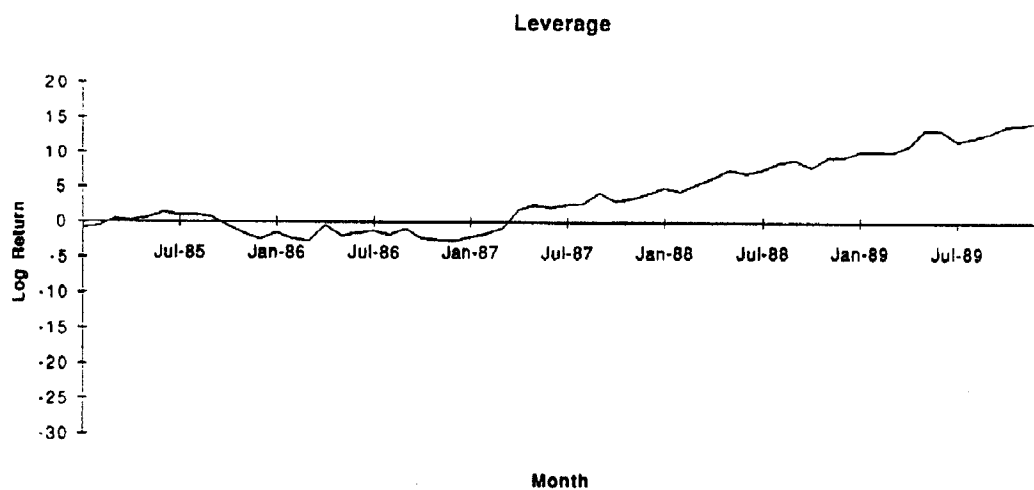


Figure 3: Cumulative Time Series of Leverage Factor Return.



In sum, the fundamental factor model has identified 20 characteristics which were shown to have a discriminatory impact on stock returns. The relevance of these findings obviously depends on the stability and robustness of these characteristics through time. In section 7 we will address these questions in the context of the comparison with the statistical model.

5. The Statistical Factor Model

As mentioned in section 2 the statistical model takes as a starting point the historical covariance matrix V which is decomposed using a factor analysis procedure. The implementation of factor analysis on large data sets often causes significant problems (see SHANKEN (1982)) and a detailed discussion of the statistical intricacies is beyond the scope of this paper. (For an introduction to Factor Analysis see: LAWLEY/MAXWELL (1971) or JACKSON (1991)).

A number of these empirical estimation problems arise from the fact that typically the number of assets (250 in our case) largely exceeds the number of data points (60 months) which may cause singularity problems in the historical matrix V . Intuitively it should be obvious that estimating matrix V - which attempts to capture the interrelationships between 250 different stocks - using only 60 data points tries to extract too much information from a limited data set. Therefore assets are gathered into 30 mutually exclusive groups using nearest neighbour cluster analysis and group returns are calculated.

A maximum likelihood factor analysis was then performed on the covariance matrix of group returns, resulting in a matrix of group factor loadings X and a time series of factor returns. The maximum likelihood convergence criterion selected 9 factors. The added explanatory power contributed by the remaining is negligible in a statistical sense.

Historical time series regressions of the excess stock returns against the statistical factor returns were used to obtain the individual asset factor

exposures. The variance of the error term of these regressions is again a measure of the specific risk of each stock.

The average R-Squared of these regressions was 0.68, which compares favourably with the fundamental model's average of 0.46. This result in itself is not surprising since the factor analysis has the opportunity of fully 'mining' the data to obtain both the best factor return series and factor exposures [5].

6. Fundamental versus Statistical Model: A First Comparison

Before addressing the empirical robustness of both approaches, it is worthwhile to compare the conceptual differences. The undisputed advantage of the fundamental model lies in its intuitive appeal: it is couched in a terminology and applies concepts which are directly familiar to portfolio managers and investment analysts. The approach studies and validates those decision criteria which are traditionally used in stock selection and portfolio construction. The identifiability of the factor exposures also facilitates error checking and model diagnostics.

The relevance of the approach depends however on an exhaustive enumeration and quantification of all relevant dimensions: to the extent that important company characteristics are overlooked or incorrectly measured, a regression analysis of equation (1) will suffer from omitted variables or errors in variables problems. Although one hopes to guard against these problems by specifying all relevant characteristics using objective information variables, there will always be a lingering doubt that a statistically useful construct has been overlooked. A second criticism of the fundamental modelling approach addresses the possible multicollinearity problems in specifying the factor exposure matrix X . It is obvious that a number of the descriptive statistics in the Appendix are highly collinear (i.e. they essentially measure the same attribute). The cluster analysis which reduces the descriptors to a limited set of risk indices (as in Table 1a) significantly reduces this problem. In fact the highest cross-

sectional correlation between the resulting risk indices, (company attributes) is .39 (between Growth and Earnings Variability). [6]

Another significant advantage of the fundamental model lies in its adaptability through time: the exposure matrix (i.e. the way in which companies are described) will change from month to month and will immediately reflect new information. For instance a merger or acquisition will obviously affect the fundamental characteristics of a company. This will be picked up immediately by the revised balance sheet and income statement ratios for the new company. Similarly when a company changes its leverage or cuts its dividend, this information will immediately be reflected by the changed risk indices. Finally, the approach easily accommodates new issues. In fact, given information about the financial ratios of the company, a newly introduced company can be integrated into a fundamental model with virtually no market history at all. Not surprisingly, the strengths of the fundamental model are the weaknesses of the statistical one (and vice versa). In particular, the statistical model assumes a constant exposure matrix (i.e. it implies that the attributes of a company do not change through time). Since the approach only uses price information to estimate the model, any changes in the risk profile of a company will only feed through slowly into the estimation process (since a sufficiently long period will have to elapse before this change is reflected in the price history in a statistically significant way). In addition the statistical approach cannot accommodate newly issued stocks since the vital ingredient (a price history) is lacking.

Another weakness of the statistical model arises from its lack of intuitive appeal : in its pure statistical form the factors and the stock exposures to these factors have no straight economic meaning. As pointed out, the factor exposures can be correlated with company attributes (or the factor returns can be correlated with changes in macro variables) but this is not a rigorous and uniquely defined process. On the other hand the statistical model is guaranteed to be 'complete': the estimation procedure will identify all statistically relevant dimensions. In ad-

dition, the resulting factors will be orthogonal (i.e. independent of each other) by design.

Finally it is worth mentioning that the statistical model (as all such procedures) is highly sensitive to data errors and outliers: the method will try to fit the input panel exactly, irrespective of whether the data is correct or representative. For example if - for spurious reasons - Nestle and Swissair performed highly similarly over the estimation interval, the statistical procedure will tend to group them together and classify them as having a similar profile. Since the statistical model only uses return information as an input it will never be able to discriminate between stocks which have similar returns for different reasons. The fundamental model on the other hand uses the known company profile to avoid these problems. In addition, it guards against the eventuality of the statistical analysis being dominated by outliers through a winsorization process (see footnote 2) which minimises the impact of extreme observations.

7. Fundamental versus Statistical Model: Empirical Tests

A fair and direct test of both models is not easy to devise. In this section we will discuss the results of a comparison based on a Monte Carlo simulation (a 'virtual reality' test), as well as the accuracy of the out-of sample predictions of the model.

7.1 The 'virtual reality' comparison

Let us assume that the true matrix V is known and well behaved [7]. Assuming that the stocks follow a multivariate normal distribution with covariance matrix V , we can generate a set of 250 returns (which would be the equivalent of one month's worth of returns for 250 stocks). We can therefore create however many monthly returns as we like. In our tests we created a short (36 months) and a long (84 months) simulated history.

The simulated return series can now be used as inputs into the two estimation procedures. For the

fundamental model we used the factor exposure matrix of December 1989 (X_f) in combination with the simulated returns to estimate the (36 or 84 month) time series of 20 factor returns and stock specific returns. The time series of factor and stock specific returns in turn were used to estimate matrices F_f and Ω_f . This results in an estimated 'fundamental' covariance matrix:

$$\hat{V}_f = X_f F_f X_f^T + \Omega_f$$

Alternatively we can use the simulated return series to compute the corresponding sample covariance matrix (based on 36 or 84 observations). The factor analysis is used to identify the 20 most relevant factors and an estimated 'statistical' covariance matrix is reconstructed:

$$\hat{V}_s = X_s F_s X_s^T + \Omega_s$$

Since we know the true original covariance matrix V which generated the return series, we can directly compare the estimated matrix with the true one.

Let

V_{ij} = true covariance between security i and security j

\hat{V}_{ij} = estimated covariance between security i and security j

then the following test metric

$$T = \frac{\sum_{i=1}^N \sum_{j=1}^N |V_{ij} - \hat{V}_{ij}|}{\frac{N(N+1)}{2}}$$

gives an idea how closely the estimated matrix approximates the true one. Each simulation was performed 30 times. Table 4 summarises the values of the test metric.

On average the fundamental and statistical model perform very similarly. However the statistical model has a slight upper hand over a longer estimation period. This is inevitable since - by design - the

statistical model will match V ever more closely as the number of observations increases. The fundamental model on the other hand has a tendency to do better over short observation intervals since it is less sensitive to return outliers. Overall however, both approaches seem to provide robust in-sample estimates of covariance matrices.

7.2 The out-of-sample test

Although both approaches give qualitatively similar results within sample, their accuracy during an out of sample period will provide more relevant insights. The strength of the model during the out of sample period will crucially depend on the stationarity of the factor exposure matrix: the fundamental model uses continually updated company characteristics whereas the statistical model assumes that the factor exposures will remain largely the same into the future. If indeed the characteristics of the stocks do not change through time, the statistical and fundamental model will give similar results - as evidenced by the virtual reality test. If however the nature of companies evolves through time, this effect will be overlooked by the statistical model. It is an empirical question how significant this exposure nonstationarity may be. The out of sample test covers the period January 1990 through December 1991 and involves the following procedure: given the predetermined company characteristics (estimated using up-to-date information for the fundamental model and using the data from January 1985 through December 1989 for the statistical model),

Table 4: Test metric: Estimated versus true covariance matrix.

	36 monthly returns		84 monthly returns	
	Statistical	Fundamental	Statistical	Fundamental
Minimum	8.79	7.84	7.27	8.15
Maximum	16.63	18.14	13.18	12.53
Average	12.29	11.26	8.98	9.66

Table 5: Goodness of fit of statistical versus fundamental Model: In-sample versus out of sample average adjusted R-squared (Monthly regressions).

	Fundamental	Statistical
In sample	46%	68%
Out of sample	42%	38%

regression (1) is rerun every month. Table 5 summarises the accuracy of each approach.

Note the significant deterioration of the performance of the statistical model: it succeeded exceptionally well in fitting the data in sample but performed rather poorly out of sample. The fundamental model on the other hand maintained approximately the same explanatory power out of sample as in sample. The same insights are obtained when concentrating on the Root Mean Squared error of regression (1). Let

$$RMSE_t = \sqrt{\sum_{i=1}^N W_i (R_{it} - \hat{R}_{it})^2}$$

where

$$\begin{aligned} R_{it} &= \text{actual excess return of asset } i \text{ in month } t \\ \hat{R}_{it} &= \text{fitted excess return of asset } i \text{ in month } t \\ W_i &= \text{weight of asset } i \end{aligned}$$

The $RMSE$ can be calculated using either capitalisation or equal weights. Table 6 summarises the

Table 6: Goodness of fit of statistical versus fundamental model: in sample versus out of sample; Average Root Mean Squared Error (Monthly Regressions).

	Capitalisation Weight	Equal Weight
Fundamental Model		
- In sample	5.13	6.44
- Out of sample	5.42	7.37
Statistical Model		
- In sample	3.78	4.44
- Out of sample	7.21	9.69

results. For the statistical model, again a significant deterioration of the goodness of fit during the out of sample period should be noted.

We must therefore conclude that for the specific periods of this study, the statistical model, estimated on monthly data, significantly underperformed the fundamental model. This must be due to the fact that the company factor loadings as estimated by the statistical model were not sufficiently flexible to pick up changes in company profiles.

8. Fundamental versus Statistical Model: A Round Up

Both the statistical model and fundamental model performed well within sample and proved to be useful for risk and return analysis purposes. However, the statistical model is couched in a terminology which is statistical rather than economic and therefore lacks intuitive appeal.

The fundamental model will provide a better fit to the data out of sample in those periods when company characteristics are subject to change. Only if companies remain unchanged through time, will a statistical model estimated on the basis of long data series provide a better fit to the data than a fundamental one. The fundamental model will always maintain the advantage of being able to accommodate newly issued stocks.

On balance the advantages of the fundamental model seem to outweigh the disadvantages. The risk breakdown and attribution on the basis of the intuitive fundamental factors can also provide useful insights to the portfolio manager. This is briefly illustrated in the following paragraphs.

9. Risk Analysis Using a Fundamental Factor Model

The fundamental factor model can be used to analyse and compare the risk of one portfolio versus another. Let us briefly illustrate this exercise using the SMI index as our portfolio and the SPI index as

Table 7: Factor Exposures of SMI versus SPI August 1992.

	SMI		SPI	
	1 Standardised Factor Exposure	2 Percentile Ranking	3 Standardised Factor Exposure	4 Percentile Ranking
Size	.81	79	0.0	50
Success	.18	58	0.0	50
Yield	-.03	48	0.0	50
Volatility	.06	53	0.0	50
Earnings Var	-.09	46	0.0	50
Growth	-.15	44	0.0	50
Leverage	.02	51	0.0	50
Value	-.04	48	0.0	50

the benchmark. Table 7 summarises the factor exposure information.

The SMI index is biased towards larger, slightly lower yielding companies with slower growth than the typical company in the SPI index. A similar comparison on the sector distribution (Table 8) reveals that the SMI index is significantly overweight on the food and chemical sector and underweight in insurance and other services in comparison to the SPI index.

Table 8: Sector distribution of SMI versus SPI August 1992.

Factors	SMI 1	SPI 2	DIFF 3
Banks	23.31	19.48	3.83
Insurance	3.41	9.49	-6.08
Transport	0.41	0.71	-0.30
Department Stores	0.00	1.31	-1.31
Other Services	0.30	7.22	-6.92
Machinery	0.00	2.89	-2.89
Utilities	1.34	1.63	-0.29
Chemicals	37.63	30.95	6.68
Food & Beverages	27.57	15.30	12.27
Electronics	3.43	5.46	-2.03
Building & Constr.	1.36	2.05	-0.69
Miscellaneous	1.25	3.51	-2.27

Given the differences in profile between SMI and SPI and given our knowledge about the factor return covariance matrix F , we can infer the differential risk (sometimes also called active risk or tracking error) of the SMI versus the SPI.

Let

Z = vector of differential factor exposures between SMI and SPI (i.e. the combination of column 1 in Table 7 and column 3 in Table 8)

F = factor covariance matrix

W_i^P = weight of stock i in the SMI index

W_i^I = weight of stock i in the SPI index

ω_i = specific risk of stock i

then the tracking error variance is given by

$$Z' F Z + \sum_{i=1}^N (W_i^P - W_i^I) \omega_i^2 \quad (3)$$

The intuitive explanation of equation (3) rests on the following observations:

- if two portfolios have a similar profile, they will perform in line with each other. The factor exposure vector quantifies the similarity between SMI and SPI.
- the fact that two portfolios have a significantly different profile does not necessarily imply that they will perform very differently. The expected differential performance is also a function of the riskiness of the dimensions on which the portfolios differ. In particular:
 - a small overweighting in the chemical sector may be a more important cause of differential performance between portfolio and benchmark than a large overweighting in the food sector, given that the food sector is less volatile;
 - although the transport sector in itself is very volatile, this will not have an important impact on the differential performance since both SMI and SPI have a similar exposure to this industry.
- the fact that the SPI has many stocks which are not part of the SMI implies that specific events associated with these companies will have an

impact on the return of the SPI but not of the SMI.

- the fact that the stocks in the SMI have a higher relative weight than the same stocks in the SPI implies that stock specific events for these companies will affect the SMI more heavily than the SPI.

Applying formula (3) results in a tracking error of 3.94% for the SMI relative to the SPI. In other words in any given year one would expect (with 66% probability) the return on the SMI to equal the return on the SPI plus or minus 3.94% (i.e. if the return on the SPI is 8% one would expect the return on the SMI to lie between 4.06% and 11.94%).[8] Equation (3) also allows us how to analyse exactly the sources of this tracking error : we can identify how each differential characteristic impacts the active risk. This exercise could be thought of as the sequential elimination of all the differential factor exposures and a recalculation of the tracking error. For example one could ask the question "How much would the tracking error decrease if the portfolio remained exactly the same except for the over-weighting in the food sector, which would be neutralised?". Table 9 identifies the relative importance of the different sources of tracking error.

It can be inferred that the large company, low growth bias of the SMI is a relatively more important source of tracking error than the industry over- and underweightings. The fact that the SMI only contains 21 stocks and therefore is less perfectly diversified (more subject to stock specific risk) also comes through strongly in the risk breakdown.

It is obviously up to the portfolio manager to decide whether the amount of tracking error and the way it is made up is acceptable in the context of his investment strategy and his expectations for stock, sector or market returns.

Table 9: Breakdown of tracking error.

	% contribution
Risk indices	58%
Sector weightings	11%
Stock specific risk	31%

10. Portfolio Construction Using a Fundamental Factor Model

Whereas the risk analysis uses a static portfolio as input to compare against the benchmark, a risk model can also be used as an aid in portfolio construction. The simplest application in this context is portfolio indexation i.e. create a portfolio which will match the benchmark on all relevant dimensions.

Technically speaking the optimisation will minimise the tracking error of equation (3) by making sure that the factor exposures of the portfolio match those of the index. This is easily achieved if one can buy all stocks in the index in exactly their index weights. If however not all stocks can be bought, then a selection will have to be made such that the portfolio will only deviate from the benchmark profile on the less important (in a risk sense) dimensions. If only a few stocks can be bought, then the optimisation exercise has to ensure that portfolio and benchmark match each other on the most important dimensions.

Minimisation of equation (3) subject to constraints such as a maximum number of stocks which can be held can be achieved through a statistical technique called quadratic programming. For example an optimised portfolio of 50 stocks would be able to track the SPI index within 0.78%. (Its return would - with 66% probability - approximate the return on the SPI within .78 %) Figure 4 displays graphically the tracking error which can be expected for optimised portfolios with different numbers of stocks [9]. The application of optimisation techniques obviously need not be restricted to the (rather boring) case of indexation: equation (3) can be extended to include a return component as well as a transaction cost element.

For example, let

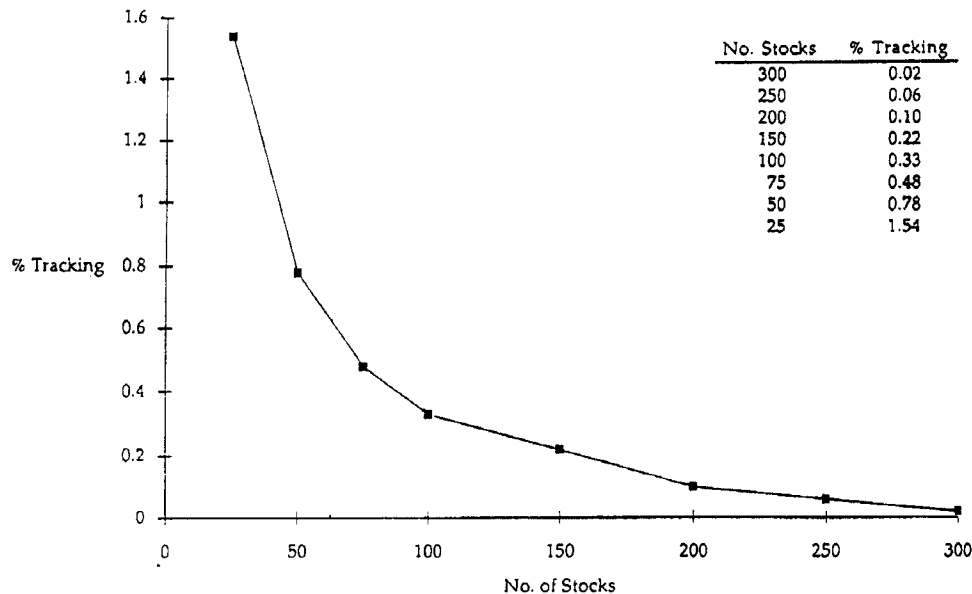
α = vector of expected stock specific returns
[10]

W_p = vector of stock weights (decision variable)

W_i = vector of stock weights in the index

c = vector of per stock transaction costs

λ = penalty for risk (also called risk aversion

Figure 4: Tracking Error versus the SPI Index.

parameter). This parameter transforms units of tracking error into units of return. For example with a risk aversion parameter of .1 one would expect at least .1% return for 1% tracking error (or one would be willing to pay .1% to get rid of 1% tracking error)

ω^2 = vector of stock specific variances

$$\begin{aligned} \text{Max } W_p^T \alpha - \lambda [(W_p - W_I)^T X F X^T (W_p - W_I) \\ + \sum_{i=1}^N (W_p^i - W_I^i) \omega_i^2] - W_p^T C \end{aligned}$$

The optimisation exercise ensures that the existing portfolio gets transformed into a more optimal one by

- buying stocks with high expected specific return
- selling stocks with low expected specific returns
- only buying and selling to the extent that the increase in return more than exceeds the transaction costs involved
- making sure that the overall risk profile of the portfolio remains in line with that of the index

11. Performance Analysis and Attribution Using a Fundamental Factor Model

Portfolio management is a game of luck and skill. Ex post we would like to know which part of the return can be explained by the exceptional insights of the portfolio manager and which part must be attributed to the whims of the market. Unfortunately the human mind has a tendency to associate all positive outcomes with skill and to blame bad luck for all negative ones. Ideally we would like a more objective measure of luck and skill. The risk model can play a useful role in this exercise. To illustrate, let us again assume that the SMI is the managed portfolio whereas the SPI is our benchmark. We would like to know whether, how and to what extent the strategic biases (as well as the tactical changes resulting from reweightings of the stocks) in the SMI have contributed to its relative performance. Note that from the cross-sectional regression (1), we know each month what the return associated with each of the company attributes is. If a portfolio differentiates itself from the benchmark along one of these characteristics, we can attribute part of the differential return between portfolio and benchmark to the differential (or active) factor exposure

and the corresponding factor return. For example in July 1992, the SMI outperformed the SPI by .1948%. The factor return associated with size in July was .03% (i.e. all other things equal a portfolio with a unit exposure to size would have outperformed the SPI by .03%) whereas the SMI had a size factor exposure of .86. Therefore, .025% (.86 times .03%) of the .1948% outperformance can be explained by the size bias in the SMI. Table 10 gives a complete return attribution of the SMI-SPI differential return for July 1992.

The return attribution can be repeated for a number of months and related to the inherent risks. For

Table 10: Return attribution SMI versus SPI July 1992.

	(1) Active Exposure	(2) Factor Returns	(3) Active Return Contri- bution (i.e. co- lumn 1 * column 2)
Size	.86	.03	.0258
Success	.14	1.76	.2464
Yield	-0.01	-0.36	.0036
Volatility	0.05	-1.06	-.053
Earnings Variability	-0.09	0.65	-.0585
Growth	-0.11	0.14	-.0154
Leverage	-0.01	-1.42	.0142
Value	-0.03	-2.39	.0717
		Sum	.2348
Banks	3.59%	-1.20	-.0431
Insurance	-4.99%	0.38	-.0189
Transport	-0.32%	-3.48	.0114
Department Stores	-1.33%	-0.60	.0080
Other Services	-7.34%	-6.83	.5013
Machinery	-3.03%	-4.56	.1382
Utilities	-0.38%	-3.34	.0126
Chemicals	6.54%	-4.73	-.3093
Food & Beverages	12.54%	-3.58	-.4489
Electronics	-2.00%	-7.02	.1404
Building & Const.	-0.92%	-1.72	.0158
Miscellaneous	-2.34%	-3.10	.0725
		Sum	.08
	Specific Asset Selection		-.12
	Total		.1948

example, if the size bias in the SMI by itself entails 1% annual tracking error but contributed to an annual outperformance of 2%, we would conclude that the portfolio manager had skill in identifying the periods in which an exceptionally large size factor return could be expected.

We could imagine that somebody would have followed a strategy of replicating the SMI over the period January 1991 through July 1992. Over this period the SMI cumulatively outperformed the SPI by 10.02%. As evidenced in Table 11, most of this outperformance was due to the risk index characteristics (see also Figures 5 and 6).

Translating these relative performance numbers to an annual basis, we can compare them to the corresponding risk characterization. Table 12 summarizes this information.

We would therefore conclude that the outperformance of this strategy could not be due to luck. Given a T-Stat of 1.99, one would attribute active management skill to this particular portfolio manager. Most of the outperformance arises from the portfolio style (i.e. the risk index attributes). A slight outperformance derives from the sector biases whereas stock specific events adversely affected this portfolio in comparison to the SPI. (Neither of these two effects is statistically significant though). Table 13 summarizes the 5 best and 5 worst policies. It is immediately obvious that the large company bias was by far the most advantageous differentiating characteristic (or in other words there was an extremely significant large capitalization effect in Switzerland over this period) [11]. In other words a portfolio which would have been identical to the SPI except for a large company bias (similar to the one in the SMI), would have outperformed the SPI by 5.36% per year over the period January 1991 through July 1992. Similarly, a portfolio with

Table 11: Cumulative performance SMI versus SPI. January 1991 - July 1992.

Cumulative Active Return	10.02%
Risk Index Contribution	11.82%
Industry Contribution	2.02%
Stock Specific Contribution	-3.82%

Figure 5: Cumulative Active Performance Summary.

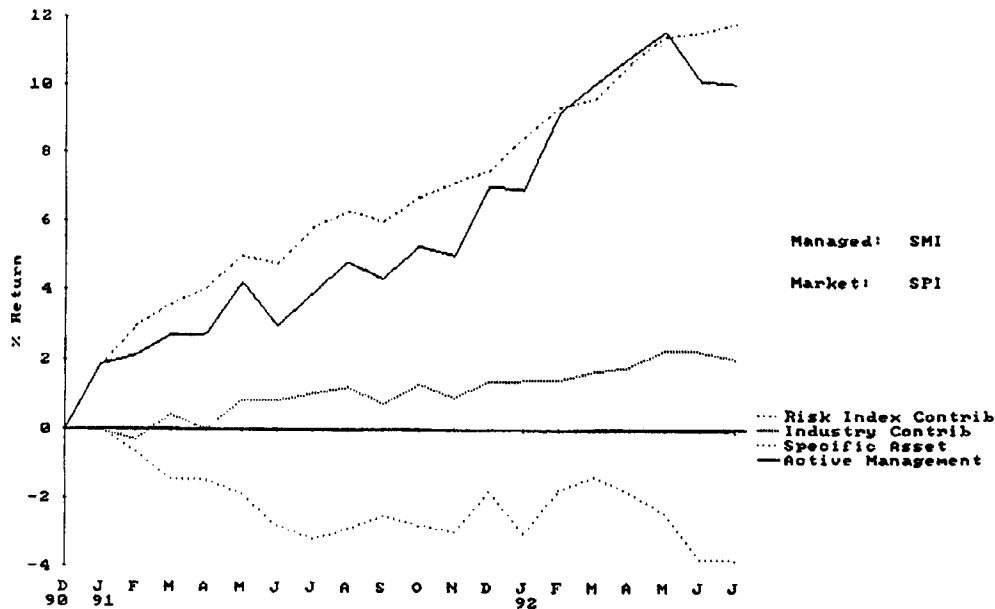


Figure 6: Active Returns.

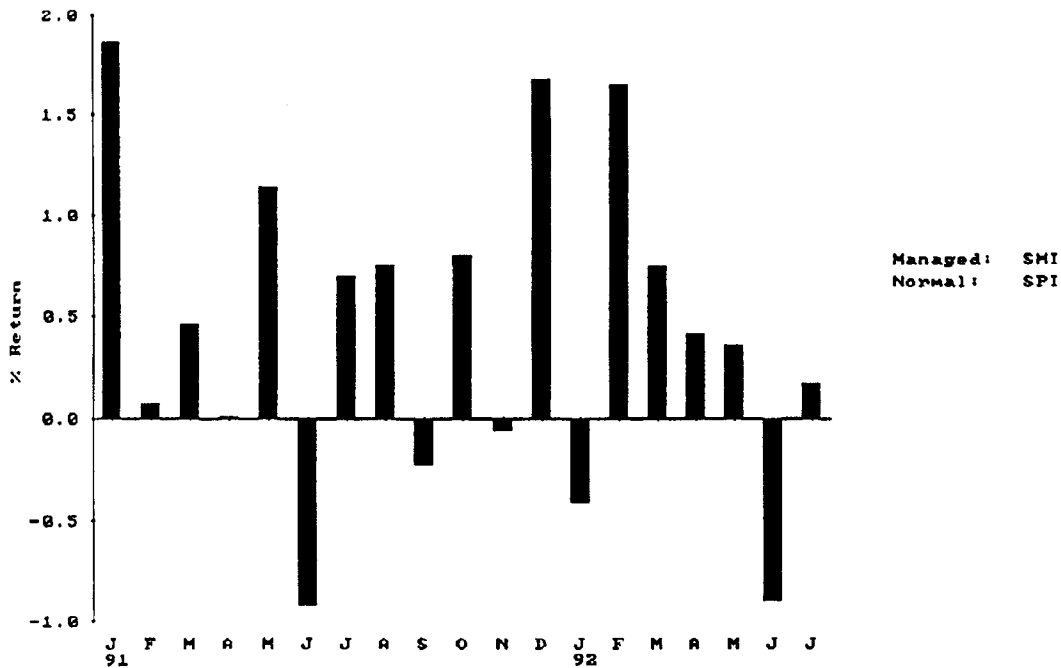


Table 12: Annualized performance SMI versus SPI; January 1991- July 1992.

	% Return 1	% Risk (Std Deviation) 2	Information Ratio a* 1/2	T-Stats**
Total From Active Management	5.81%	3.67%	1.58	1.99
From Risk Indices	6.87	2.64	2.60	3.28
From Industry Selection	1.19	1.37	0.87	1.09
From Stock Selection	-2.20	2.34	-.94	-1.19

* The information ratio is the reward/risk ratio (% return divided by % risk). It indicates how much return on average the manager achieved per unit risk.

** There is a straightforward relationship between the information ratio and the T-Statistic:

$$T.Stat = \text{Info Ratio} * \sqrt{\text{number of periods}}$$

In our example our study extends over 1.58 years (1 year and 7 months). So therefore,

$$T.Stat = \text{Info Ratio} * \sqrt{1.58}$$

If the T.Stat is higher than 2, one would typically conclude that the result is unlikely to be due to chance alone.

Table 13: Best & Worst Policies SMI versus SPI; Annualized Returns.

Best Policies	Annualized Contribution
Size	5.36%
Success	0.785
Chemicals	0.71%
Growth	0.52%
Other Services	0.37%
Worst Policies	Annualized Contribution
Specific Asset Selection	-2.20%
Banks	-0.34%
Volatility	-0.17%
Earnings Variability	-0.15%
Electronics	-0.12%

an overweighting on the Bank Sector (similar to the overweighting of the SMI on the Banks) would - ceteris paribus - have underperformed the SPI by 0.34% per year over this period. Finally the stock specific events had the most detrimental impact on the relative performance.

The performance attribution and analysis allows us to study more closely the differential performance between managed portfolio and benchmark: they help identify managerial strengths and weakness and - given a sufficiently long performance history - help separate luck from skill.

12. Conclusion

In this paper we have discussed alternative approaches to Multiple Factor Models for the Swiss Equity Market. We reported on the estimation of a fundamental factor model as well as a purely statistical one.

Both factor models provided a good characterization of risk and return within sample (i.e. over the estimation interval). However during the out of sample period the fundamental model proved more robust than the statistical one. We therefore used the fundamental model as the starting point for a number of portfolio applications. The static risk analysis allowed us to compare and quantify the importance of salient differences between portfolio and benchmark. It proved to be useful as a tool to measure a portfolio manager's aggressiveness and business risk.

The fundamental risk model was also used in the context of portfolio construction. We illustrated how an SPI index fund could be constructed and discussed the possible application for active portfolio management.

Finally, the model also provided interesting insights into comparative performance analysis and attribution. An SMI replicating portfolio was shown to have significantly outperformed the SPI over the 18 month period between January 1991 and July 1992. This outperformance was almost exclusively due to a strong "large cap" effect over this period

(whereas stock specific events affected the portfolio more adversely than the SPI).

Multiple Factor Models are straightforward extensions of the single factor Capital Asset Pricing Model. They are more powerful in their explanatory powers and can use concepts and a terminology which is directly intuitive to the portfolio manager.

Appendix

Descriptors tested for the Swiss Fundamental Factor Model

AGRO Asset growth rate, equal to the annual trend in total assets divided by average value, last five years

$$AGRO = \frac{\text{regression coefficient of } X \text{ on } t}{|\bar{X}|}$$

where

X = total assets

t = yearly index; $t = 1, 2, 3, 4, 5$

ASSI Logarithm of total assets

BLEV Book leverage

$$BLEV = \frac{\text{long-term debt}}{|\text{common equity}|}$$

BTSG Product of beta and sigma; see HBET and HSIG.

$$BTSG = (HBET)(HSIG)$$

CMRA Cumulative range, last 12 months, in logarithms.

$$CMRA = \ln(\text{max price}) - \ln(\text{min price}),$$

where max price = the maximum price over the last 12 months

min price = the minimum price over the last 12 months and

prices are adjusted for capital transactions.

CTOP Cash flow to price ratio.

$$CTOP = \frac{\text{latest gross cash flow}}{\text{market capitalization}}$$

DELE Delta earnings; a measure of proportional changes in adjusted earnings per share in the last two fiscal years where earnings/share are adjusted for capital transactions.

$$\frac{\text{earnings}_t - \text{earnings}_{t-1}}{|\text{earnings}_t| + |\text{earnings}_{t-1}|} = 0$$

2

when earnings are zero in both years.

DELE = 0.33 when DELE > 0.33, and

DELE = -0.33 when DELE < -0.33

	idual mean square in beta regression.
	$HSIG = \sqrt{\frac{1}{T-2} \sum_{t=1}^T \hat{e}_t^2}$
	where
	e_t = the residual in the regression for historical beta in month t $\hat{e}_t = r_t - \hat{r}_t$ T = the number of data points available.
LNCAP	An indicator of size, equal to the natural logarithm of the market value of common equity.
PCTAS	LNCAP = Ln [Market capitalisation] Increase in assets (one year).
	$PCTAS = \frac{\text{current total assets}}{\text{total assets, previous year}}$
RSTR	Logarithmic rate of return over last year (relative strength).
	$RSTR = \sum_{t=1}^{12} \ln(1 + r_t)$
	where
	r_t = asset return over month t. Note: Normalizing converts this to a relative strength measure.
SGRO	Sales growth rate.
	$SGRO = \frac{\text{regression coefficient of } X \text{ on } t}{ \bar{X} }$
	where
	X = total sales t = yearly index; t=1,2,3,4,5,
STOP	Sales to price
	$STOP = \frac{\text{total sales}}{\text{market capitalization}}$
VERN	Variability (coefficient of variation) of annual earnings in last five years.
	$VERN = \sqrt{\frac{\sum_{t=1}^T (\text{earnings}_t - \text{average earnings})^2}{T-1}}$
	average earnings

where

average earnings include earnings over the preceding 5 years

T is the number of years that data exist

$VERN = 1.5$ when $VERN > 1.5$

VFLO

Variability (coefficient of variation) of cash flow.

$VFLO =$

$$\sqrt{\frac{\sum_{t=4}^T (\text{cash flows}_t - \text{average cash flows})^2}{T - 1}} \div |\text{average cash flows}|$$

YLD

$\text{Current divid. yield} = \frac{\text{most recent dividend}}{\text{most recent price}}$

YLD 5

Yield, average value for last five years.

Footnotes

- [1] This is standard procedure. See for instance FAMA/FRENCH (1992).
- [2] To facilitate the regression analysis and to make the regression coefficients directly comparable, the risk indices are standardised and winsorized. The standardisation involves subtracting each month the cross-sectional capitalisation weighted average of all stocks from the raw variable. The resulting value is then divided by the equal weighted standard deviation. In practice this means that the units in which risk indices are measured are expressed as the number of standard deviations away from the mean.
This procedure would for instance result in an earnings variability index of 2.75 for Alusuisse, implying that relative to the market average, Alusuisse positions itself 2.75 standard deviations above the mean. A transformation using standard normal distribution tables implies that about 98% of the Swiss stocks have a more stable earnings pattern than Alusuisse.
The winsorization process refers to a truncation of the outliers at a fixed number of standard deviations (usually 3) from the mean. This prevents stocks with extreme characteristics from having an inordinate impact on the regression results. For instance SIG has even more erratic earnings growth than Alusuisse. However giving this exceptional behaviour its full weight would bias the insights we could get on the relevance of the earnings variability index for the other companies. Rather than saying that SIG is a 7 standard deviation event, it is positioned at the 3 standard deviation mark (which implies that there are no other companies in the sample with even more erratic earnings).
- [3] This portfolio also would not have any industry exposure and would be neutral with respect to all other characteristics (i.e. it has zero exposure on all other risk indices). It would therefore involve both long and short positions. Although it would be perfectly possible to construct such a portfolio, needless to say it would not be one that would typically be held. The factor return displays the pure return associated with that attribute, net of all other effects. In contrast, the return to a high leverage portfolio - constructed by screening out all lowly leveraged stocks - is not only affected by the leverage factor but also by the industry, size, growth and other biases which this screening entails.
- [4] This is due to the fact, that in periods when the market moves strongly upward or downward, this effect will be impacting all stocks more or less uniformly. Therefore, the high R^2 in periods with large market returns is attributable to the intercept term (or in our regressions to the industry dummies which replace the intercept).

- [5] As a comparison, CUENOT/REYES (1992) estimated a multiple factor model for a universe of 179 Swiss stocks over the period August 1988 through August 1991 using weekly data. They chose to pre-specify the economic factors as the changes in the long interest rate, the spread of long rate over short rate, exchange rates, oil prices and residual market return. The average R^2 they obtain for the 179 stocks is 25.96%. Note that this R^2 is not directly comparable to the R^2 in this study since they only use 5 predetermined factors.
- [6] The cross-sectional regressions can also be checked for the occurrence of significant multi-collinearity. Two test-statistics are available: the variance inflation factors which - for the fundamental model - are always below 5 (with 10 indicating multicollinearity problems). Alternatively the condition number for the model is 8.3 (with a value of 30 signifying serious collinearity).
- [7] We use as a starting point the historical covariance matrix V estimated for the 250 stocks over the period January 1985 through December 1991 using monthly data. (i.e. we assume that this matrix correctly describes the relationships between all stocks).
- [8] Note that the tracking error can also be thought of as a measure of the business risk borne by the investment manager. The tracking error captures the extent to which the portfolio could out - or underperform the benchmark. If the client would be desperately unhappy with an underperformance of 3.94% relative to the SPI - an event which could be due to pure chance - it would behoove the portfolio manager to reduce his tracking error to avoid being fired for having bad luck. Conversely it would be extremely unlikely that a portfolio manager with a tracking error of 0.50% would outperform the benchmark by 1.5%. If a significant outperformance of the SPI is expected, then in that case a higher tracking error will have to be accepted.
- [9] An optimised portfolio containing 21 stocks (i.e. the same number of stocks as the SMI index) would have a tracking error of 1.80% or less than half the tracking error of the SMI.
- [10] Optimizers can just as easily handle other return inputs such as return forecasts for industries or risk indices. Note that in this example we have only entered stock specific returns. Since we have not expressed any views on the behaviour of sectors or types of companies (large, high yielding etc.), there is no reason why we should take a position which differs from the index on any of these dimensions.
- In the case where the return input does express preferences for certain sectors or types of companies, the optimiser will bias (tilt) the portfolio in the desired direction (but only to the extent that the increase in

return more than offsets the (subjective) cost associated with the resulting tracking error).

- [11] This behaviour is in direct contradiction with the small company anomaly which implies that over the long run small companies outperform large ones. The small capitalization effect was discovered by BANZ (1981). However, the anomaly observed by Banz and others holds over the long term. Here we observe a "reverse small cap" effect over a relatively short period.

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