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Portfolio Management Using Portfolio Theory Techniques and the PMSP Professional Software Package

1. Overview of the Portfolio Management Process

When reading about modern portfolio management techniques, we usually see three commonly used variants of asset allocation theory:
Strategic asset allocation is the optimal asset mixture which provides the best risk/return performance taking into account investor risk tolerance and investment horizon.
Tactical asset allocation is the process of adjusting the optimal mix to reflect changes in forecasted economic conditions. This represents a proactive or active management based on the forecasting ability of the manager.
Dynamic asset allocation is the short run adjustment of the portfolio's risk profile in response to market moves by using options and futures (also known as "portfolio insurance"). Dynamic allocation is an adaptive or reactive policy based on adapting to changes in market conditions.
The problem with this descriptive approach is that it defines asset allocation as three distinct approaches when they are simply components of a more complex process. Portfolio management is a multistep interrelated system that reflects these three variants as individual steps in a systematic approach to portfolio management. This system can be described as follows:

1. Screening investments for their risk/return characteristics. Statistical measures such as variance, beta and lower partial moment are used to measure the amount of risk inherent in an investment. The ranking of assets by their risk/return statistics provides an initial screening.

2. Adjusting input data to reflect expected economic conditions. While statistical measures may at first be estimated using historic data, the portfolio manager may wish to adjust the statistics to reflect future expectations and not just past history.

3. Strategic and tactical optimization of portfolio funds. Optimization may be carried out at a strategic level using general asset classes and it may be carried out at the tactical level with individual securities within a general asset class. Optimization provides the individual asset allocations for portfolios with the best expected risk-return performance.

4. Utility selection of the appropriate portfolio. Optimization techniques only provide tradeoffs between risk and return. There will be optimized high return-high risk portfolios, optimized medium return-medium risk portfolios, and optimized low return-low risk portfolios. At this point, the portfolio manager has to decide which portfolio will maximize the economic satisfaction (utility) of the investor. In other words, which portfolio will best meet the needs of the investor. At this point, the portfolio may also be manipulated to respond to short term market conditions by using options and futures or to reflect forecasted market conditions by switching into and out of the money (cash) markets.

5. Evaluation of ongoing portfolio performance. After the appropriate portfolio has been selected and purchased, then it has to be monitored to see whether it is providing the expected performance.

6. Revising portfolio allocations to reflect changes in economic conditions and portfolio performance. As noted above, the short term performance of the portfolio can be manipulated by switching to/from cash balances (other general asset classes) and by using options and futures. However, with changing economic expectations and conditions, the allocations within the portfolio may have to be revised by going back to step 1.

The Portfolio Management Software Package - Professional Edition was developed by Computer Handholders, Inc. to assist the portfolio manager with the implementation of this systematic approach [1]. The purpose of this paper is to provide an overview of this systematic approach to portfolio management and how PMSP Professional (PMSPP) implements this system.

2. Measuring Risk and Return

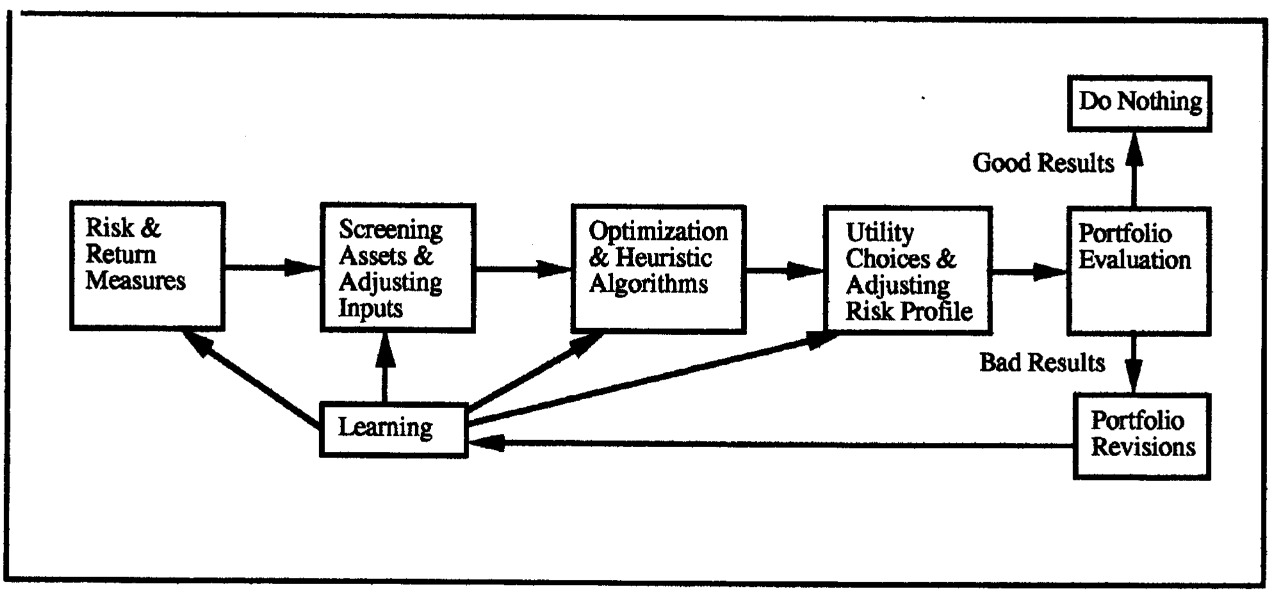
2.1 Statistical Measures

Statistical tools are used in conjunction with historic data to estimate future variability and return for an investment. Since the optimization techniques maximize return and minimize risk, the selection of the measurement tool is important. Flawed estimates of return and risk lead to flawed optimization. The most common statistical tools used to measure risk and return are as follows:

2.1.1. Geometric Average and Final Wealth (Measuring Return)

The PMSPP program utilizes relative percentage changes which are consistent with the compound (future) value of one dollar tables found in all finance textbooks. By basing security return on the investment of one dollar, the returns follow the compound value of one dollar formula, (1+i) and provide a generality that is independent of the

Figure 1: Portfolio management as a multi-step interrelated system.



amount of money invested in the portfolio. Investment returns are calculated using relative percentage changes and the geometric mean.

$$R_t = \frac{P_t + D_t}{P_{t-1}}$$

$$\text{Geometric Mean} = \left(\prod_{t=1}^{k} R_t\right)^{1/k} \qquad (1)$$

2.1.2 Variance (Risk Measure)

The major source of perceived risk to the investor during the investment process is the variability of returns over time. Therefore, various statistical measures of variability of asset returns over time are used. Most of these are based on probability distributions with the most commonly used measure being the variance. It is calculated as follows (for a population statistic):

$$V_i \text{ or } \sigma_i^2 = \frac{1}{k} \sum_{t=1}^{k} [(R_{it} - E(R_i)]^2 \qquad (2)$$

where R_{it} is the relative return for asset i in period t and $E(R_i)$ is the expected geometric mean return for asset i. There are k observations. The variance measures the width or dispersion of a distribution. The greater the width of the distribution, the greater the perceived risk. Given two distributions with the same expected geometric mean rate of return, the investment with the smaller variance is the better investment.

2.1.3 Beta (Risk Measure)

The beta is calculated as the slope of a simple linear relationship between an asset and the general market index. The line that describes this relationship is estimated using regression analysis and a logarithmic transformation.

$$Log\ (R_{it}) = \alpha_i + \beta_i\ Log\ (R_{mt}) + e_t$$

$$\sigma_e^2 = \frac{1}{k} \sum_{t=1}^{k} e_t^2 \qquad \text{where}$$

$$e_t = Log(R_{it}) - \alpha_i + \beta_i\ Log(R_{mt}) \qquad (3)$$

where R_{it} is the relative return for asset i for period t, R_{mt} is the return relative for the market m for period t, α is the intercept of the line, β is the slope of the line and e is the error or deviation that R_{it} is from the regression line during period t. There are k observations, or $t = 1,2, \ldots, k$.

The beta or ß is a measure of an asset's variability relative to the general market index. This is usually called the volatility of the asset. When ß = 1.0, then the asset is as risky as the market and the two assets will tend to move in tandem. If ß < 1.0, then the asset is less volatile than the market, therefore, it is less risky. If ß > 1.0, then the asset is more volatile than the market, therefore, it is more risky. Because beta measures only volatility due to the market (market risk) not total risk, the beta or ß can be used as a risk measure only if the individual assets will be going into a well diversified portfolio (usually 15-20 assets) where the non-market risk is diversified away. The beta should not be used to screen or rank assets unless the final portfolio will have more than 15-20 assets. Do not use it if the final portfolio will have less than 15 assets.

2.1.4 Lower Partial Moment or LPM (Risk Measure)

Both the beta and the variance work best when the distribution of security returns is normally distributed. Both also make assumptions as to the specific type of the investor's utility function. In order to reduce estimation errors and avoid ranking errors, the lower partial moment was developed. The lower partial moment is also known as the semivariance and also as the below-target variance. The lower partial moment traces back to MARKOWITZ (1959), who suggested that semivariance analysis be used to handle skewed security return distributions and to handle investors who do not have quadratic utility functions specific to the variance. BAWA (1975) and FISHBURN (1977) provide a proof that the semivariance is a special case of lower partial moment analysis. They derive the n-degree LPM where the semivariance is a special case (n=2) of the LPM. The variable n is the degree to which deviations below the target return are raised. In the case of the semivariance, the below-target deviations are squared. If n=3, then the below-target deviations are cubed.

The first problem encountered in measuring asset riskiness is the problem of nonnormal distributions as shown in Figure 2. Given two distributions, B and A, one positively skewed (B) and the other negatively skewed (A). Both distributions can have the same mean and variance. Hopefully, it is clear that the positively skewed distribution B is the preferred investment, however, the variance measure will not make a differentiation between the two distributions.

The n-degree lower partial moment can handle nonnormal distributions and is mathematically defined by the following equation:

$$LPM_n(h) = \frac{1}{k}\sum_{t=1}^{k} Max\,[0,(h - R_t)]^n \qquad (4)$$

where n is the degree of the lower partial moment ($n \geq 0$), h is the target return that the investor does not wish to go below, R_t is the return for a security for period t, and k is the number of periods used to calculate the LPM. Note that the above-target returns ($R_t > h$) provide negative numbers.

Given the choice of a zero or a negative number, the maximization (Max) function will select the zero. Only below-target returns ($R_t < h$) will provide a positive deviation that is raised to the n power and added into the LPM calculation. In the n-degree LPM model, n=1.0 is the boundary line between risk averse behavior and risk seeking behavior. When n>1.0, the investor is averse to risk and attempts to minimize it. When n<1.0, the investor seeks to add additional risk to a portfolio. When n=2, the semivariance results.

A Demonstration of the N-Degree Lower Partial Moment

Consider the following example of two investments, Company A and Company B. Both have the same expected return and variance (Table 1a). However, they have different skewness values and below target returns. If the target return (h) is 7.5%, the LPM can be calculated for different values of n (Table 1b).

Figure 2: Nonnormal distributions.

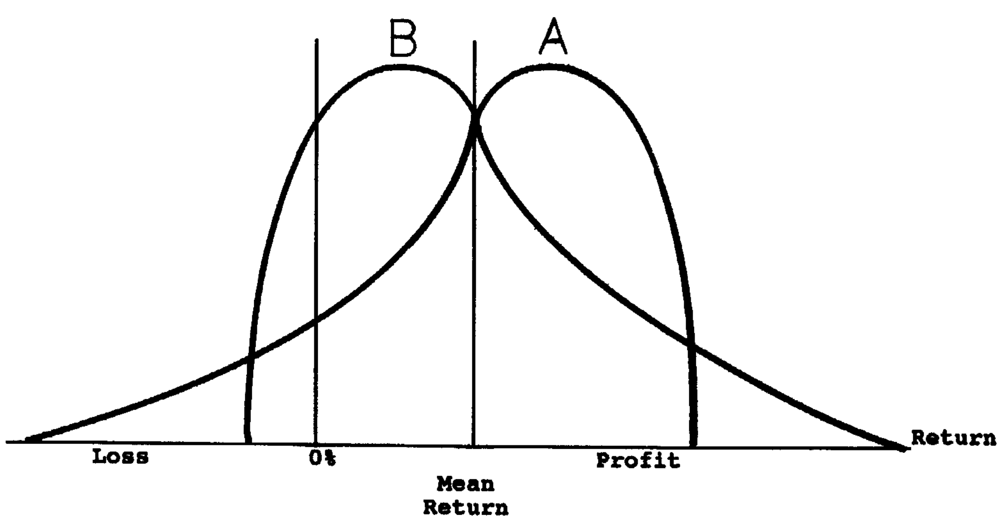


Table 1a: Returns and probabilities of the two investments.

Company A		Company B	
Return	Prob.	Return	Prob.
-2.50	0.20	5.00	0.80
7.50	0.10	17.50	0.20
10.00	0.70		

Table 1b: Mean Returns, variances, skewnesses and LPMs of the two investments.

Investment	A	B
Mean Return	7.50	7.50
Variance	25.00	25.00
Skewness	-1.51	1.50
LPM n=0.5 h=7.5	0.63	1.26
LPM n=1.5 h=7.5	6.32	3.16
LPM n=2.0 h=7.5	20.00	5.00
LPM n=3.0 h=7.5	600.00	12.50

Notice that when $n < 1$, investment A is superior to investment B, i.e. LPM_A is less than LPM_B. When $n > 1$, investment A is now considered by LPM to be riskier than investment B. As n increases, A receives a heavier utility penalty from the LPM measure. The utility choice clearly becomes more risk averse as n increases. Meanwhile, the variance measure does not differentiate between the two investments. Recent papers by HARLOW (1991), NAWROCKI (1990, 1991a), NAWROCKI/STAPLES (1989), and SORTINO (1991) provide empirical support for the use of LPM as a risk measure in asset allocation.

2.2 Adjusting Input Statistics and Scenario Analysis

Historic data for different assets provides a historic perspective on the assets' variability and correlation with other assets. Portfolio managers and security analysts have to look forward into time and formulate expectations of future returns and variability. The historic measures provide a starting point, however, the analyst may wish to adjust some or all of the inputs to reflect the result of their economic analysis. There are three types of adjustments.

2.2.1 Subjective Adjustments (Excogitation)

This is where analysts will prop their feet up on their desk and ponder the universe, specifically, forecasts of economic conditions, security analyst reports, and other qualitative factors (intuition) and determine new estimates for the asset's return, variance and possibly correlations between pairs of assets.

2.2.2 Other Estimates

The analyst may have estimates of asset return and variability from other sources (security analysts, experts, consultants, financial databases, etc.). These can be used to override the estimates obtained from a historic data set. A mixture of historic estimates and other estimates can be utilized, i.e. not all estimates for returns, variances and correlations have to be changed. Only a few critical assets are targeted with the remainder relying on historic estimates.

2.2.3 Scenario Analysis Using Econometric Relationships

Regression analysis can be used to estimate the relationship between each individual asset and some common factor like a market index. The common factor can be changed to reflect different scenarios and then the returns and variability of the individual assets can be changed using the estimated relationship. The appropriate equations are shown below.

$$E(R_i) = \alpha_i + \beta_i E(R_m)$$

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_e^2$$

$$r_{ij} = \beta_i \beta_j + \sigma_{e,ij} \quad \textit{where}$$

$$\sigma_{e,ij} = r_{ij}(\sigma_i^2 \sigma_j^2)^{0.5} - \beta_i \beta_j \sigma_m^2 \qquad (5)$$

where σ is the standard deviation, α is the intercept of a regression relationship, ß is the beta of a regression relationship, e is the error term from a regression relationship and r_{ij} is the correlation coefficient between two assets i and j. Because of these relationships, simply change the expected return, $E(R_m)$, and variance, σ_m^2 of the market index and all of the returns, variances and covariances for the individual assets will be changed. For example, there are two assets, X and Y and a market index as shown in table 2.

Now set up two scenarios for the general market. First, poor markets conditions might result in a market return and variance of 4% and 18%, respectively. A good market might have a return and variance of 22% and 20%, respectively. These two scenarios would result in the numbers shown in tables 3a and 3b.

By setting up different scenarios for the market index, the rest of the assets can have their returns and variances adjusted for the new market conditions. Portfolios can be generated for both poor and good market conditions. The portfolios can be analyzed for similarities and differences, thus providing insight as to portfolio performance under different market conditions.

Table 2: Parameters for asset X and Y.

Asset	Return	Variance	α	ß	e
X	10	18	0.4	0.8	8.4
Y	16	32	1.6	1.2	10.4
Market	12	15			

Table 3a: Poor market conditions.

	Return	Variance
X	3.6 = 0.4 + 0.8(4) and	19.92 = (0.8)2(18) + 8.4
Y	6.4 = 1.6 + 1.2(4) and	36.32 = (1.2)2(18) + 10.4

Asset	Return	Variance
X	3.6	19.92
Y	6.4	36.32
Market	4.0	18.00

Table 3b: Good market conditions.

	Return	Variance
X	18.0 = 0.4 + 0.8(22) and	21.20 = (0.8)2(20) + 8.4
Y	28.0 = 1.6 + 1.2(22) and	39.20 = (1.2)2(20) + 10.4

Asset	Return	Variance
X	18.00	21.20
Y	28.00	39.20
Market	22.00	20.00

3. Optimization Algorithms

3.1 Strategic and Tactical Optimization of Portfolio Allocations

Typically, implementations of modern portfolio theory (MPT) allocate funds between different classes of assets such as stocks, bonds, real estate, foreign currency, etc.. Indeed, most commercial computer implementations of MPT (asset allocation) utilize general asset classes. However, there is more to asset allocation theory than merely allocating funds between mutual funds that replicate the market average for each asset class. Asset allocation should also refer to the optimal portfolio derived from a set of individual investments within an asset class such as corporate equity. Therefore, we should think of the optimization of general asset allocations as strategic optimization and the optimization of individual asset allocations as tactical optimization. There is evidence to support the concept of tactical optimization. For example, HAUGEN (1990a,1990b) has shown that an equity market index with optimized allocations will outperform equity market indexes with equal weighted or value weighted allocations.

Optimization algorithms are mathematical procedures that solve multiple variable problems simultaneously. The answers are optimal given the information provided in the formulation of the problem. In portfolio theory, we are trying to allocate funds into different investments in such a way so that return is maximized and variability is minimized. The general tool used is called quadratic programming, because the mathematical equations used to formulate the problem are quadratic equations. The method usually minimizes/maximizes a quadratic objective function subject to linear constraints. The most efficient algorithms typically will only have one constraint equation, i.e. the budget constraint where all allocations will sum to 100% of the available funds. The objective function is formulated such that return (geometric mean) is maximized and risk (variability) is minimized. Since risk can be measured by different statistical measures, quadratic programming is used for more than the standard mean-variance analysis. The beta and the LPM can also be used in quadratic programming formulations to minimize the risk of the portfolio.

3.2. Markowitz Variance-Covariance Analysis

MARKOWITZ (1959) developed the basic variance-covariance analysis. The covariance is used to measure the relationship between financial assets. This allows the portfolio to use low or negative correlations between assets to reduce the overall variability or risk of the portfolio. Quadratic programming is used to minimize variability and maximize return through the following formulation:

$$Min\ z = V_p - lambda\ E_p \tag{6}$$

where E_p is the expected return and V_p is the variance for portfolio p. Lambda is the slope of the objective function. It can be varied from infinity to zero in order to solve for different points on the efficient frontier. The algorithm used in PMSPP is the critical line algorithm. It starts with the highest return portfolio which, by definition, includes the highest return asset. Each asset is then evaluated using a critical value (pivot conditions) to see which is the next asset to enter into the portfolio. As assets enter into the portfolio, it becomes more diversified and will have lower risk as well as return. Each portfolio derived by PMSPP is called a corner portfolio. A corner portfolio is when an asset either enters or exits the portfolio. The result of these corner portfolios is that they trace out the efficient frontier, where each portfolio represents the lowest risk for a given return or the highest return for a given risk (or mean-variance efficient). The efficient frontier shown in figure 3 derives its name from the fact that it is the frontier between feasible portfolios and infeasible portfolios and that every point on the frontier is mean-variance efficient. As a computational algorithm, the quadratic programming approach provides us with the tradeoff between higher return-higher risk portfolios down to lower return-lower risk portfolios.

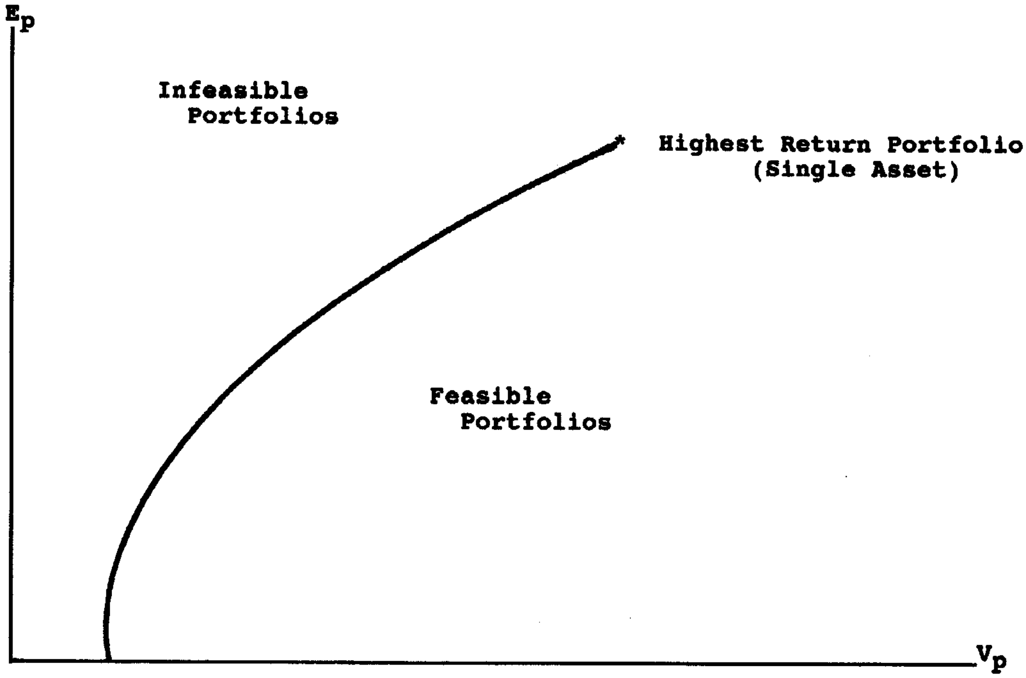
3.3 Lower Partial Moment (LPM) Analysis

LPM Analysis simply replaces the variance with the lower partial moment. The same expected return and risk equations hold true as does the quadratic programming formulation.

$$LPM_{2p} = \sum_{i=1}^{k} \sum_{j=1}^{k} X_i\ X_j\ SD_i\ SD_j\ r_{ij}$$

$$Min\ z = LPM_{2p} - lambda\ E_p \tag{7}$$

Figure 3: Efficient frontier.



Legend:
Ep expected return of portfolio p
Vp variance of portfolio p

where $LPM_{2,p}$ is the Semivariance (LPM with degree = 2) of the portfolio p, k is the number of assets, SD_i is the semideviation (square root of the semivariance) for asset i, and r_{ij} is the correlation coefficient between assets i and j [2]. This formulation defines a symmetric matrix that is easier to solve than the asymmetric matrix developed by HOGAN/WARREN (1972). Again, the critical line algorithm is used.

3.4 Elton, Gruber and Padberg Beta Analysis

The ELTON, GRUBER and PADBERG (1976) analysis provides a simple algorithm for determining optimal portfolios using the beta measure. The algorithm starts by ranking (sorting) assets by their reward-to-volatility (beta) ratio.

$$\frac{R}{\beta_i} = \frac{[E(R_i) - R_f]}{\beta_i} \qquad (8)$$

where R/β_i is the reward-to-beta ratio for asset i, R_f is the riskless rate of return (actually a base interest rate), and β_i is the beta of asset i. Assets are ranked (sorted) from highest (R/β) ratio to (R/β) lowest ratio. Critical values (pivot conditions) are computed for each asset. When the R/β_i ratio is less than the critical value (C_i), then security allocations are computed from the R/β ratios, the last critical value, C^*, the beta (β_i) and the standard error of the regression ($\sigma_{e,i}$).

$$z_i = \frac{\beta_i^2}{\sigma_{e,i}^2}\left[\frac{R}{\beta_i} - C^*\right] \qquad (9)$$

After the z_i is computed for each asset where $R/\beta > C^*$, then the allocations are computed from summing all of the positive z_i and dividing into the individual z_i. The efficient frontier is generated by varying the R_f from a high value to a low value. The higher the R_f rate, the fewer assets in the portfolio.

A diversified portfolio is generated by lowering the R_f value in order for additional assets to enter the portfolio. This algorithm is an optimal quadratic programming algorithm like the original SHARPE (1963) algorithm and is used to determine mean-variance efficient portfolios. Because of estimation error when calculating the beta, β , this algorithm will only approximate an actual mean-variance frontier computed using covariance analysis. LEVY/ SARNAT (1984) [3] found that this method will provide portfolios with higher variances and lower returns than the variance-covariance optimization method. In addition, this method will require additional assets in the portfolio in order to achieve a given level of diversification. This should not discourage a potential user since this algorithm represents a simple algorithm that has been shown to be a better forecaster of future investment performance than the covariance matrix [4].

4. Sorting (Heuristic) Algorithms

Heuristic algorithms are mathematical procedures that solve problems for solutions that are approximately optimal. They typically are sequential processes based on sorting processes that use less information, are computationally less complex, are faster, and are less subject to statistical biases [5]. Because they are simpler models, they tend to be better forecasters than more complex models [6]. PMSPP contains four heuristic programming algorithms: Elton, Gruber, and Padberg fixed correlation model, reward-to-variance, reward-to-LPM, and reward-to-beta.

4.1 Elton, Gruber, and Padberg Fixed Correlation

ELTON, GRUBER and PADBERG (1976) developed a simple algorithm utilizing the reward-to-variability ratio along with their single index model utilizing the reward-to-volatility (beta) ratio. They computed the reward-to-variability ratio for each individual asset and ranked (sorted) the assets from highest ratio to lowest ratio.

$$\frac{R}{V_i} = \frac{[E(R_i) - R_f]}{\sigma_i} \tag{10}$$

Next, a critical value (C_i) is computed for each individual asset using the average correlation coefficient for the group of financial assets. Again, the allocations are computed using all assets where $R/V_i > C^*$.

$$z_i = [\frac{R}{V_i} - C^*] \quad for\ all \quad \frac{R_i}{V_i} \rangle C^* \tag{11}$$

4.2 Nawrocki Return-to-Risk Ratio Heuristics

The key concept is that sorting reward-to-risk ratios can provide near optimal or heuristic portfolios that are close to being mean-variance efficient. The Elton, Gruber and Padberg heuristic algorithm suggests the following algorithm developed by NAWROCKI (1983):

$$z_i = \frac{[E(R_i) - R_f]}{Risk\ Measure_i} \tag{12}$$

Portfolios can have weighted allocations (as above) based on z_i, or each asset with a positive z_i can be given an equal weight (allocation). The Risk Measure for each asset i can be the variance, beta, or the LPM.

4.3 Asset Screening to Reduce Matrix Invertibility Errors

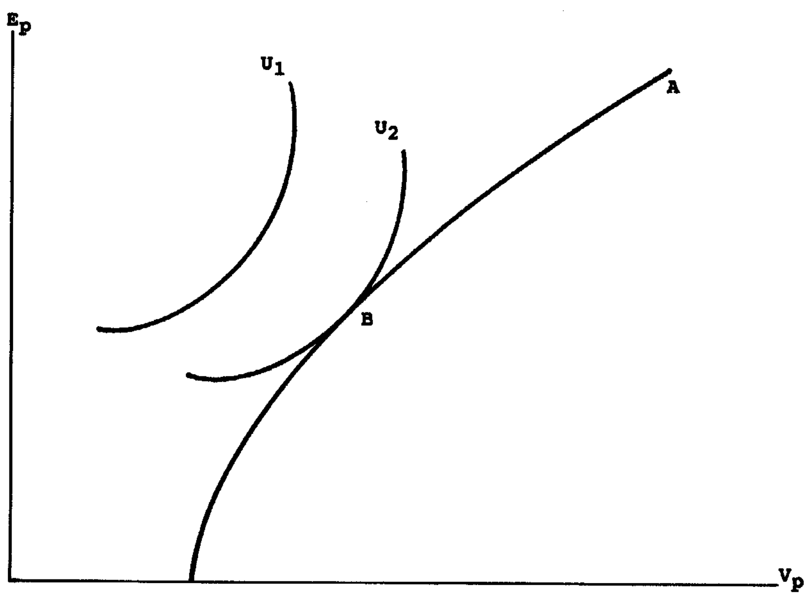
The covariance matrix is susceptible to statistical errors resulting from rank of matrix or invertibility of matrix errors. Simply stated, there should be more observations used to compute the statistical inputs (means, variances, and covariances) than there are assets in the covariance matrix. This is analogous to the positive degrees of freedom requirement taught in basic multiple regression analysis, i.e. the number of observations have to exceed the number of independent variables. With the 48-60 months of data typically used to compute betas or covariances, the number of assets in the covariance matrix should not exceed 25-30 assets. Even with 120-180 observations, the covariance matrix should not exceed 50-60 assets. It is for this reason that the PMSPP program is limited to a maximum of 50 assets in the covariance matrix optimization section.

Because the reward-to-risk ratios have proven to be important alternative (albeit heuristic) algorithms to the covariance optimization algorithm, they are valuable when encountering the invertibility of the matrix problem. Assets can be sorted from highest reward-to-risk ratio to the lowest reward-to-risk ratio. The 25-50 assets with highest ratios can then be submitted to the optimal covariance matrix algorithm. This should reduce some of the invertibility error that results from a larger covariance matrix. The PMSPP package will sort up to 125 assets using the reward-to-variance, reward-to-LPM, and reward-to-beta ratios. The top 25-50 assets then can be optimized by the variance-covariance, EGP Beta, or LPM optimization algorithms.

5. Picking the Best Portfolio

Up to this point, we have been concerned with deriving either an optimal efficient frontier or an approximate (heuristic) efficient frontier. The efficient frontier simply provides the tradeoff between higher risk-higher return portfolios and lower risk-lower return portfolios. Economic utility theory is employed to pick the best portfolio from the frontier. Investors, after much soul searching, have to determine the best tradeoff in risk/return that will maximize their satisfaction (utility) from the portfolio.

Figure 4: Utility Curves.



Legend:

U1, U2	utility curves
A	efficient frontier
Ep	expected return
Vp	variance

5.1 Utility Analysis

The classic utility analysis from economics would have us deriving a utility curve and using it to pick the best portfolio [7].
This approach assumes that the investor can derive a utility curve that fits the individual investor. This has proven to be difficult in practice. Fortunately, the quadratic utility measure, which is inherent in variance-covariance analysis, can be approximated through a couple of simple techniques. The benefit of these approaches is that they are easy for investors to understand and to conceptualize.

5.1.1 Utility Using Roy Safety First Techniques

Safety first techniques were first developed by ROY (1952). They require that the investor set a rate of return that the investor prefers not to go below. Investors would prefer safety first and would prefer to avoid disaster. The rate of return set by the investor is called the disaster level. Investors would prefer to minimize the probability of a return less than the disaster level. Assuming normality, the probability of going below the disaster level is:

$$Z = \frac{d - E(R_p)}{\sigma_p} \quad or$$

$$\frac{R}{V}\ Ratio = \frac{E(R_p - d)}{\sigma_p} \tag{13}$$

where d is the disaster level return set by the investor and Z is the standard normal deviation that can converted to a probability through the use of normal Z-value probability tables. The disaster level can be set to zero return in order to minimize the probability of a loss, or it can be set to the riskless rate of return, R_f, or it can be set to some other target return set by the investor. The best portfolio will have the highest Z or R/V ratio value.

5.1.2 Utility Using Risk Tolerance

The investor can also set a level of risk tolerance in order to pick the optimal portfolio [8]. If you divide investment into two components: a risky investment and a riskless investment, the level of risk tolerance can be defined as the amount of the portfolio to be placed in the risky investment. If the risk tolerance is 80, then 80% of the portfolio's assets should be in the risky component and 20% in the riskless component. A lower risk tolerance of 30 would place 30% in risky assets and 70% in riskless assets. Given the investor's risk tolerance, the utility is computed as below. The portfolio with the highest utility is the best portfolio.

$$Utility = Port.\ Return - \frac{Port.\ Variance}{Risk\ Tolerance}$$

5.2 Adjusting the Risk Profile of the Portfolio

Once the optimal portfolio has been picked, the risk profile of the portfolio can be adjusted (increased risk or decreased risk) using two techniques.

5.2.1 Adjusting Risk Level Using Margin and Riskless Assets

The risk of a portfolio can be reduced by reducing the allocation in the risky portion of the portfolio and increasing the allocation in the riskless asset. This is the same as reducing the risk tolerance of the portfolio. Increasing the risk of the portfolio can be accomplished by increasing the allocation in the risky component. After 100% in the risky component has been reached, the risk can continue to be increased through the use of margin or borrowing. Funds can borrowed and used to invest in the risky component. In the USA, borrowing can be accomplished up to a maximum of 50% of the portfolio (also known as the margin requirement). Margined portfolios will have greater returns than a unmargined portfolio. However, they will also have greater risk.

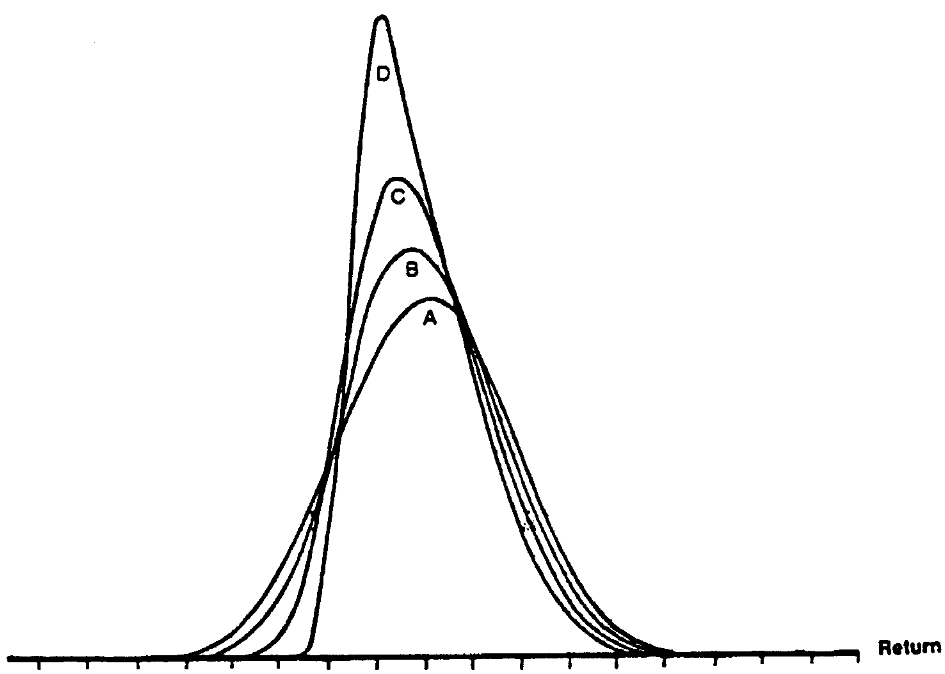
5.2.2 Adjusting Risk Level Using Options (Puts and Calls)

Options to buy assets (Call options) and to sell assets (Put options) can also be used to affect the risk level of the portfolio [9]. Put options can be purchased in order to reduce the risk of the portfolio. The effect of purchasing put options is shown in figure 5.
The distribution for portfolio A represents a normal symmetric distribution. Portfolio B represents the purchase of puts equivalent to 25% of the portfolio. Portfolio C represents the purchase of puts equivalent to 50% of the portfolio. Portfolio D represents the purchase of puts equivalent to 75% of the portfolio. As the number of puts is increased, the variance of the portfolio is reduced and the skewness of the portfolio is increased. Both factors represent reduced risk to the investor. This effect on the portfolio distribution can also be obtained by short positions in future contracts. The risk level of the portfolio can be increased (increased variance and decreased skewness) by buying call options or by taking long positions in future contracts. Put and Call options can be used to increase the margin positions beyond the USA's 50% margin requirement to 10-15% margin positions. (This strategy should work for both American and European options.)

5.3 Extrapolating the Investment Horizon

It was noted earlier that the geometric mean provides the best measure of an investment's economic return. As the investor holds a portfolio for longer periods of time (increasing the investment horizon), liquidity risk becomes less of a factor. Under

Figure 5: Distribution of portfolio returns with and without options.



Legend:
A portfolio without options
B purchase of puts equivalent to 25% of the portfolio.
C purchase of puts equivalent to 50% of the portfolio.
D purchase of puts equivalent to 75% of the portfolio.

these conditions, the variability of return becomes less important. The result is that an investor will be more concerned with maximizing return or maximizing the geometric mean return. While an investor with a short term investment horizon is interested in a tradeoff between risk and return, a longer term investor simply wants to maximize return. The mean and standard deviation reported by the portfolio analysis is a one period horizon. It is not difficult to extrapolate this one period performance to a longer investment horizon. Then we can see which portfolio provides the best long term performance. The long term simulation is computed as follows:

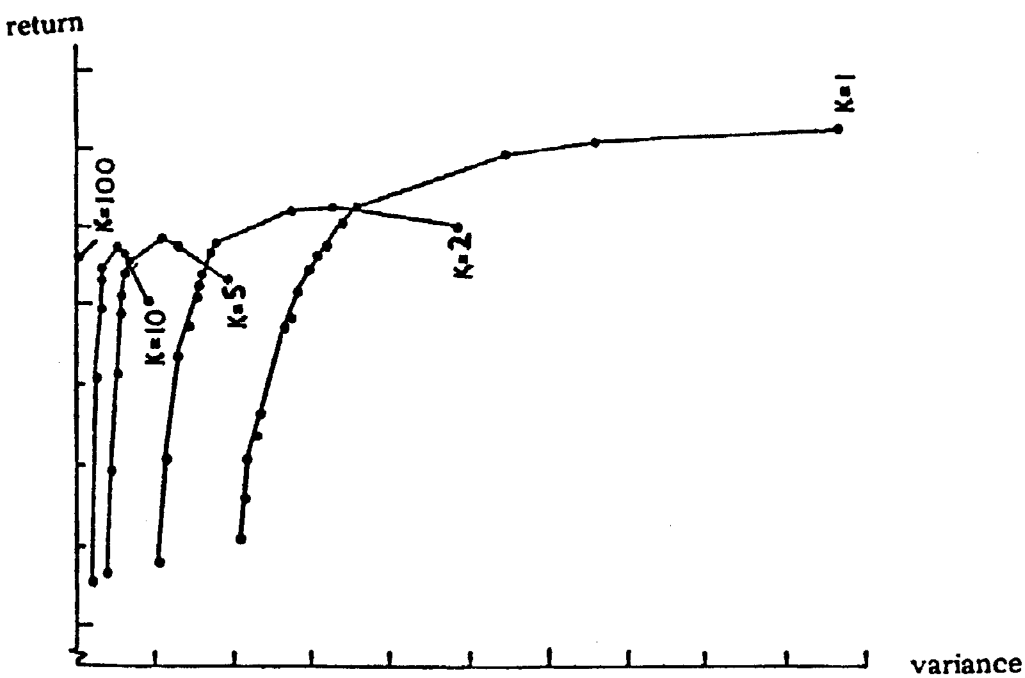
$$U_k = \frac{U_1}{(1.0 + \sigma_1^2/U_1^2)^{1/k - 1/2k}}$$

$$\sigma_k^2 = U_k^2 \left[(1.0 + \sigma_1^2/U_1^2)^{1/k} - 1.0\right] \qquad (14)$$

where U_1 is the expected geometric mean return for a one period (k=1) investment horizon, σ_1 is the one period standard deviation/-variance and k is the number of periods in the actual investment horizon [10]. As k increases, the standard deviation, σ, decreases. Therefore, the probability of the portfolio being below some benchmark return will decrease. The best portfolio will have the highest expected return, U, and the lowest probabilities for a given investment horizon, k. The effect of this simulation on the one period efficient frontier as k increases is shown in figure 6.

While this analysis is useful, it has an important problem. It assumes that the distribution of portfolio returns derived for a one period horizon will remain stable throughout the longer holding period. In other words, the mean return, U_1, and the standard deviation, σ_1, will retain the same values during the longer investment horizon holding pe-

Figure 6: Efficient frontiers for different investment horizons.



Legend:
k investment horizon

riod. If the distribution changes, then the portfolio may have to be changed (revised).

6. Portfolio Evaluation

After a portfolio has been selected, its performance has to be monitored and compared to performance benchmarks. As long as the performance is within a target range of performance, it is not necessary to revise the portfolio. If the performance is deteriorating, then a change may be necessary. In order to monitor the performance of a portfolio, performance measures that account for both risk and return have to be computed.

6.1 Terminal Wealth

The terminal wealth measure answers the question, "How much money did I make?". It is a ratio that shows how much money is in the investor's portfolio for each one dollar of initial investment. A ratio value of 1.25 tells us that for every $1.00 initially invested in the portfolio, the portfolio ended with a balance of $1.25. The terminal wealth ratio is computed as part of the geometric mean calculation.

$$\textit{Geometric Mean} = \left(\prod_{t=1}^{k} R_t\right)^{1/k},$$

$$\textit{Terminal Wealth} = \prod_{t=1}^{k} R_t \qquad (15)$$

where k is the number of periods. The geometric mean is the k th root of the terminal wealth which is computed as the product of the individual return relatives. With a long term investment horizon, the terminal wealth is the only performance measure that is relevant when evaluating portfolio performance.

6.2 Sharpe's Utility Measure

The first of the risk-return measures is the Sharpe utility measure. This measure does not compute a return-to-risk ratio as do the others. Additionally, it uses an estimate of the investor's risk tolerance instead of the riskless rate of return as an indicator of the investor's utility function. The higher the investor's risk tolerance (ranging from 0 to 1), the higher the proportion of the portfolio invested in risky assets [11].

6.3 Sharpe, Treynor and Jensen Measures

The Sharpe and Treynor measures utilize the reward-to-risk ratio. The Sharpe measure employs the standard deviation as the risk measure and the Treynor measure utilizes the beta as the risk measure. Both measures use the riskless rate of return, R_f, as a safety first return.

$$\textit{Sharpe} = \frac{(R_p - R_f)}{\sigma_p},$$

$$\textit{Treynor} = \frac{(R_i - R_f)}{\beta_p} \qquad (16)$$

The Jensen measure utilizes the CAPM market line equilibrium to measure investment performance. It is computed from the following regression:

$$(R_p - R_f) = \alpha_p + \beta_p (R_m - R_f) + e_t \qquad (17)$$

When the portfolio performance is in equilibrium with the market's risk-return performance, then the intercept of the regression line, α_p, is equal to zero ($\alpha_p = 0$). When the portfolio outperforms the market, $\alpha_p > 0$, and when the portfolio is underperforming the market, $\alpha_p < 0$ [12].
There are a number of studies that conclude that these measures are statistically biased. The most complete study was by ANGH/CHUA (1979). The bias is sufficient that there is a great deal of doubt about empirical studies using these measures [13].

The effect of the bias is that each of the measures may rank the performance of a group of portfolios differently from the other measures. The portfolio manager is then in doubt whether the portfolio really did outperform the benchmark portfolios that have been chosen.

6.4 Reward-to-Semivariance

Because of the problems with the above measures, the reward-to-semivariance ratio was proposed as an alternative [14]. ANGH/CHUA (1979) report that the reward-to-semivariance is statistically unbiased. In addition, research that derives the CAPM using semivariance or lower partial moment provides the necessary economic interpretation of the ratio [15]. This ratio should be the preferred return-to-risk ratio for evaluating portfolio performance.

$$\textbf{\textit{Reward-to-Semivariance}} = \frac{(R_p - R_f)}{SD_p} \qquad (18)$$

where SD_p is the semideviation of the portfolio. The semideviation being the square root of the semivariance. The drawback to the reward-to-semivariance measure is that it assumes a specific investor utility function (n=2). In this case, the more general reward-to-LPM ratio should be used since the degree of the LPM measure, n, can be altered to match the investor's utility function.

7. Portfolio Revision

Portfolio revision is necessitated by changing information in the environment and, therefore, by changing investor expectations in the market. As expectations change, the probability distributions (underlying the risk and return estimates on which the portfolio is based) will change. These changes should show up in the portfolio evaluation process. The performance of the portfolio will deviate from the expected performance.

These conditions will necessitate changes in the portfolio's allocations. New assets will enter the portfolio. Other assets will leave. The allocations will now be derived from new expectations. Revision constantly evaluates portfolio performance and makes the necessary changes.

Portfolio revision is also the answer to the long run investment horizon problem (See Extrapolating the Investment Horizon). Even though changing expectations reduces the relevance of the results of the long run simulation analysis, the analysis can be redone in order to find a new portfolio in which to invest. Revision is a dynamic process that involves the continuous evaluation and revision of portfolios and portfolio analysis techniques.

Commission costs and market efficiency are factors in portfolio revision. There are costs associated with revising a portfolio. Commission costs result from the trades that bring in new assets and sell off old undesired assets. Market efficiency refers to the competitiveness of the market information process. If the market's information process is very efficient, continuous trading based on changing expectations will not improve portfolio performance. Any improvement will likely go only to the broker making the trades. Less information efficient markets will reward trading based on changing expectations [16]. In an efficient market, short run trading will usually only make the broker rich. However, even in an efficient market, long term adjustments in the portfolio's allocations may become necessary. SMITH (1968) provides four categories of portfolio revision strategies.

Buy-and-Hold - This strategy is based on an information efficient market where there are no benefits to continuous trading. As a result, simply buying the portfolio and holding it for the long run will provide the best results.

Rebalance - This strategy assumes that the original analysis deriving the portfolio's allocations remains correct. As time elapses, some assets go up in value increasing their allocation within the portfolio. Other assets go down in value which decreases their allocation. After a while, the portfolio does not resemble the original portfolio. Periodically, the rebalance strategy rebalances the portfolio back to the original allocations by selling the assets that have gone up in value and buying the assets that dropped in value. In the long run, this strategy becomes a dollar averaging strategy where the average sale prices will exceed the average purchase prices.

Complete Transition - This strategy assumes an inefficient information market and will make trades every time expectations change. Because of poor dissemination of information, revision strategies based on quick updates can be profitable.

Controlled Transition - This strategy is more middle-of-the-road. Transactions are made only if the expected benefit of the revision is greater than the transaction costs generated by the revision.

8. Summary

This paper has covered a number of statistical and mathematical techniques available from modern portfolio theory (MPT). These techniques have been integrated into a systematic approach to dynamically manage an investment portfolio. In addition, I hope that the reader will note that both portfolio theory and the PMSPP computer program presented in this paper have resulted from the work of a large number of researchers. I have tried to provide them with the appropriate credit and would like to acknowledge my debt to them.

Footnotes

[1] NAWROCKI (1991b).
[2] NAWROCKI (1991a).
[3] LEVY/SARNAT (1984), pp. 370-376.
[4] ELTON/GRUBER/URICH (1978).
[5] SHARPE (1967).
[6] ELTON/GRUBER/URICH (1978).
[7] LEVY/SARNAT (1984).
[8] SHARPE/ALEXANDER (1990).
[9] BOOKSTABER (1985).
[10] GRESSIS/HAYYA/PHILIPPATOS (1974).
[11] SHARPE/ALEXANDER (1990).
[12] LEVY/SARNAT (1984), pp. 521-533.
[13] SAUNDERS/WARD/WOODWARD (1980).
[14] KLEMKOSKY (1973).
[15] HOGAN/WARREN (1974) and NANTELL/PRICE (1979).
[16] GROSSMAN/STIGLITZ (1980).

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