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Optimal Currency Hedging and International Asset Allocation: An Integration

1. Introduction

The benefits of international diversification are well documented and the modern approach to allocating assets internationally has been described by the author in an earlier paper in this journal [1].

The management of the currency risk attendant on international equity portfolios is the subject of this paper.

A number of issues relating to currency risk management policy arise in the context of the optimal allocation of assets across countries. Firstly, at which stage of the process should hedging be assumed to occur? Some managers assume all assets to be hedged prior to the determination of the asset allocation and therefore subject only hedged assets to the optimisation routines. Other managers impose the hedge on the assets of an optimal portfolio determined on a set of unhedged assets. Secondly, how much of the currency risk should be hedged? Some argue for full hedging others argue for minimum variance hedging. Thirdly, in the case of minimum variance hedging should the hedge ratio be determined on a single currency or on a multiple currency basis?

Finally how should currency management policy be coordinated with equity management policy? Two aspects of equity management that may potentially affect currency management are (1) the degree to which a manager is active and (2) the manager's (or client's) degree of risk aversion [2]. This paper attempts to answer these questions and in so doing provide the practising portfolio manager with some guidelines to assist in formulating a coherent set of currency management policies.

The approach taken here is to analyse extensively the empirical behaviour of the world's major stock and currency markets in the decade January 1980 to December 1989, in order to illustrate the general principles of risk management.

The body of the paper comprises six sections. The next section presents a decomposition analysis of stock market risk and return into equity and currency components and their interrelationship. The third section presents a decomposition analysis of the various currency markets' risk and return characteristics. Section 4 analyses the impact of fully hedging on the risk return characteristics of international equities and develops a framework within which to formulate currency hedging policy. This is followed by a section describing minimum variance hedging and the penultimate section presents a method for determining the optimal allocation of assets across equity markets and the simultaneous determination of optimal multi-currency hedge ratios. The method is illustrated with extensive empirical analysis of various hedging strategies.

The paper concludes with a brief summary which includes some portfolio policy recommendations in the light of the empirical and theoretical analyses presented.

* I thank Walter Wasserfallen for useful comments.

Throughout the numeraire currency is the Swiss franc. The term local currency refers to non-Swiss currency in the context of this paper. A technical appendix to this article formally describes the methods and statistics referred to in the main body of the text.

2. Decomposition of Risk and Return

Consider an investment in the Japanese equity market by a Swiss-based investor. Figure 1 reports the behaviour of the monthly returns on the Tokyo Stock Exchange in Swiss francs over the period January 1980 to December 1989.

During this period, the annualised average monthly return was 27.89% [3]. It is apparent from figure 1 that the returns varied considerably around this average on a monthly basis. For example the return in September 1981 was -16.88% [4] while in March 1986 the return was a staggering 26.15%. The average within this range was 2.32% per month.

The fact that the market has historically fluctuated through such large amplitudes reduces our confidence in making ex-ante estimates. Ex-post, or after the event, we are able to observe that approximately 66% of the monthly returns on the Tokyo stock market were between -3.53% and + 8.17%. Most of the monthly returns, therefore, had a value within 5.85% of the average return. In statistical terms this measure is called the standard deviation of return which is 20.28% on an annualised basis [5]. It is obvious that after the event all that matters is the investment yield. In considering future investments, however, it is necessary to estimate the expected return and to judge the range around this estimate within which the actual future return is likely to fall. A knowledge of the ex-post volatility of returns is, therefore, an important starting point in assessing the ex-ante risks inherent in an particular investment.

In order to measure the effect of currency fluctuations, the Swiss franc returns are decomposed into an equity element and a currency element. Figures 2 and 3 show the monthly return behaviour of the Tokyo Stock Exchange in Yen and the Yen market in Swiss francs respectively.

Figure 1: Japanese Equity Market Returns in Swiss franc.

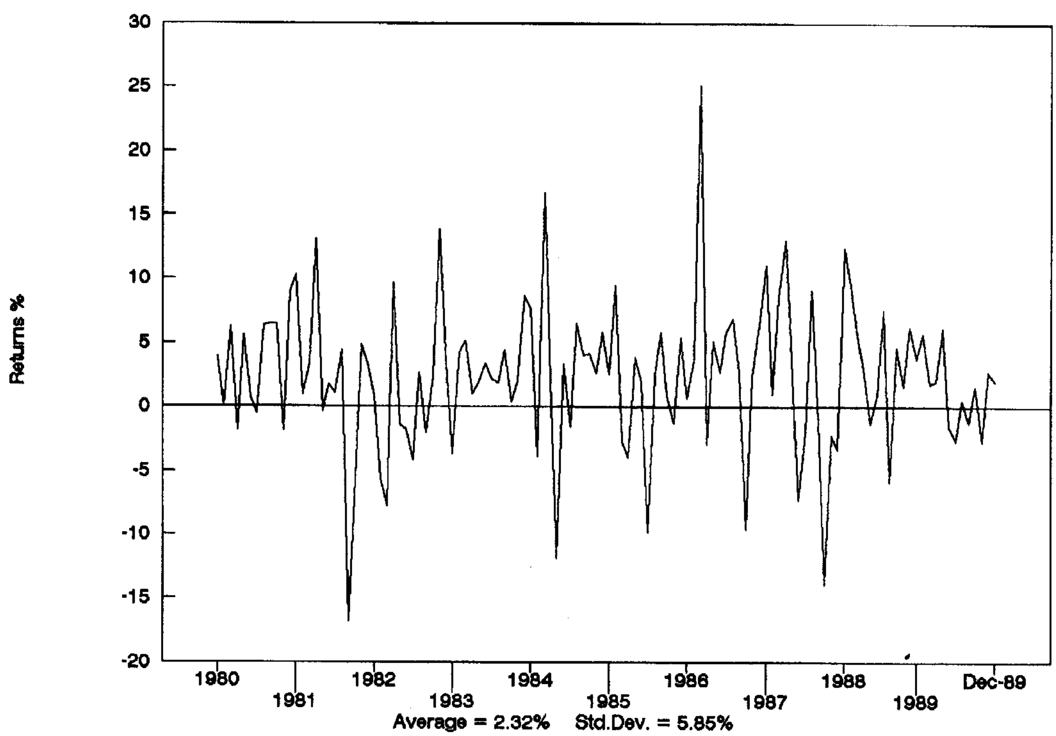


Figure 2: Japanese Equity Market Returns in Yen.

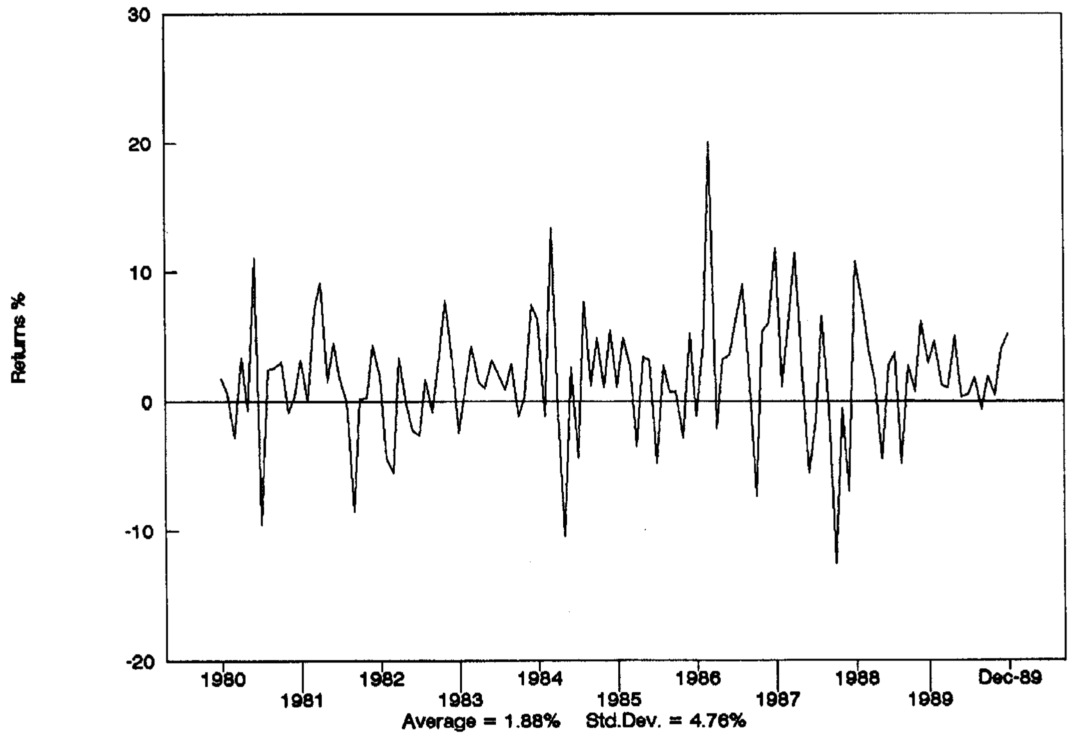


Figure 3: Returns on Yen in Swiss franc.

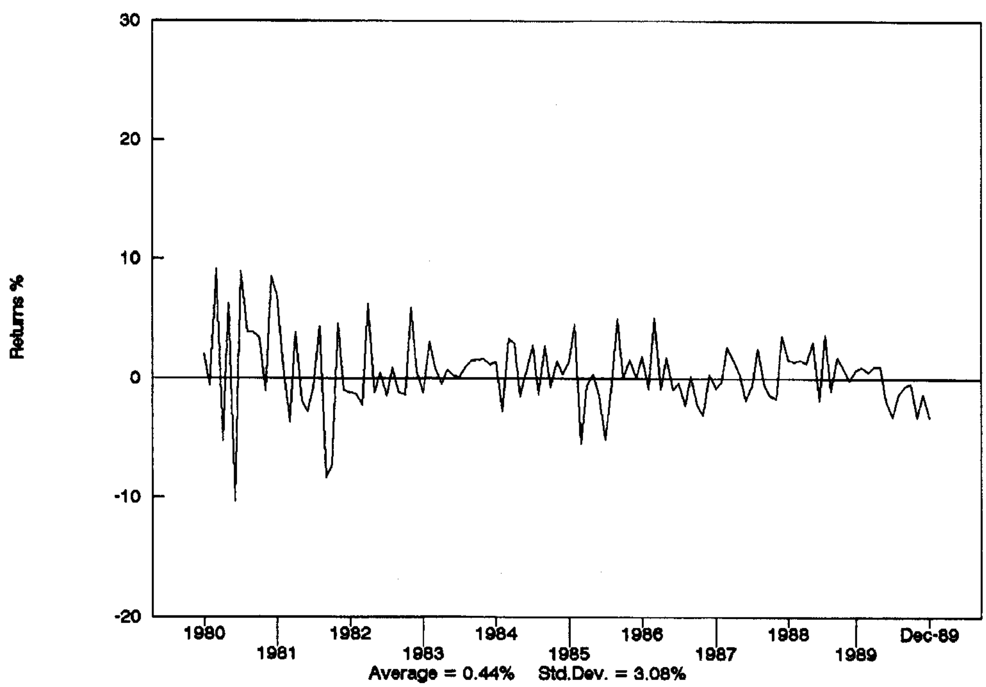


Table 1: Decomposition of Return in Major Equity Markets in Swiss Franc Terms.

Equity Market	Equity Returns Local Currency (R)	Return to Risk Ratio Local Currency	Currency Returns v Swiss Franc (e)	Equity Returns Swiss Francs	Return to Risk Ratio Swiss Franc
France	23.03	1.07	-3.73	19.30	0.83
Germany	15.62	0.81	1.10	16.72	0.82
Japan	22.61	1.37	5.28	27.89	1.37
Switzerland	12.11	0.77	0.00	12.11	0.77
UK	23.27	1.19	-2.84	20.43	0.89
US	17.24	1.06	0.81	18.05	0.84

Note:
All statistics are estimated over the period January 1980 to December 1989 using monthly data.
All return statistics are annualised averages.

Table 2: Decomposition of Risk in Major Equity Markets in Swiss Franc Terms.

Equity Market	Equity Returns Local Currency Standard deviation	Currency Returns v Swiss Francs Standard deviation	Correlation R & e	Equity Returns Swiss Francs Standard Deviation	Residual Currency Risk
France	21.62	6.28	0.10	23.13	24%
Germany	19.32	5.92	0.05	20.51	20%
Japan	16.50	10.70	0.07	20.28	35%
Switzerland	15.75	0.00	0.00	15.75	n.a.
UK	19.60	10.99	0.04	22.85	30%
US	16.26	13.49	0.04	21.54	39%

Note:
All statistics are estimated over the period January 1980 to December 1989 using monthly data.
All risk statistics are annualised averages.

Tables 1 and 2 report the average performance of each of the six stock markets over the ten year period under study.

In Swiss franc terms Japan out-performed all other markets in both return and in return to risk. Switzerland exhibited the lowest variability but also the lowest return to risk ratio. The other four markets were closely matched on the return to risk ratio with the UK slightly ahead of the others.

An examination of the decomposed performance, reported in tables 1 and 2, on return to risk ratio reveals that in local currency terms the rankings were identical. Japan exhibits exactly the same return to risk ratio in Yen and Swiss franc. Curiously the increased volatility introduced via the currency markets was compensated for at the same rate in return as for the Japanese equity market (in Yen terms). This occurred despite the fact that the return to risk ratio was almost three times larger in the Japanese equity market (in Yen terms) than in the Yen/Swiss franc currency market. This curiosity highlights an important feature of international investing.

It will be noticed that the return in Swiss franc terms is a simple addition of local equity market return and the local currency/Swiss franc market return [6]. However only a portion of the currency risk increments the local equity market risk. So although the currency markets exhibit considerable volatility the marginal impact of currency volatility on portfolio risk for a foreign investor in a particular stock market is relatively small. The proportion of currency risk to which the Swiss investor was exposed in the various markets is reported in the last column of table 2 headed "Residual Currency Risk".

The reason "Residual Currency Risk" is so small is that a large proportion of currency variability is naturally diversified due to a lack of pairwise correlation between each equity market (in local currency terms) and each local currency/Swiss franc market. This correlation is reported in column 3 of table 2. "Residual Currency Risk" is defined as equity risk in Swiss franc less equity risk in local currency divided by currency risk.

The relationship between the variabilities of the various return elements reported in table 2 is described by equation (9) in the appendix. There the effect of the correlation between currency markets and local equity markets can been seen directly.

This phenomenon of natural diversification in currency markets is a crucial element of the determination of optimal hedge ratios and the correlation statistic alluded to here will be again encountered in section 5 below.

Without this phenomenon, unhedged international investing would have been extremely expensive for the Swiss investor. Firstly, the returns on the currency markets (except for Japan) were at best modest, at worst negative and secondly an increase in risk due to currency exposure was experienced. As will be shown with further analysis the diversification benefits of international investment far outweigh these costs since the magnitude of the impact of currency risk is low and can be relatively inexpensively removed with hedging.

Section 4 analyses the properties of the hedged returns of these markets, however before proceeding to this aspect of international investing the next section reports an analysis of the properties of the currency markets returns.

3. Decomposition of Currency Risk and Return

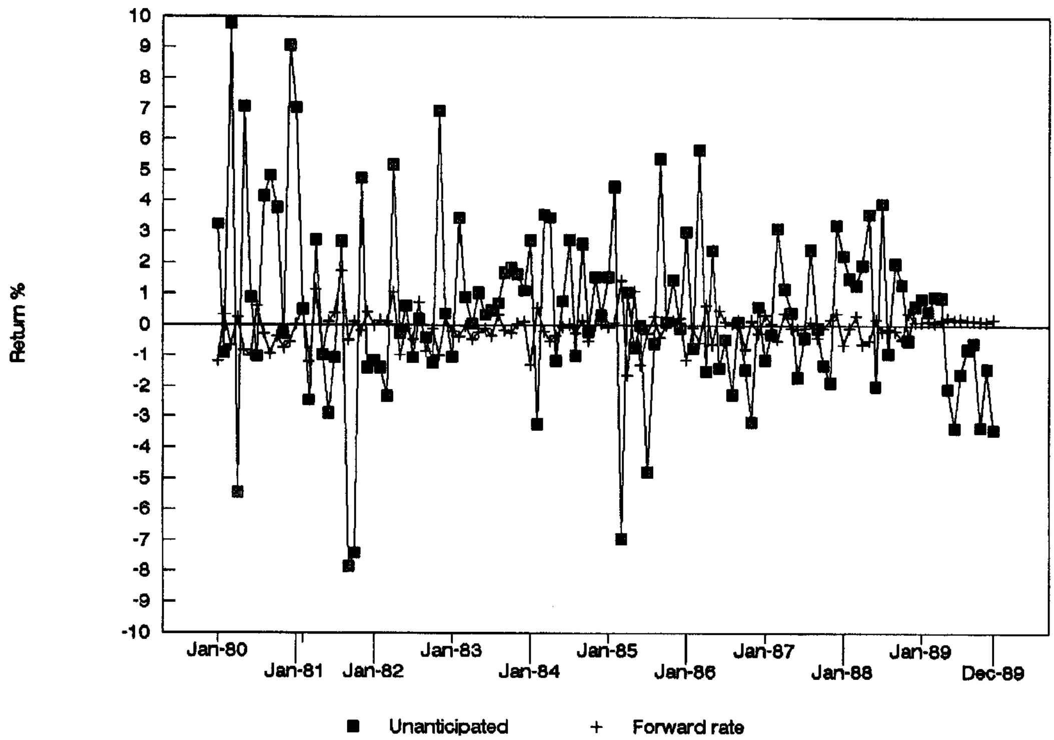
An important component of the world foreign exchange market is the forward market, where currencies can be traded for future delivery [7]. This market together with the more recently developed currency futures and options markets enable the international investor to hedge the currency exposure attendant on offshore investing [8]. Since the price behaviour of these contracts determines the risk and return characteristics of hedged equity returns, it is useful to decompose these currency fluctuations facing our Swiss investor, over the last decade, into a forward rate and an unanticipated rate.

Figure 4 shows the evolution of these two rates over the last decade for the Japanese Yen in Swiss franc terms. Clearly the unanticipated component is the more volatile although the forward rate does vary. This variation in the forward rate cannot be hedged and is known as basis risk. In this study the forward rate is computed with direct reference to euro-currency deposit interest rate differentials. The formula used is described as equation (13) in the appendix.

Tables 3 and 4 report the mean and standard deviation for each element of currency return. The last column in table 4 reports the proportion of currency volatility contributed by the unanticipated element. As suggested in figure 4 this element accounts for virtually all the currency volatility. Again notice that the standard deviations of the two elements are not additive and the exact relationship is described in equation (14) of the appendix. These volatility results are tautological in the sense that unexpected changes in exchange rate are what cause exchange rate fluctuations.

The returns reported on table 3 are somewhat disquieting for forward rate parity protagonists. On average, over the decade of the eighties, all currencies reported were at a discount to the Swiss franc [9], but the unexpected currency returns were all positive and most of them significantly so. Under forward

Figure 4: Japanese Yen in Swiss franc.



rate parity one would expect the mean of the unexpected element over such a long period to be close to zero. It appears that the forward rate may not be any better a predictor of future spot rates than the current spot rate [10].

If an individual Swiss investor had developed a personal expectation of the evolution of the currency markets that diverged from the forward rate to the extent that actual rates did, over the eighties, as represented in table 3, what should his or her hedging policy have been? Firstly because the unanticipated element was positive our Swiss investor would have been bullish foreign currencies versus the Swiss franc and therefore would have faced a positive trade off between risk and return when considering hedging policy. The lower the hedge ratio the higher the return and the risk. How much exposure our investor should take depends mainly on his or her taste for risk, or degree of risk aversion. Clearly the most important determinant of hedging policy is the investor's beliefs regarding the process generating exchange rates. In the next section the properties of hedged returns are analysed and the linkages between the investor's degree of risk aversion and his/her beliefs about the process generating exchange rates are examined in a return-risk (mean-variance) context.

4. Full Currency Hedging in a Mean-Variance Context

The annual returns on the Tokyo Stock Exchange over the decade of the eighties are reported in figure 5 for three classes of investors viz. (1) the local Japanese investor (2) the international Swiss investor with currency risk fully hedged (i.e. hedge ratio = -1) and (3) the unhedged international Swiss investor. It is clearly shown that the hedged return in Swiss francs exhibits the characteristics of the local equity portfolio. The Tokyo Stock Market returns are less volatile in Yen than Swiss franc as demonstrated in

Table 3: Decomposition of Return in Major Equity Markets in Swiss Franc Terms.

Currency Market	Currency Returns Total	Monthly Forward Rate	Currency Returns Unanticipated in Forward Rates	Proportion of Currency Return Unanticipated
France	-3.73	-7.27	3.54	-0.95
Germany	1.10	-0.47	1.57	1.43
Japan	5.28	-1.26	6.54	1.24
Switzerland	n.a.	n.a.	n.a.	n.a.
UK	-2.84	-6.70	3.86	-1.36
US	0.81	-5.01	5.82	7.21

Note:
All statistics are estimated over the period January 1980 to December 1989 using monthly data.
All return statistics are annualised averages.

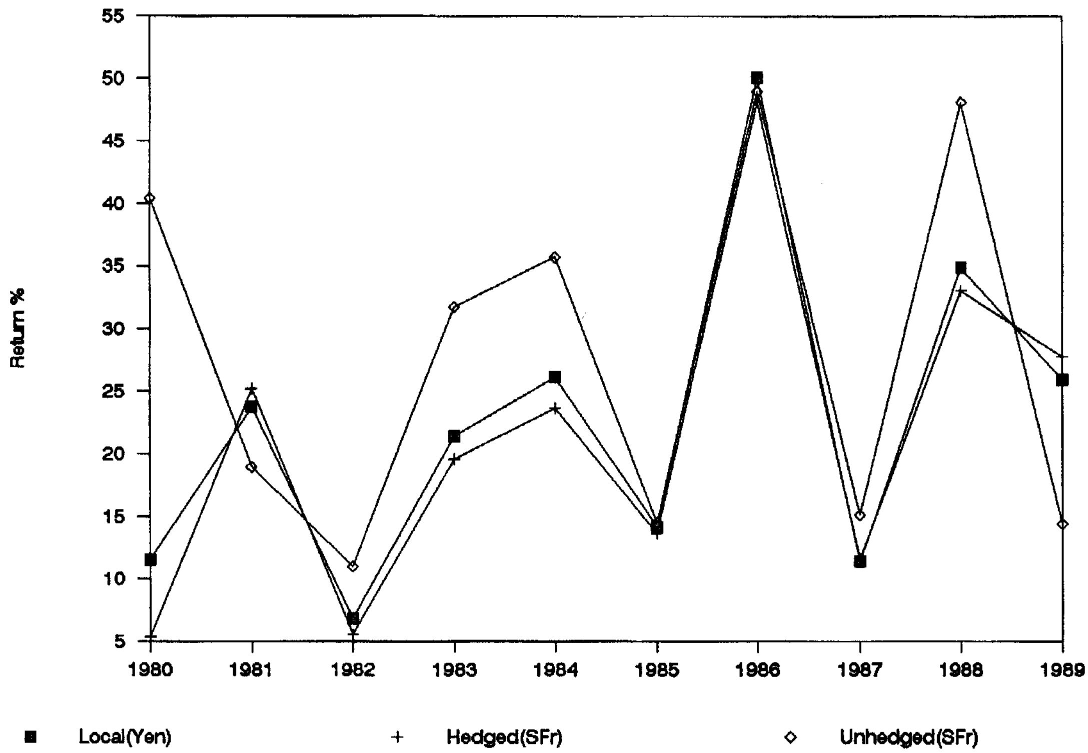
Table 4: Decomposition of Risk in Major Equity Markets in Swiss Franc Terms.

Currency Market	Currency Returns Total Standard Deviation*	Monthly Forward Rate Standard Deviation	Currency Returns Unanticipated Standard Deviation	Proportion of Currency Risk Unanticipated
France	6.28	2.03	6.13	0.98
Germany	5.92	3.32	4.99	0.84
Japan	10.70	5.10	10.12	0.95
Switzerland	n.a.	n.a.	n.a.	n.a.
UK	10.99	3.92	10.58	0.96
US	13.49	2.30	13.82	1.02

Note:
All statistics are estimated over the period January 1980 to December 1989 using monthly data.
All risk statistics are annualised averages.
* Covariance component not reported.

section 2 above. It is shown in figure 5 that hedging goes some way to removing the effect of residual currency risk and consequently the return on the hedged Swiss franc portfolio displays similar characteristics to the local currency returns. This is of course the purpose of hedging, to transform the unhedged return into an investment return which exhibits the same volatility characteristics of the investment available to local investors. However this risk reduction is often costly in return resulting in a positive trade off between the two. After the event it is easy to measure the cost of hedging and examples are reported in table 5. The cost of hedging, in terms of return, is reported as the difference between the hedged return and the unhedged return (see the last column of table 5). This magnitude exactly equals the difference between the currency return and the forward rate as reported in table 3.
If forward rate parity holds (see equation (18)) the expected difference between the hedged and unhedged

Figure 5: Japanese equity market returns.



return on a foreign equity market will be zero, since this difference is not directly dependent on the forward rate. Thus, under conditions of forward rate parity, hedging reduces risk with no concomitant reduction (or increase) in return. PEROLD and SCHULMAN (1988) refer to this phenomenon as the "free lunch" in currency hedging. Naturally, hedging is associated with certain transaction costs but these are likely to be negligible relative to the magnitude of the risk reduction.

Prior to progressing further into a discussion of currency hedging policy, some explanation and comment on tables 5 and 6 is offered. As mentioned in the previous section the unanticipated return in all the reported currency markets relative to the Swiss franc was positive and consequently all the hedged returns on the respective equity markets are less than their unhedged counterparts. Thus, over the last decade, the international Swiss investor has faced a positive trade off between risk and return in the currency markets.

This is illustrated in figure 6 where the risk and return characteristics of the various markets reported in tables 5 and 6 are plotted in the risk/return plane. The following set of abbreviated equations illustrate the key relationships:

$$R_u = R_l + e \qquad (1)$$

$$R_h = R_l + f \qquad (2)$$

R_u = Swiss franc return on foreign equity market unhedged.
R_l = local currency return on foreign equity market.
R_h = Swiss franc return on foreign equity market hedged.
e = Swiss franc return on Yen/Swiss franc currency.
f = Swiss franc forward rate against the Yen.

$$(R_u - R_h) = (e - f)$$

Table 5: Decomposition of Return in Major Equity Markets in Swiss Franc Terms Hedged.

Equity Market	Equity Returns Local Currency	Forward Returns v Swiss Franc	Equity Returns Swiss Francs Hedged	Retutn to Risk Ratio Swiss Francs (h)	Equity Returns Swiss Francs Unhedged (u)	Difference between Hedged& Unhedged = Alpha
France	23.03	-7.27	15.76	0.73	19.30	3.54
Germany	15.62	-0.47	15.15	0.78	16.72	1.57
Japan	22.61	-1.26	21.35	1.29	27.89	6.54
Switzerland	12.11	n.a.	12.11	0.77	12.11	0.00
UK	23.27	-6.70	16.57	0.85	20.43	3.86
US	17.24	-5.01	12.23	0.75	18.05	5.82

Note:
h = hedged
u = unhedged
All statistics are estimated over the period January 1980 to December 1989 using monthly data.
All return statistics are annualised averages.

Table 6: Decomposition of Risk in Major Equity Markets in Swiss Franc Terms Hedged.

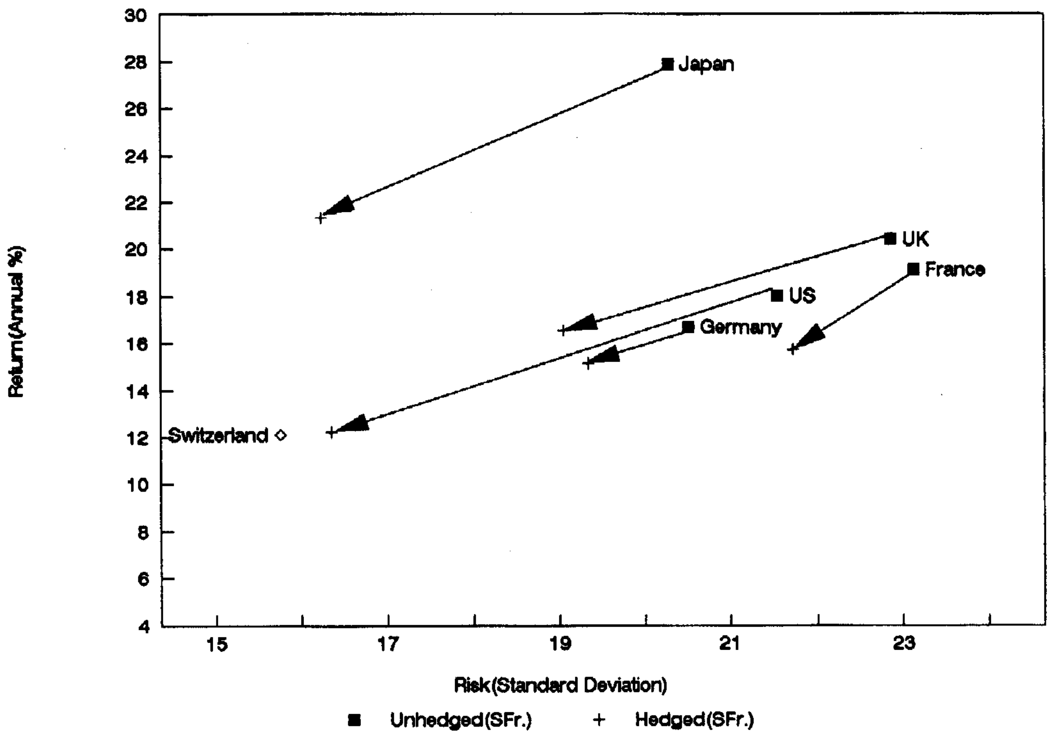
Equity Market	Equity Returns Local Currency Standard Deviation	Forward Returns v Swiss Franc Standard Deviation	Correlation Local Equity Returns & Forward Rate	Equity Returns Swiss Francs (h) Standard Deviation	Equity Returns Swiss Francs (u) Standard Deviation
France	21.62	2.03	0.00	21.72	23.13
Germany	19.32	3.32	-0.08	19.34	20.51
Japan	16.50	5.10	-0.23	16.10	20.28
Switzerland	15.75	n.a.	n.a.	15.75	15.75
UK	19.60	4.13	-0.24	19.04	22.85
US	16.26	2.39	-0.04	16.34	21.54

Note:
h = hedged
u = unhedged
hedge ratio = 1
All statistics are estimated over the period January 1980 to December 1989 using monthly data.
All return statistics are annualised averages.

In this paper (*e-f*) will be defined as α (alpha) to facilitate the discussion. Alpha represents the currency return not anticipated in the forward rate and as such represents the ex-post difference between a hedged and an unhedged return. In an ex ante context α represents an investor's private expectations of the deriation between the currency return and the forward rate.

In table 6 it will be noticed that the standard deviation of the hedged equity returns are almost identical to

Figure 6: Hedging in a Mean-Variance Context.



the standard deviation in the corresponding local currency returns, despite the existence of basis risk (variability in the forward rate). The hedged counterpart of equation (9) in the appendix is equation (23) which demonstrates the effect of the correlation between the forward rate and the corresponding equity returns in local currency units. These correlation statistics are reported in table 6 and as with the correlations reported in table 2, they are not significantly different from zero and if anything slightly negative. The implication is of course that the effect of basis risk is negligible and may be ignored for all practical purposes.

The relative risk-adjusted performance of the markets as measured by ranking the return to risk ratios on a hedged and unhedged basis reveals some differences. While Japan remained top of the league the Swiss market performance improves from bottom of the league to fourth rank behind the UK and German markets. This suggests that, ceteris paribus, asset allocations among hedged assets would be quite different than those among unhedged assets. Things may not of course remain the same, in particular the pairwise correlations among hedged assets may be quite different than the corresponding correlations among unhedged assets. This issue will be addressed again in section 6 below.

Returning now to figure 6 where the data from tables 5 and 6 are plotted in the risk return plane, it is clearly seen that each arrow is downward pointing illustrating the reduction in risk and return associated with the hedging of currency risks in these markets over this interval. Consequently no particular hedging policy dominates another on the risk return plane and no specific guidance can be offered in this regard without explicit reference to the investors idiosyncratic degree of risk aversion. It should be of practical significance to specify under which conditions a universal hedging policy is appropriate. In this context a universal hedging policy is determined to be a policy which results in the unambiguous domination of hedging over not hedging, or vice versa, and therefore a policy which does not require reference to the investor's degree of risk aversion. These conditions are identified with the aid of a series of general cases illustrated in figures 7a through 7f, where the full set of permutations of relationships among hedged returns (R_h), unhedged returns (R_u) and local returns (R_l) are examined with a view to establishing a hedging policy framework.

Cases 1 through 4 (figures 7a through 7d) illustrate conditions where alpha is non-zero, i.e., the investor expects the currency return to be different to the return implied in the forward rate. These diagrams represent the private expectations of an active currency forecaster. Universal dominance is apparent only in case 1. Here irrespective of the sign of the forward rate hedging is preferred regardless of the investor's degree of risk aversion since expected return is increased and risk reduced by hedging. The key determinant of this condition is the sign of alpha, in this case negative.

It is emphasised that this condition arises independently of whether the forward rate is at a premium or a discount. Strategies based on the sign of the forward rate apply to cases 2 and 3 (figures 7b and 7c). In both cases hedging is preferred if the forward rate is at a premium, since alpha is impliedly negative. Notice how this can arise whether the overall expected return on the currency is negative or positive as in cases 2 and 3 respectively. However in cases 2 and 3 where the forward rate is at a discount (negative forward rate), no dominance is apparent and hedging policy must be made with explicit reference to the investor's degree of risk aversion. Case 4 (figure 7d) illustrates the conditions where the sign of the forward rate, as with case 1, provides no guidance on hedging policy. In this case alpha is positive and a trade off between risk and return must be made with reference to the investor's risk preferences.

These examples illustrate the following generalisations for active currency forecasters:

(1) Reference to the sign of the expected currency return is not sufficient to determine hedging policy.

Figure 7a: Expected Equity Returns, Hedged, Unhedged & Local Currency.

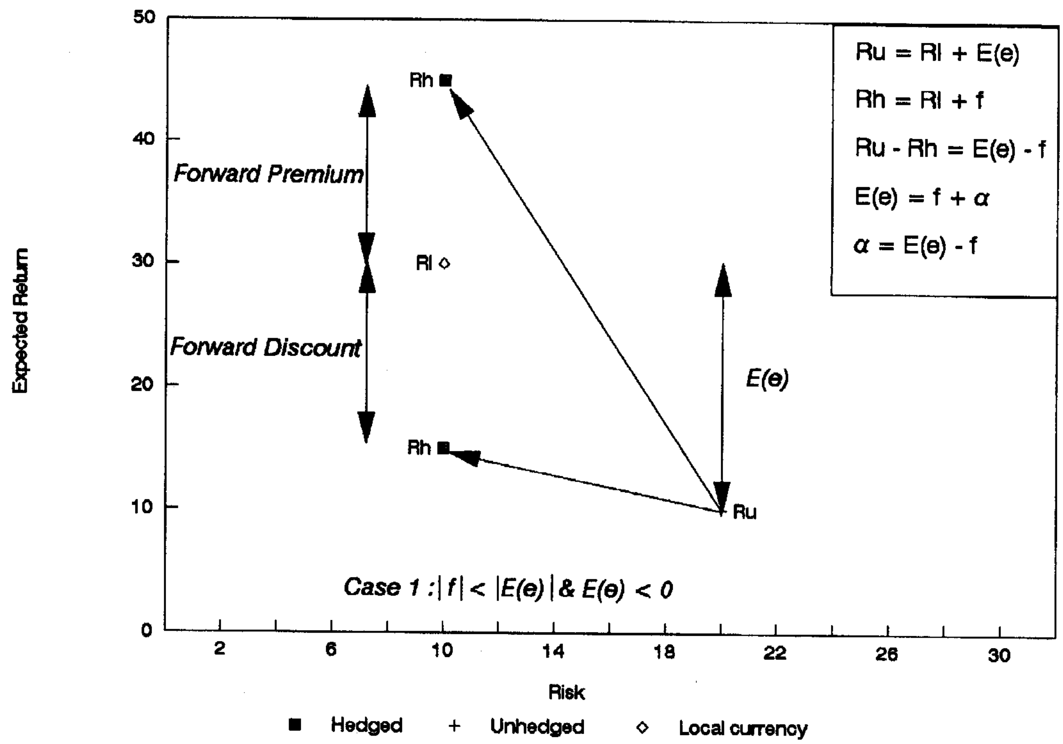


Figure 7b: Expected Equity Returns, Hedged, Unhedged & Local Currency.

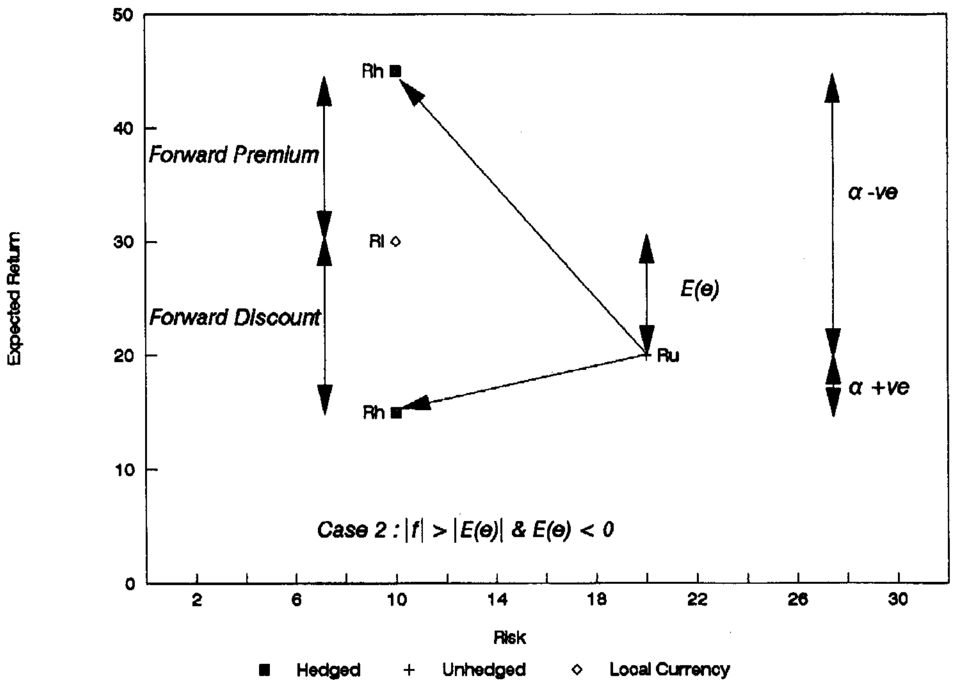


Figure 7c: Expected Equity Returns, Hedged, Unhedged & Local Currency.

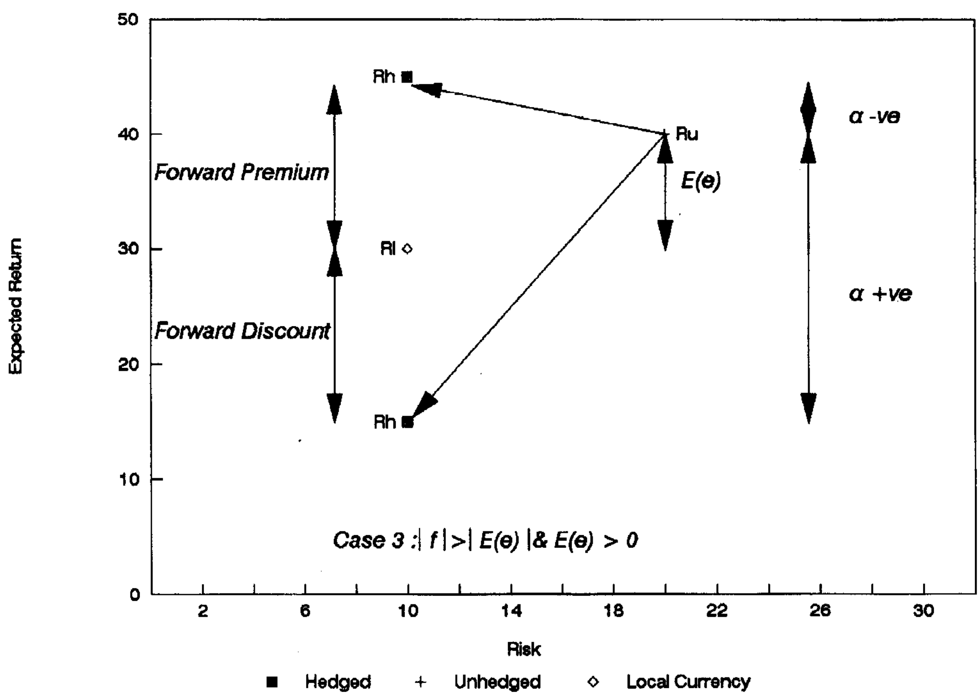


Figure 7d: Expected Equity Returns, Hedged, Unhedged & Local Currency.

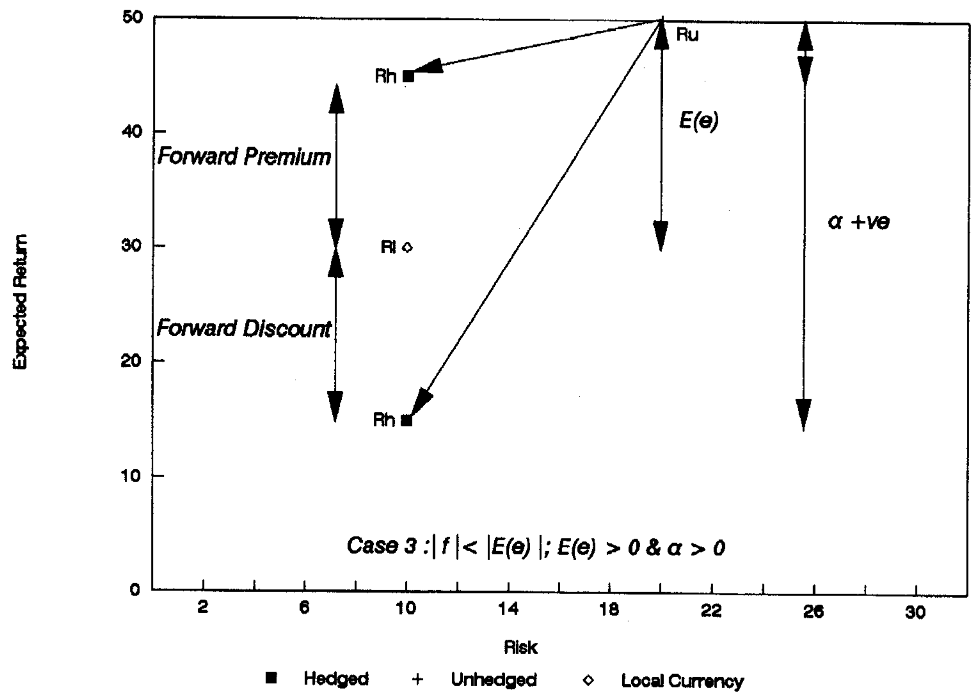


Figure 7e: Hedging Policy if Exchange Rates follow a Random Walk.

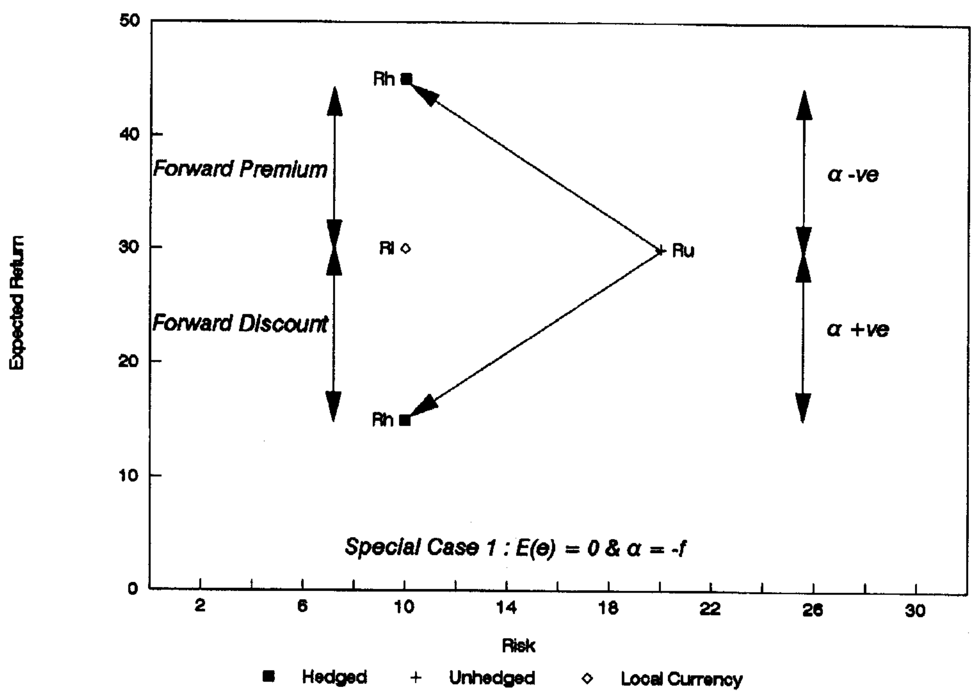
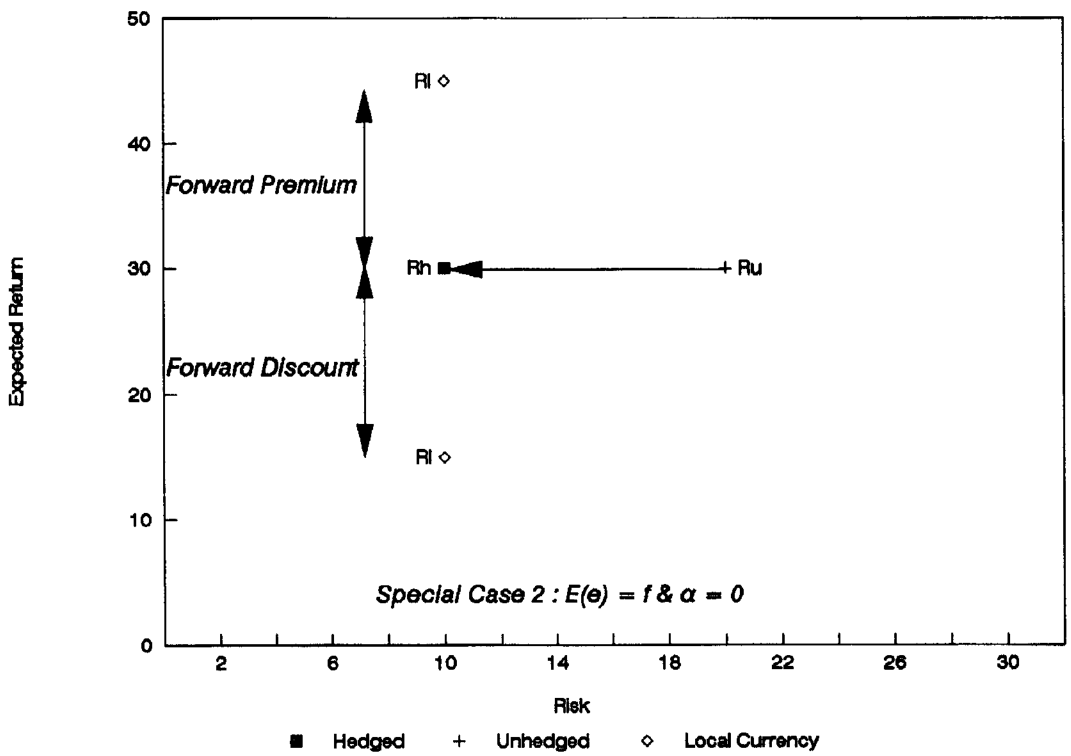


Figure 7f: Hedging Policy if Forward Rate Parity Holds.



(2) Reference to the sign of the forward rate is not sufficient to determine hedging policy.

(3) Reference to the sign of both the forward rate and the expected currency return will yield a universal hedging policy of full hedging viz. case 1. This case holds if and only if the sign of the expected currency return is negative and the absolute magnitude of the expected currency return is greater than the absolute magnitude of the forward rate. In all other cases an optimal hedging policy requires explicit reference to the investor's degree of risk aversion.

(4) The only reliable indications to guide hedging policy is the sign of alpha or the difference between the individuals expected currency return and the forward rate. If alpha is negative full hedging is optimal, independent of risk tolerance but if alpha is positive a trade off between risk and return is necessary.

(5) Although the sign of alpha is the only consistent guide to hedging policy it is asymmetrical in the sense that it does not lead to a hedge/no hedge decision but rather a hedge/trade off decision. Thus full hedging may be optimal for very risk averse investors despite a positive alpha.

The cases analysed are summarised in table 7.

The relevant equations for the four cases are equations (15) and (16) in the appendix.
There are two special cases in addition to those covered to this point.
Special case 1 (figure 7e) reflects the expected returns of an investor who believes that exchange rates follow a random walk and therefore the expected currency return is zero. Here, as indicated in point (5) above, the investor faces an asymmetric policy. If the forward rate is at a premium (and alpha is negative) full hedging is optimal, if the forward rate is at a discount (and alpha is positive) a risk return trade off is necessary and consequently reference to the investor's degree of risk aversion is required.

Table 7: Hedging Policy Implications of Expected Currency Returns and Forward Returns. The Cases Refer to Figures 7a through 7d.

Expected Currency Return	Forward Rate	Alpha Positive	Alpha Negative
Positive	Premium	Case 4	Case 3
Positive	Discount	Case 3&4	Not feasible
Negative	Premium	Not feasible	Cases 1& 2
Negative	Discount	Case 2	Case 1
Implied Hedging Policy		Risk Return trade of Optimal hedging policy depends on investor's degree of risk tolerance	Full hedging is optimal The minimum variance hedges dominates all others

Thus the assumption of a random walk in exchange rates does not yield simple hedging rules independent of the investor's risk tolerance.

Special case 1 is described in equation (19) of the appendix and represents that case where alpha and the forward rate are exactly equal in size and opposite in sign.

Special case 2 (figure 7f) described in equations (17) and (18) of the appendix reflects the set of currency expectations consistent with a belief in forward rate parity. Under this condition the investor adopts the consensus expectation and alpha is zero. Full hedging is the optimal policy regardless of the sign of the forward rate.

It is clear from the above analysis that a key determinant of currency hedging policy is the belief the investor holds regarding the process generating exchange rates. The issue of which process best describes the behaviour of exchange rates is highly controversial in the international finance literature. Even beyond this controversy there exists only one class of model viz. forward rate parity which resolves the hedging question in an unambiguous way. In all other cases reference has to be made to the implied sign of alpha and when positive reference has to be made to the investor's degree of risk aversion.

Two further complications may arise in the implementation of currency hedging policy. Firstly, notice that the standard deviation of both R_l and R_h are the same in all cases illustrated on the figure 7 illustrations, which assumes that no basis risk exists. Although the forward rate does vary over time the effect is generally not significant for practical purposes. Secondly, the whole analysis is based on one asset in isolation. It is quite conceivable that even where hedging appears to dominate in a single asset case some diversification potential of the asset, in the context of a multi-asset/currency portfolio, may be lost. In other words, even if the hedged asset dominates the unhedged asset on the risk return plane, given the pairwise correlations between these two assets and all other assets in a portfolio the unhedged asset may be more desirable as a diversification vehicle.

It is therefore inadvisable to determine hedging policy at an individual asset (country) level.

The determination of a coherent currency hedging policy in practice obviously has the potential for considerable controversy.

Prior to proceeding to the presentation of an integrated approach to currency hedging and asset allocation in section six, the next section provides a brief exposition of minimum variance hedging.

5. Minimum Variance Hedging

The hedged return of each equity market in Swiss franc represented on figure 8 does not necessarily represent the minimum risk hedged position in each case. There exists a minimum variance hedge ratio for each country/equity market. This minimum variance hedge ratio represents that proportion of hedging that expunges both direct and indirect currency exposure from an equity investment. Our Swiss investor could have established a hedge ratio such that the returns in Swiss franc (hedged) on the various equity markets exhibited a lower volatility than the corresponding return in local currency units. The minimum variance hedge ratio for each country, estimated over the ten year period ending December 1989, are reported in table 8.

The three regression parameters described in equations (28), (29) and (33) in the appendix are likewise reported. These slope coefficients (beta factors) are estimated independently on a single currency basis. It should be noted that β_1 is determined primarily by the correlation between local equity market returns and currency market returns in Swiss franc. This correlation, reported in table 2, was shown to be not significantly different to zero. If this correlation is assumed to be zero the minimum variance hedge ratio is -1 as defined in equations (30) and (31). The impact of different hedge ratios on equity returns in Swiss franc for different values of ρ (R_i, e_i^{sf}), described as rho in the figure, is illustrated in figure 8 [13].

Notice the following features of figure 8:

(1) Where the hedge ratio is zero and;
  (1) *rho* = 1, the standard deviation of the equity return in Swiss franc is the sum of local equity risk and currency risk i.e. $S(R)$ and $S(e)$ respectively on the diagram.
  (2) *rho* = 0, the incremental impact of currency risk on the local currency equity market risk is considerably less than the currency risk ($S(e)$).
  (3) *rho* = -1, the marginal impact of currency risk is negative.

Figure 8: Minimum Variance Hedge Ratios.

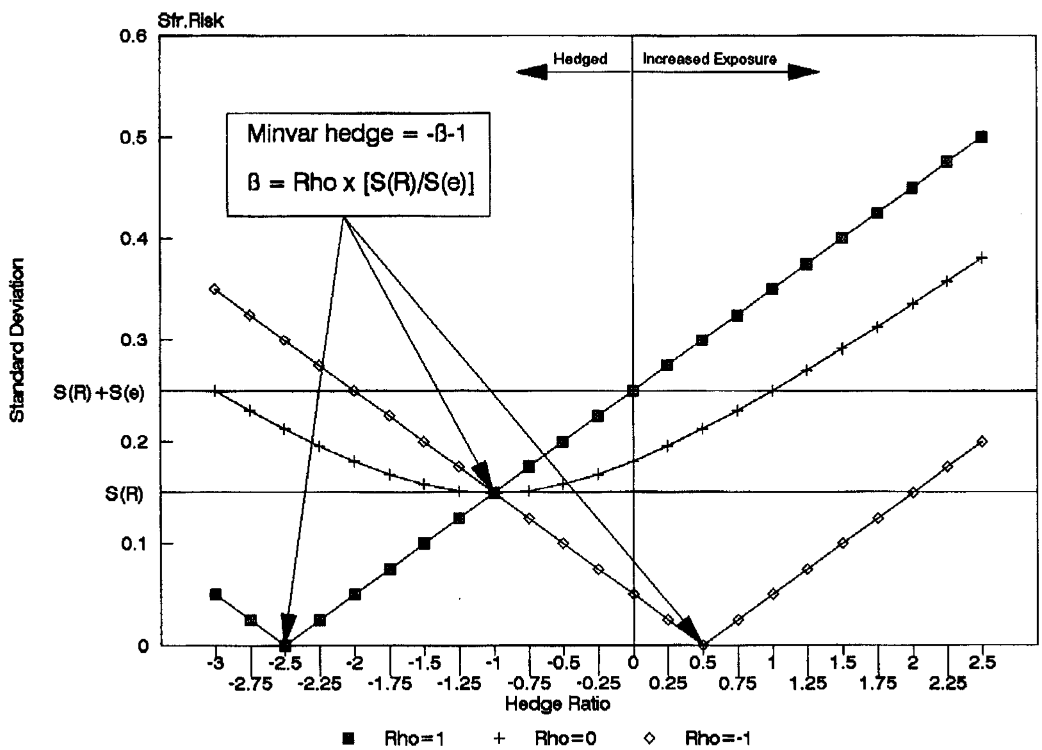


Table 8: The Estimation of Minimum Variance Hedge Ratios. Regression Slope Coefficients with Alpha as the Independent Variable.

Dependent Variable	Equity Return (local)	Currency Return	Equity Return (SFr.)	Minimum Variance
Slope Coefficient	β_1*	β_2	β_3	Hedge Ratio
France	0.35	0.96	1.31	-1.31
Germany	0.37	0.98	1.35	-1.35
Japan	0.33	0.93	1.26	-1.26
Switzerland	n.a.	n.a.	n.a.	n.a.
UK	0.19	0.97	1.15	-1.15
US	0.02	0.96	0.98	-0.98

Note:
Alpha represents the currency movement unanticipated in forward rates.
All statistics are estimated over the period January 1980 to December 1989 using monthly data.
* None significantly different to zero.

(2) Where the hedge ratio is -1 (fully hedged) the standard deviation of the Swiss franc return is exactly equal to the standard deviation of the local currency return regardless of the correlation between currency and equity markets. Again this assumes no basis risk.
(3) The minimum variance hedge ratio is less than -1 for all correlations above zero.
(4) Where rho = +1 or -1 the variance of the minimum variance hedged return equals zero.
(5) If rho is non-zero, currency assumes the role of a hedging asset and overall risk can be reduced with some currency exposure.
(6) If rho is zero, currency risk is all "noise" and a fully hedged position dominates all others in the risk dimension.

The data reported in table 8 suggests that the markets under observation in this study exhibit the properties associated with rho = 0 on figure 8.
The empirical behaviour of different hedging strategies on the risk and return on the Japanese market in Swiss franc is illustrated in figures 9 and 10.
The impact of different hedge ratios on the standard deviation in the return on the Japanese market in Swiss franc, over the study period, is illustrated in figure 9. The standard deviation in the unhedged return on the Japanese market ($S(u)$) and the standard deviation in the fully hedged ($S(h)$) returns are likewise illustrated for comparison.
Notice in figure 9 how the standard deviation in the return on the Tokyo market in Swiss francs, hedged using the minimum variance hedge ratio, is not significantly different to the fully hedged return or the Yen return counterparts. This is because the minimum hedge ratio is close to -1 or fully hedged. The major determinant of this phenomenon is the lack of correlation between the returns on the Japanese equity market in Yen and the return on the Yen in Swiss francs. The hedge ratios and various coefficients described in section 5 of the appendix are reported in table 7.
The implications of this general lack of correlation between local equity market returns and exchange rate returns is that the single currency minimum variance hedge ratio can be assumed to be -1.
Figure 10 adds the return function for the various hedging policies reported in figure 9 [14]. This diagram illustrates that the minimum hedge ratio is not optimal for all classes of investors since other hedge ratios are associated with higher returns. The slope of the return function is positive because alpha (e-f) is

Figure 9: Hedging Japanese Equities 1980-1989.

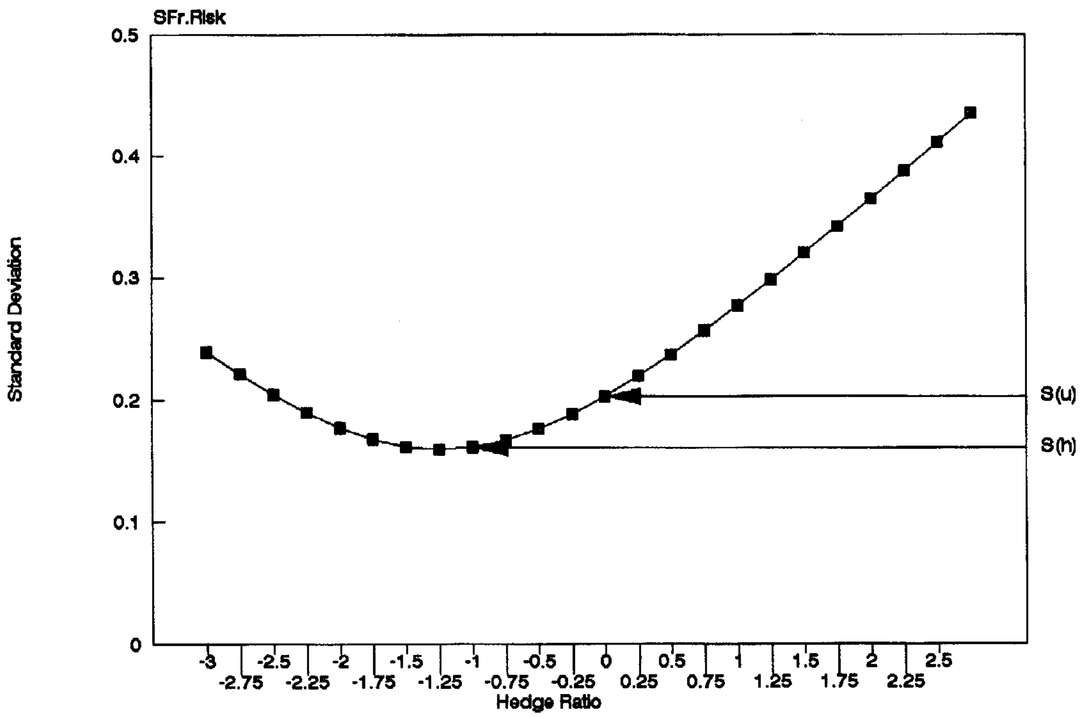
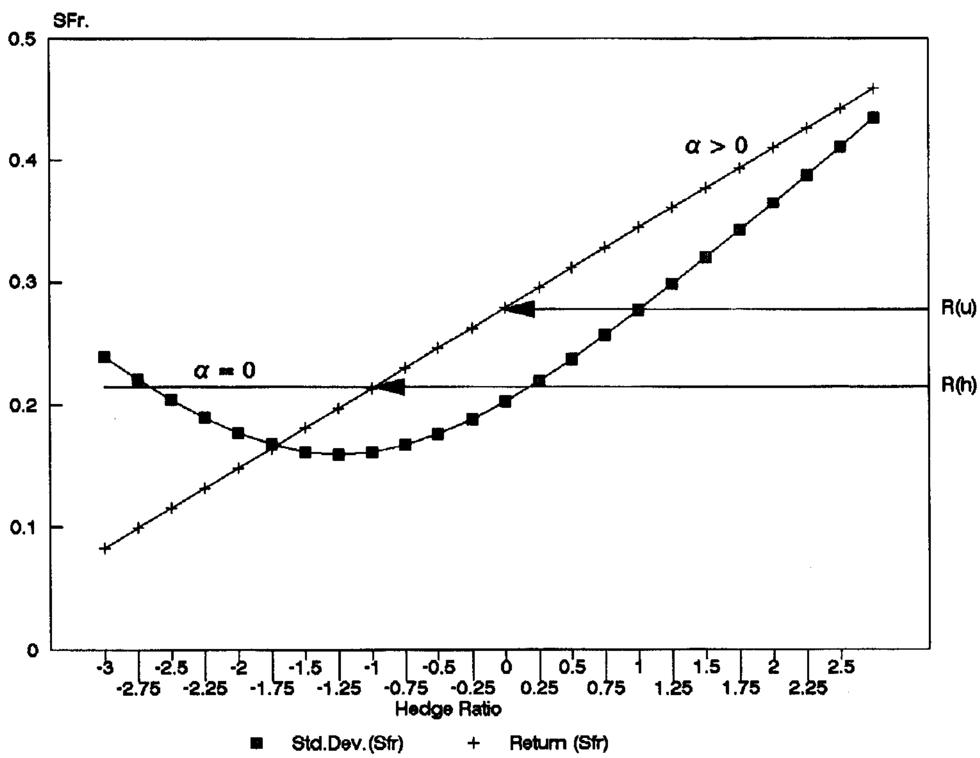


Figure 10: Hedging Japanese Equities 1980-1989.



positive. If alpha (e-f) is set to zero the return function is flat and therefore invariant to hedging policy. This represents the forward rate parity case described on figure 7f which results in an optimal hedging policy regardless of risk aversion. This optimal is of course the minimum hedge ratio and this strictly dominates all others since all others have the same expected return but greater risk.

A recent paper by Fischer BLACK (1990) suggests that in equilibrium rational investors will select a universal hedge ratio which is between 0 and -1. His conclusion revolves around the observation that currency returns are a positive sum game due to Siegel's paradox. This implies that the return schedule in figure 10 is positively sloping and a positive trade off between risk and return exists. The exact magnitude of the implied universal hedge ratio then turns out to be a function, inter alia, of the average degree of risk aversion.

The relevant magnitudes of the fully hedged return (h = -1) (R_h), and the unhedged return (h = 0)(R_h) are highlighted in figure 10.

It is again emphasised that the exposures here reported may be different when estimated in the context of a multicurrency portfolio when the minimum variance hedge ratios for each currency are determined simultaneously for a given portfolio [15].

The following section describes a method which integrates the estimation of optimal multi-currency hedge ratios and the optimal allocation of assets across countries.

6. Integrating Currency Hedging and Asset Allocation

So far an analysis of currency risk, and the impact of the hedging thereof, on the risk and return characteristics of individual assets denominated in a foreign numeraire has been presented. In this section a procedure is suggested which integrates currency risk management into the process of determining the optimal allocation of assets internationally.

In order to determine an optimal portfolio of international assets our Swiss investor must first generate a set of expectations regarding the risk and return for each potential asset in Swiss franc terms for the investment universe. This involves forecasting returns in currency markets in Swiss francs and equity market returns in local currency units. Secondly, an estimation must be made of the pairwise correlation between each asset and all other assets in the investment universe.

The historic magnitudes of all these variables for the six markets are reported in table 9 as a surrogate for forward looking estimates. These historic statistics were based on a ten year interval of monthly data commencing January 1980. Obviously, in practice our investor would develop his or her own set of forward looking estimates and the historic statistics would merely constitute a benchmark case [16].

These inputs are then subjected to a Markowitzian optimisation to generate a set of efficient portfolios of the underlying assets [17]. Efficient merely means that a portfolio has the maximum return for a given risk and thereby it dominates all other portfolios and assets with the same risk.

Four key portfolios from the efficient set are reported in table 10. These were generated by subjecting the table 9 data to the optimisation procedures.

These four portfolios represent optimal portfolios for four different investors classified by their degree of risk aversion [18]. The minimum variance portfolio, which represents the first point at the lower end of the efficient frontier, is optimal for the most risk averse investor since it represents that combination of the six markets with the lowest risk. Notice that this portfolio has less risk (13.61%) than the lowest risk asset, Switzerland (15.75%), and a considerably larger return 17,74% against 12.11%. This reflects the benefits of international diversification for the Swiss investor as described by KNIGHT (1989). Interestingly, the minimum risk portfolio suggests almost 40 % of assets should be invested abroad. Even if we assume that our investor has invested 50 % of the portfolio in Swiss franc bonds and therefore the allocations implied in the minimum variance portfolio pertain only to 50% of the overall portfolio - it is clear that the portfolio is out of the bounds of the new Federal Pension Scheme regulations (BVG/

Table 9: Inputs for International Asset Allocation Unhedged Strategy (Swiss franc).

Swiss Franc Unhedged	France	Germany	Japan	Switzerland	UK	US
Return	19.30	16.72	27.89	12.11	20.43	18.05
Risk	23.13	20.51	20.28	15.75	22.85	21.54
Correlation Matrix						
France	1.00	0.54	0.31	0.52	0.47	0.46
Germany	0.54	1.00	0.23	0.71	0.40	0.41
Japan	0.31	0.23	1.00	0.21	0.36	0.35
Switzerland	0.52	0.71	0.21	1.00	0.49	0.56
UK	0.47	0.40	0.36	0.49	1.00	0.64
US	0.46	0.41	0.35	0.56	0.64	1.00

Note:
All statistics are estimated over the period January 1980 to December 1989 using monthly data.
All risk and return statistics are annualised averages (%).

Table 10: International Asset Allocation Unhedged Strategy. Efficient Portfolios.

	Efficient Portfolios			
	Minimum Variance	Risk Tolerance Factor		Maximum Return
		10	30	
Degree of Risk Tolerance	Minimal	Moderate -	Moderate +	Maximal
Performance				
Return	17.74	20.51	25.05	27.89
Risk	13.61	14.11	16.83	20.28
Asset Allocations (%)				
France	1.97	3.86	6.26	0.00
Germany	1.11	7.92	13.56	0.00
Japan	32.71	45.42	69.62	100.00
Switzerland	60.44	35.09	0.00	0.00
UK	1.97	5.34	10.56	0.00
US	1.80	2.37	0.00	0.00
Funds allocated (%)	100.00	100.00	100.00	100.00
Performance Hedged				
Return	14.29	15.93	18.76	21.35
Risk	13.21	13.01	14.37	16.10

Note:
All statistics are estimated over the period January 1980 to December 1989 using monthly data.
All risk and return statistics are annualised averages (%).

LPP). The new regulations permit a maximum of 25 % in foreign equities (up from 10%) [19]. This implies that the constraints imposed by Federal legislation may be forcing certain conservative pension funds to take on more risk than they need to! The efficient portfolios are unhedged and therefore are influenced by currency movements. The effect of hedging these efficient portfolios is reported in the last two rows of table 10 and described in figure 11 as the post-optimisation hedge strategy [20]. The minimum variance portfolio has very little currency risk and therefore hedging reduces total volatility a mere 0.40%, from 13.61% to 13.21%, however return is reduced by 3.45%. Given these hedging alternatives our risk averse investor would clearly prefer to adjust his portfolio to the allocations which were optimal for risk tolerance levels of 10% on an unhedged basis. This portfolio (risk tolerance 10 now hedged) has a lower risk and higher return than the minimum variance portfolio hedged (post-optimisation).

It is clear from figure 11 that the minimum variance portfolio hedged (post-optimisation) is no longer the minimum risk portfolio. The set of portfolios derived by hedging the efficient set of unhedged assets is itself not efficient.

A more effective way of determining the optimal hedged portfolio is to optimise over the set of hedged assets. The efficient set so derived is illustrated on figure 11 and described as hedge pre-optimisation. The inputs and outputs for this optimisation are reported on tables 11 and 12 respectively.

The new minimum variance portfolio exhibits less risk and higher return than the lowest risk of the hedged portfolios (post-optimisation) reported in table 10. In fact as shown on figure 11, the frontier derived from the set of hedged returns (pre-optimisation) dominates the set of hedged portfolios

Figure 11: Efficient Frontiers with Hedging.

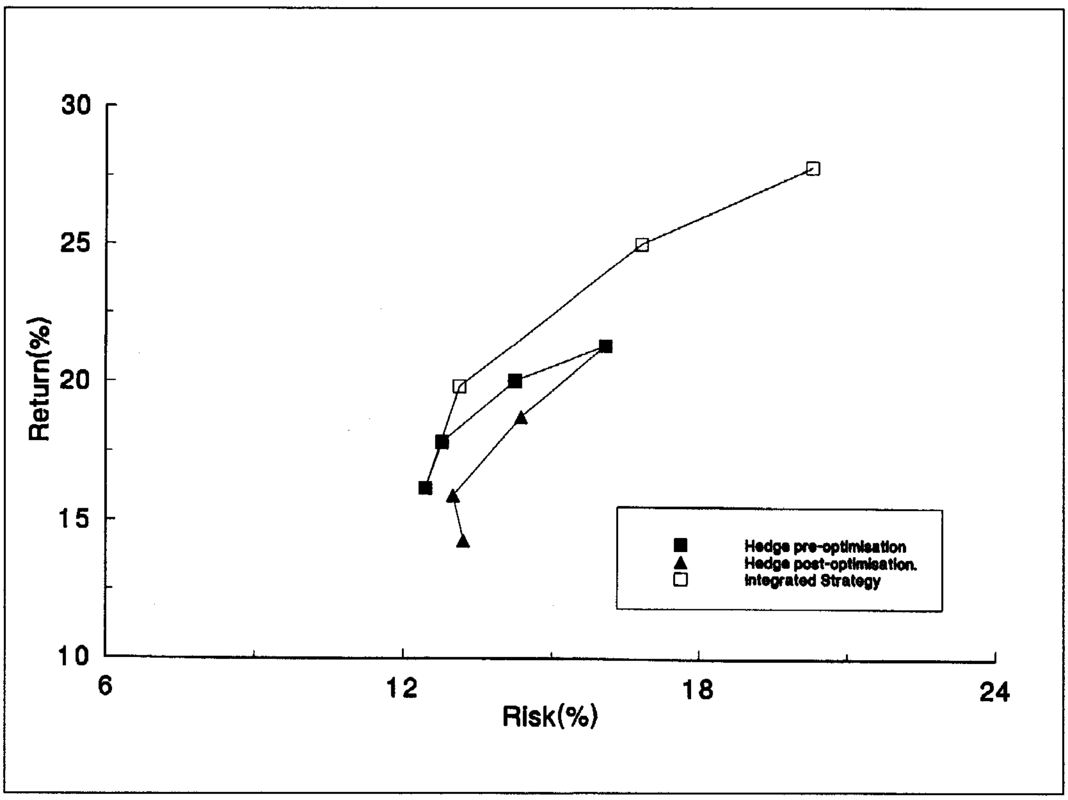


Table 11: Inputs for International Asset Allocation Hedged Strategy.

Swiss Franc Hedged	France h	Germany h	Japan h	Switzerland	UK h	US h
Return	15.76	15.15	21.35	12.11	16.57	12.23
Risk	21.72	19.34	16.10	15.75	19.04	16.34
Correlation Matrix						
France h	1.00	0.51	0.35	0.51	0.48	0.53
Germany h	0.51	1.00	0.28	0.78	0.46	0.45
Japan h	0.35	0.28	1.00	0.27	0.38	0.36
Switzerland	0.51	0.78	0.27	1.00	0.56	0.64
UK h	0.48	0.46	0.38	0.56	1.00	0.67
US h	0.53	0.45	0.36	0.64	0.67	1.00

Note:
h denotes hedged return relative to Swiss franc (h = -1).
All statistics are estimated over the period January 1980 to December 1989 using monthly data.
All risk and return statistics are annualised averages (%).

Table 12: International Asset Allocation Hedged Strategy. Efficient Portfolios.

Swiss Franc hedged	Efficient Portfolios			
	Minimum Variance	Risk Tolerance Factor		Maximum Return
		10	30	
Degree of Risk Tolerance	Minimal	Moderate -	Moderate +	Maximal
Performance				
Return	16.18	17.84	20.08	21.35
Risk	12.45	12.78	14.25	16.10
Asset Allocations (%)				
France h	0.00	0.65	0.00	0.00
Germany h	4.10	8.36	11.57	0.00
Japan h	41.94	55.04	76.62	100.00
Switzerland	33.50	21.05	0.00	0.00
UK h	0.88	7.94	11.82	0.00
US h	19.59	6.96	0.00	0.00
Funds allocated (%)	100	100	100	100

Note:
h denotes hedged return relative to Swiss franc (h = -1).
All statistics are estimated over the period January 1980 to December 1989 using monthly data.
All risk and return statistics are annualised averages (%).

derived from the set of unhedged returns (post-optimisation). This occurs because the optimal allocations to the various assets are different between the two cases for each level of risk tolerance except the maximum risk tolerance portfolio which consists of one and the same asset being the highest return asset.

This pattern of dominance of one strategy over the other, at all levels of risk tolerance, and coincidence at the maximum return point, is generalisable. Since the dominated set is a potential solution in the optimisation over unhedged assets; the former (post-optimisation) strategy represents a constrained solution to the more general (pre-optimisation) strategy. At worst they could be identical. The conditions which would result in the same solution via both methods are fairly restrictive and therefore unlikely in practice. These conditions include a uniform return to risk reduction when hedging across assets and an absence of basis risk. This ensures that the relative pairwise correlation between assets would remain unchanged whether hedged or not. Notice on figure 11 how the spread between the two strategies diminishes as risk tolerance increases.

If international asset allocations are to be determined in a fully hedged currency regime it is important to follow the pre-optimisation hedge approach, particularly where risk tolerance is low. However, deciding upon a full hedging policy prior to the asset allocation decision is often hazardous with respect to efficiency even for the most risk averse investor. It is submitted that the most effective method to estimate an optimal allocation of assets and an optimal currency policy is to determine both asset allocations and currency hedge ratios simultaneously. In this way currency risk and return are treated symmetrically with equity risk and return to reflect the investor's degree of risk tolerance. The approach suggested to achieve the higher frontier reported in figure 11 (integrated strategy) is based on an optimisation over the pooled set of hedged and unhedged assets. This approach permits the investor to use the standard asset allocation optimiser to estimate optimal hedge ratios and international asset allocations simultaneously. The inputs for the integrated strategy are the combined inputs reported in tables 9 and 11 and in addition the correlation statistics reported in table 13. The results for the key efficient portfolios are reported on table 14.

The optimal portfolio for a Swiss investor with the minimal degree of risk tolerance consists entirely of hedged assets. The minimum variance portfolio is thus identical to that reported in table 10. This is an intuitively appealing but not general result. Clearly, one expects the most risk averse investor to eschew currency risk and therefore select fully hedged portfolios. However there is no reason to suppose that the minimum variance portfolio should not have some currency exposure. It is conceivable that a minimum variance portfolio is not fully hedged. In

Table 13: Additional Inputs for International Asset Allocation Integral Strategy.

Correlation Matrix	France	Germany	Japan	Switzerland	UK	US
France h	0.965	0.503	0.295	0.511	0.426	0.394
Germany h	0.494	0.970	0.216	0.728	0.378	0.356
Japan h	0.313	0.256	0.870	0.273	0.325	0.278
Switzerland	0.522	0.706	0.212	1.000	0.492	0.556
UK h	0.477	0.436	0.310	0.556	0.889	0.579
US h	0.513	0.439	0.264	0.642	0.565	0.767

Note:
h denotes hedged return relative to Swiss franc (h = -1).
All statistics are estimated over the period January 1980 to December 1989 using monthly data.

Table 14: International Asset Allocation Integral Strategy. Efficient Portfolios.

Swiss Franc unhedged	Efficient Portfolios				Implied Optimal Hedge Ratio	
	Minimum Variance	Risk Tolerance Factor 10	Risk Tolerance Factor 30	Maximum Return	(1)	(2)
Degree of Risk Tolerance	Minimal	Moderate -	Moderate +	Maximal	Moderate -	Moderate+
Optimal Hedge Ratio	-1	See column (1)	See column (2)	0		
Return	16.18	19.23	25.05	27.89		
Risk	12.45	13.13	16.83	20.28		
Asset Allocations (%)						
France	0.00	2.49	6.26	0.00	0.00	0.00
Germany	0.00	8.53	13.56	0.00	0.00	0.00
Japan	0.00	15.57	69.62	100.00	-0.71	0.00
UK	0.00	0.38	10.56	0.00	-0.94	0.00
US	0.00	3.92	0.00	0.00	-0.55	0.00
France h	0.00	0.00	0.00	0.00		
Germany h	4.10	0.00	0.00	0.00		
Japan h	41.94	38.17	0.00	0.00		
Switzerland	33.50	19.76	0.00	0.00		
UK h	0.88	6.38	0.00	0.00		
US h	19.59	4.81	0.00	0.00		
Funds allocated (%)	100	100	100	100		

Note:
h denotes hedged return relative to Swiss franc (h = -1).
All statistics are estimated over the period January 1980 to December 1989 using monthly data.
All risk and return statistics are annualised averages (%).

cases where the currency component of an asset's unhedged return lacks correlation with, or is negatively correlated with, the returns on other assets, overall portfolio risk may be reduced via a natural diversification. This diversification benefit could be lost if the currency risk in question was directly hedged [21]. It is noted that for a given correlation structure and risk estimation for a set of assets the constitution of the minimum variance portfolio is independent of return estimates. In the context of international asset allocation this implies that the minimum variance portfolio is the same regardless of the sign of alpha, ceteris paribus.

Moving to the other end of the frontier, given that alpha is positive for all currencies the highest return asset must be unhedged and thus the most risk tolerant investor will take a position in one asset unhedged, which over the period under study is Japan. Again this is not a general result, the highest return asset may well be hedged if alpha is negative. The top end of the frontier coincides with the maximum return portfolio in table 12.

However between the two extremes exist portfolios with a certain degree of currency exposure depending on the investor's risk tolerance.

The investor with a risk tolerance of 10 would choose a combination of assets which imply a set of optimal currency hedge ratios reported in the second column from the end in table 14.

The proportion of a portfolio's value to be allocated to a particular equity market is the sum of the percentages allocated to the country hedged and it's unhedged counterpart. The implied hedge ratio is then the percentage allocated to the hedged version divided by the total allocated to the country. For example in table 14 Japan has an implied hedge ratio of -0.71 for a risk tolerance of 10. This hedge ratio is derived by dividing 38.17% (allocation to Japan hedged) by 53.74% (the sum of 15.57% and 38.17%). Since alpha is positive as we progress up the efficient frontier i.e. increase the level of investor risk tolerance less hedging occurs [22]. At a risk tolerance of 30 the optimal hedge ratios on all markets are zero. What is striking in these results is what little impact currency movements have in the case of a Swiss based investor. This is so for two reasons, firstly it has been shown that an average over the study period hedging has been costly in all markets. Secondly, due to the lack of correlation between equity markets and their currency market counterparts the marginal impact of currency risk on portfolio risk is small and thus little hedging is required by investors with moderate to large degrees of risk tolerance.

The dominance of the integrated strategy over the other approaches discussed is demonstrated in figure 11. Notice that for any given level of risk tolerance a higher return is feasible with less than full hedging. For the data analysed in this study the critical values of risk tolerance with respect to hedging are between zero and thirty. Figure 12 reports thirty-one efficient portfolios which lie between the first three reported in table 14.

Each portfolio represents the optimal combination for a risk tolerance ranging from zero to thirty in increments of one. Figure 13 displays the asset allocation for each of these risk tolerance values. Thus each risk tolerance value and the attendant allocations across countries are represented by a corresponding point on the risk return plane shown in figure 12.

Figure 12: Integrated Currency Hedging: Efficient Frontier.

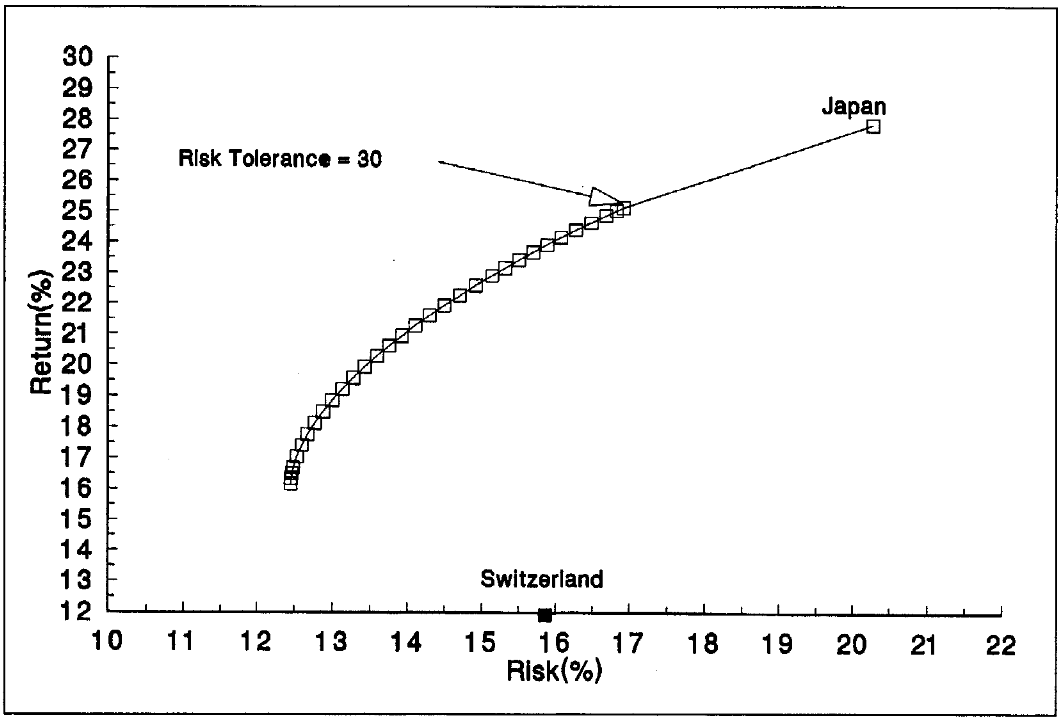
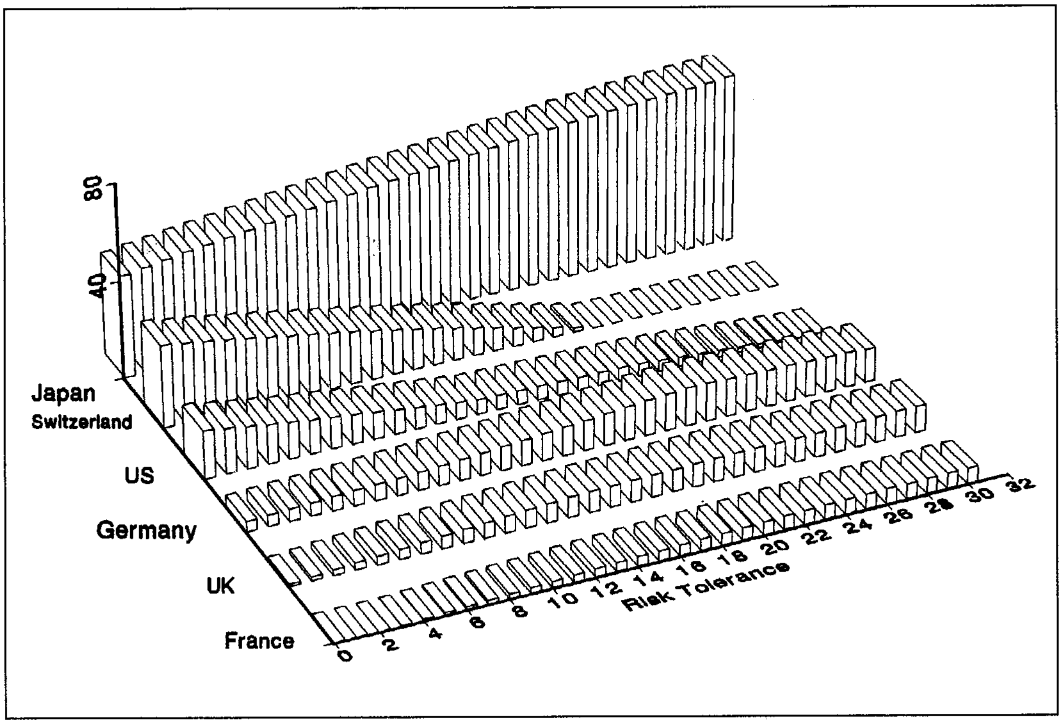


Figure 13: Optimal Asset Allocations(%) versus Risk Tolerance.



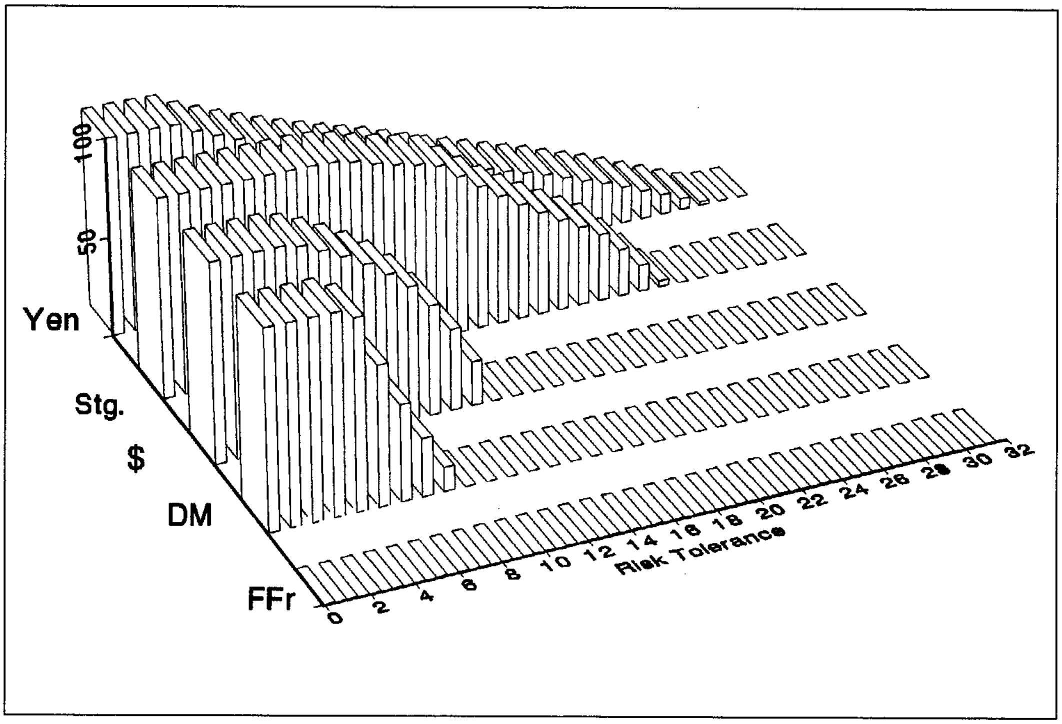
Notice how a Swiss investor who was prepared to accept the risk levels implied in the Swiss equity market (15.75%) could have improved return by almost double to 23.5% from 12.11%. This corresponds to a portfolio on the integrated frontier which would be optimal for a risk tolerance of 25. From figure 13 it is seen that this portfolio is spread across all five markets (excluding Switzerland) with a heavy concentration in Japan [23]. This illustrates the tremendous benefits to international diversification not-withstanding the high cost of hedging. Notice how funds are reallocated away from Switzerland and the US to the other four markets as risk tolerance is increased. The allocations at each risk tolerance level described on figure 13 sum to 100%.

In order to provide some insight on the behaviour of optimal hedge ratios with respect to changes in the level of risk tolerance figure 14 presents a diagram which decomposes the optimal portfolio for risk tolerance levels ranging from 0 (minimum variance) to thirty-one in increments of 1 for each currency [24].

The striking feature of this diagram is the non-uniform rate of decay in the hedge ratios from -1 (100%), at a risk tolerance level of zero, to zero at varying levels of risk tolerance up to thirty-one. The French franc (FFr) is never hedged, although the French market is not invested in for the first three levels of risk tolerance. All others reach a zero hedge at different levels of risk tolerance in increasing order, German mark (DM) 9, US dollar ($) 13, Sterling (UK) 25 and Yen 30.

Notice how the optimal hedge ratio for the Yen decreased at first at a faster rate than for Sterling, however, at a critical level of risk tolerance the Sterling hedge ratio decayed at a faster rate. The integrated approach suggested here enables the international investment manager to analyse, in detail, the sensitivity of the implied optimal hedge ratios to different levels of risk tolerance. In particular it may be useful to understand the critical range of risk tolerance over which it becomes optimal to reduce a hedge ratio from minus one to zero for a particular currency. Furthermore testing the sensitivity of the

Figure 14: Optimal Hedge Ratios(%) versus Risk Tolerance. Hedge Ratios implied in Efficient Portfolios.



implied hedge ratios to different expectations on currency and equity returns for different levels of risk tolerance may aid the decision process. The key feature of this approach is that hedging policy is now considered with explicit reference to risk tolerance in a multi-asset multi-currency context. Consequently a change in hedging policy automatically implies changes in the asset allocation ensuring that the selected portfolios are efficient. The arbitrariness in hedging policy using traditional methods is therefore removed.

The formal optimisation procedures used in this study are presented in section 6 of the appendix. There it will be noticed that the integrated approach constrains the hedge ratio to be within the bounds -1 and 0. This constraint ensures that a net long or short position in a currency (a typical constraint for most institutional investors) does not obtain. This restriction naturally precludes cross-hedging which by definition requires long or short positions in currencies. For this reason the integrated approach here presented should be characterised as optimally hedging the currency risk necessarily attendant upon investing in a foreign country.

7. Conclusions

The major conclusion to emerge from the analysis is that currency risk should be treated in the same way as other types of risk when determining optimal international portfolios.

Consequently it is crucial that hedge ratios are determined simultaneously with asset allocations. A simple method for integrating currency risk management with equity allocations using standard optimisation techniques was presented. Hedged and unhedged versions of all assets in the investment universe should be pooled and subjected to the optimisation process. The optimal hedge ratios are then inferred from the relative weights assigned to the two asset types for each country. These implied hedge ratios will therefore be a function of the technology of the returns, which determines the location of the efficient frontier on the risk return plane, and the degree of risk tolerance which determines the particular efficient portfolio an investor chooses.

Returning now to the three questions posed in the introduction. Firstly, currency hedging should be assumed to be imposed neither before (pre-optimisation) nor after (post-optimisation) the determination of asset allocations, but rather at the same time. Secondly, the degree of hedging cannot be determined independently of the investor's degree of risk aversion. Thirdly, as regards minimum variance hedging it is obvious that this should be carried out in a multi-currency context. However the minimum variance hedge portfolio will only be optimal for investors with an extreme degree of risk aversion. Fourthly, it is clear that the same risk tolerance coefficients should be applied consistently to both the equity and currency dimensions of a portfolio. It was shown that optimal hedging can be rather sensitive to the risk tolerance factor. Finally the issue of how active or passive investment managers should be on these dimensions depends on their beliefs regarding the efficiency of the two types of markets. A manager espousing efficiency in one market and inefficiency in another may create a credibility problem in a third market, the market for investment managers.

Appendix

1. Data and Statistical Measures

This study was based on monthly data for the period January 1980 to December 1989. Monthly equity and currency market returns in U.S. dollar terms were obtained from Morgan Stanley Capital International Perspective, monthly issues.

Return data in U.S. dollars were transformed into local currency units as follows:

$$R_{it} = R_{it}^{\$} - e_{it} - (R_{it}^{\$} \bullet e_{it}) \qquad (1)$$

where:

R_{it} = the return in period t on the i th equity market in local currency units.

$R_{it}^{\$}$ = the return in period t on the i th equity market in U.S. dollar.

e_{it} = the percentage change in the i th local currency, in period t, experienced by an investor in the i th equity market whose numeraire is the U.S. dollar.

$$e_{it} = \frac{S_{it} - S_{it-1}}{S_{it-1}} \qquad (2)$$

where:

S_{it} = the spot rate of exchange, at time t (the end of period t) of U.S. dollar per local currency unit i. Thus the rate is measured in U.S. dollars.

t - 1 = time point at the start of period t and the end of period t-1.

The standard deviation of equity returns was computed as follows:

$$\sigma^2(R_i) = \frac{\sum_{t=1}^{T}(R_{it}-\overline{R}_i)^2}{T-1} \qquad (3)$$

where:

$\sigma^2(R_i)$ = the standard deviation squared (variance) of R_i.

$\overline{R}_i$ = the average monthly return on the i th equity market over the period t=1 to T.
T = the number of periods, 120 months in this study.

The standard deviation of currency returns was computed in a similar way.
Standard deviation of monthly returns is annualised by multiplying it by $\sqrt{12}$, whereas monthly returns are annualised by multiplying by 12.

2. The Decomposition of Swiss Franc Equity Returns

Return data in Swiss franc terms are decomposed as follows:

$$R_{it}^{sf} = R_{it} + e_{it}^{sf} + (R_{it} e_{it}^{sf}) \quad (4)$$

where:
R_{it}^{sf} = return on the i th equity market in period t measured in Swiss francs.
e_{it}^{sf} = the percentage change in the i th local currency, in period t, experienced by an investor in the i th equity market whose numeraire is the Swiss franc.

$$e_{it}^{sf} = \frac{S_{it}^{sf} - S_{it-1}^{sf}}{S_{it-1}^{sf}} \quad (5)$$

S_{it}^{sf} = the spot rate of exchange, at time t (the end of period t) of Swiss franc per local currency unit i. Thus the rate is measured in Swiss franc units.

The variance in the Swiss franc equity returns is decomposed as follows:

$$\sigma^2(R_i^{sf}) = \sigma^2(R_i) + \sigma^2(e_i^{sf}) + \sigma^2(R_i e_i^{sf}) + 2Cov(R_i, e_i^{sf}) + 2Cov(R_i, R_i e_i^{sf}) + 2Cov(e_i^{sf}, R_i e_i^{sf}) \quad (6)$$

where:
$\sigma^2(.)$ = the variance of (.) which is the standard deviation squared.
$Cov(a,b)$ = the covariance between variable a and variable b.
$\rho(a,b)$ = the coefficient of correlation between variable a and b.

Note:

$$Cov(R_i, e_i^{sf}) = \rho(R_i, e_i^{sf})\sigma(R_i)\sigma(e_i^{sf}) \quad (7)$$

Since $(R_i \bullet e_i^{sf})$ is not significant in magnitude (4) is approximated by (8):

$$R_{it}^{sf} \approx R_{it} + e_{it}^{sf} \quad (8)$$

and (6) is approximated by (9):

$$\sigma^2(R_{it}^{sf}) \approx \sigma^2(R_i) + \sigma^2(e_i^{sf}) + 2Cov(R_i, e_i^{sf}) \quad (9)$$

where:
$\approx$ = approximates.

Tables 1 and 2 report the mean and standard deviation of the various components of equity returns for the 6 equity markets under observation. All these statis-tics are based on the reduced formulae (8) and (9). $\rho(R_i, e_i^{sf})$ values are reported in table 2.

3. The Decomposition of Currency Risk and Return

Tables 3 and 4 report the return and risk statistics for the currency markets under observation in the relevant period.

$$e_{it}^{sf} = f_{it}^{sf} + \alpha_{it}^{sf} \quad (10)$$

$$f_{it}^{sf} = \frac{F^{sf}{}_{1} - S_{t-1}^{sf}}{S_{t-1}^{sf}} \quad (11)$$

$$\alpha_{it}^{sf} = \frac{S_{it}^{sf} - F_{it-1}^{sf}}{S_{t-1}^{sf}} \quad (12)$$

where:

f_{it}^{sf} = the forward premium or discount on currency i, in Swiss franc terms, for period t.

F_{t-1}^{sf} = the forward rate of exchange, at time t-1 (the beginning of period t), of Swiss franc per local currency unit i, for delivery at time t. This rate represents the exchange rate that a Swiss investor can buy (sell) Swiss francs (local currency) for delivery and payment at time t. The rate is measured in Swiss franc units.

S_{t-1}^{sf} = the spot rate of exchange, at time t (the beginning of period t) of Swiss franc per local currency unit i. Thus the rate is measured in Swiss franc units.

α_{it}^{sf} = the change in the value of local currency (in Swiss francs) unanticipated in the forward rate at the beginning of the period.

The forward rate is determined by the interest rate differentials between the domestic and foreign markets. Interest rate parity, based on a covered interest arbitrage condition, specifies the relationship between forward and spot rates as follows:

$$F_{it}^{sf} = S_{it}^{sf}\frac{(1+i^{sf})}{(1+i)} \qquad (13)$$

where:

i^{sf} = Swiss interest rates.

i = local market interest rates.

In this study the forward rate was estimated with reference to the relevant 1 month interest rate on Euro-currency deposits.

The variance in currency returns may be decomposed as follows:

$$\sigma^2(e_i^{sf}) = \sigma^2(f_i^{sf}) + \sigma^2(\alpha_i^{sf}) + 2cov(f_i^{sf}.\alpha_i^{sf}) \qquad (14)$$

In general the process generating exchange rates is usually specified as:

$$e_{it}^{sf} = f_{it}^{sf} + \rho_{it}^{sf} + \varepsilon_{it}^{sf} \qquad (15)$$

where:

ρ_{ti}^{sf} = a time varying risk premium.

ε_{it}^{sf} = a random error or noise term with a zero mean.

If (15) is valid, rational expectations of the change in the future spot rate become:

$$E(e_{it+1}^{sf}) = f_{it+1}^{sf} + E(\rho_{it+1}^{sf}) \qquad (16)$$

Note that f_{it+1}^{sf} is known at the beginning of period t+1.

If $\rho_{it}^{sf}=0$ (15) becomes

$$e_{it}^{sf} = f_{it}^{sf} + \varepsilon_{it}^{sf} \qquad (17)$$

and rational market expectations of the change in the future spot rate become:

$$E(e_{it+1}^{sf}) = f_{it+1}^{sf} \qquad (18)$$

This condition (18) is known as forward rate parity. If the process generating exchange rates is assumed to follow a random walk $E(e_{it+1}^{sf}) = 0$ and (15) becomes:

$$e_{it}^{sf} = \varepsilon_{it}^{sf} \qquad (19)$$

4. The Decomposition of Risk and Return with full Hedging

Equation (8) describes the unhedged return on equity market i in Swiss franc terms. If the currency risk is removed by hedging, the return on the hedged equity market is given by (20).

$$_hR_{it}^{sf} = R_{it} + e_{it}^{sf} + h_i\alpha_{it}^{sf} \qquad (20)$$

where:

$\alpha_{it}^{sf} = e_{it}^{sf} - f_{it}^{sf}$

h_i = proportion of the value of the initial investment in market i that is hedged (hedge ratio).

${}_hR_{it}^{sf}$ = return on the i th equity market in period t, hedged via the forward market.

By rearranging and substitution (20) can be restated as (21).

$$ {}_hR_{it}^{sf} = R_{it} + e_{it}^{sf}(1+h_i) - h f_{it}^{sf} \qquad (21) $$

This is the general case which reduces to (22) where h_i is set to -1 (i.e. fully hedged).

$$ {}_hR_{it}^{sf} = R_{it} + f_{it}^{sf} \qquad (22) $$

The variance of the fully hedged return is:

$$ \sigma^2({}_hR_i^{sf}) = \sigma^2(R_i) + \sigma^2(f_i^{sf}) + 2Cov(R_i, f_i^{sf}) \qquad (23) $$

Tables 5 and 6 report the risk and return statistics of the fully hedged Swiss franc returns on the various equity markets.

5. Minimum Variance Hedge Ratios

If (21)is rearranged to (24)

$$ {}_hR_{it}^{sf} = R_{it} + e_{it}^{sf} + h_i(e_{it}^{sf} - f_{it}^{sf}) \qquad (24) $$

It is readily seen that the hedged return comprises 3 elements viz.

(i) local equity market returns in local currency units
(ii) Swiss franc return on local currency
(iii) the product of the hedge ratio and the unexpected return on the currency (vis à vis the forward rate).

Consequently the variance of the hedged return has 9 elements, 3 pure variance terms and 3 pairs of pairwise covariance terms:

$$ \begin{aligned} \sigma^2({}_hR_i^{sf}) = {} & \sigma^2(R_i) + \sigma^2(e_i^{sf}) + \sigma^2(e_i^{sf} - f_i^{sf})h_i^2 \\ & + 2Cov(R_i, e_i^{sf}) + 2Cov(R_i, e_i^{sf} - f_i^{sf})h_i \\ & + 2Cov(e_i^{sf}, e_i^{sf} - f_i^{sf})h_i \end{aligned} \qquad (25) $$

The minimum variance hedge ratio is derived by setting the first derivative, with respect to h_i of (25) to zero and solving for h_i:

$$ h_i^* = -\frac{Cov(R_i, e_i^{sf} - f_i^{sf})}{\sigma^2(e_i^{sf} - f_i^{sf})} - \frac{Cov(e_i^{sf}, e_i^{sf} - f_i^{sf})}{\sigma^2(e_i^{sf} - f_i^{sf})} \qquad (26) $$

or

$$ h_i^* = -\beta_{1i} - \beta_{2i} \qquad (27) $$

where:

h_i^* = minimum variance hedge ratio.

The two elements comprising this formula are directly estimable via the following separate regressions.

$$ R_{it} = \alpha + \beta_{1i}(e_{it}^{sf} - f_{it}^{sf}) + \varepsilon_{it} \qquad (28) $$

$$ e_{it}^{sf} = \alpha + \beta_{2i}(e_{it}^{sf} - f_{it}^{sf}) + \varepsilon_{it} \qquad (29) $$

where:

α = constant intercept term.
β = slope coefficient.
ε = a random error term with zero mean.

Two characteristics of (26) are of interest. If the variance in the forward rate is taken as zero (i.e. zero basis risk)

$$ \beta_{2i} = 1 \qquad (30) $$

and

$$ \beta_{1i} = \frac{Cov(R_i, e_i^{sf})}{\sigma^2(e_i^{sf})} \qquad (31) $$

From (7) the significance of the correlation between R_i and e_i^{sf} for minimum variance hedging becomes apparent.

If in addition to ignoring basis risk Cov(R_i, e_i^{sf}) 0, then $h_i = -1$.

If basis risk is not ignored but $Cov(R_i, e_i^{sf}) \neq 0$, then h_i^* becomes:

$$h_i^* = -\beta_{3i} \quad (32)$$

Where β_{3i} is the slope coefficient of the following regression

$$R_{it}^{sf} = \alpha_i + \beta_{3i}(e_i^{sf} - f_i^{sf}) + \varepsilon_{it}^{sf} \quad (33)$$

This simplification is used in multiple currency hedging. In order to establish the minimum variance hedge ratios for each currency in a given multi-currency portfolio the following multiple regression is used:

$$R_p^{sf} = \alpha + \beta_i(e_i^{sf} - f_i^{sf}) + \beta_j(e_j^{sf} - f_j^{sf}) + \beta_k(e_k^{sf} - f_k^{sf}).... \quad (34)$$

$$-\beta_i = h_i^* \quad (35)$$

where:
R_{pt}^{sf} = the Swiss franc return on a multi-country/currency portfolio.

6. Optimal Currency Hedging and International Asset Allocation

General Form:
The optimal allocation of assets across international equity markets and the simultaneous determination of optimal currency hedge ratios suggested in this paper is defined as the following optimisation problem:

Maximise

$$Z = W_d R_d + \sum_{i=1}^{N} W_{ei}\, {}_hR_i + \sum_{i=1}^{N} W_{ci}\alpha_i - \lambda \sum_{i=1}^{2N+1} \sum_{j=1}^{2N+1} Cov(R_i, R_j) \quad (36)$$

Subject to

$$\sum_{i=1}^{N} W_{ei} + W_d = 1 \quad (37)$$

$$0 \leq W_{ci} \leq W_{ei} \quad (38)$$

where:
W_d = weight in domestic equity market.
W_{ei} = weight in foreign equity market i, hedged.
W_{ci} = weight in currency market i.
R_d = return on domestic equity market.
${}_hR_i$ = hedged return on foreign equity market.
α_i = return on forward contract to sell currency i.
R_i = R_d, ${}_hR_i$ and α_i.
γ = investor specific risk aversion coefficient. This is equal to the reciprocal of risk tolerance.
h_i^* = $-(1 - W_{ci}^*)$ optimal hedge ratio for currency i. (39)
W^* = optimal weight for a particular asset for a given value of λ.

All returns are in the domestic numeraire (i.e. Swiss franc) however currency and time notations are omitted for convenience.

Alternative Form:

Maximise

$$Z = W_d R_d + \sum_{i=1}^{N} W_{ei}\, {}_hR_i + \sum_{i=1}^{N} W_{ui} R_{ui} - \lambda \sum_{i=1}^{2N+1} \sum_{j=1}^{2N+1} Cov(R_i, R_j) \quad (40)$$

Subject to:

$$\sum_{i=1}^{N} W_{ei} + \sum_{i=1}^{N} W_{ui} + W_d = 1 \quad (41)$$

$$0 \leq W_{ei} \quad (42)$$

$$0 \leq W_d \quad (43)$$

$$0 \leq W_{ui} \tag{44}$$

where:
W_{ui} = weight in foreign equity market i unhedged.
R_{ui} = return on foreign equity market i unhedged.
R_i = R_d, ${}_hR_i$ and R_{ui}.

$$h_i^* = -\frac{W_{ei}^*}{W_{ui}^* + W_{ei}^*} \tag{45}$$

Footnotes

[1] For a review of the international investments literature see SOLNIK (1988). See KNIGHT (1989) for a description of optimal international asset allocation.

[2] A portfolio manager is defined as active in this context if with respect to equities, assets are held in a different proportion than the implied weighting in a market portfolio and if, with respect to currencies, any currency exposure is not fully hedged.

[3] All stock market returns are adjusted for dividends.

[4] It is noted that this drop in value was larger than that experienced in October 1987 by some 3.13%.

[5] Standard deviation in monthly return is annualised by multiplying the statistic by the square root of 12.

[6] See equations (4) and (8) of the appendix.

[7] See WOLFF (1987) for a review of Forward Foreign Exchange.

[8] The array of heging instruments available and their various applications raise additional questions which will not be directly addressed here. See CELEBUSKI/ HILL/KILGANNON (1990) for a description of currency management strategies using different instruments.

[9] In this study the convention is adopted that the forward rate is at a premium (discount) when positive (negative).The forward rate represents the difference between the forward exchange rate and the spot exchange rate, expressed as a percentage of the spot exchange rate. See equation (11).

[10] Empirical evidence for the superiority of the spot rate over the forward rate as a prediction of future spot rates is presented by MEESE and ROGOFF (1984).

[11] From equation (7) in the appendix it will be seen that a hedge ratio of -1 would hedge only the original capital invested and not the return of a particular period. However as this return is relatively small and not known to the investor beforehand it is safely ignored here.

[12] Section 4 of the appendix sets out a full set of equations to describe the relationship described here.

[13] The functions illustrated in figure 8 are based on equation (25) in the absence of basis risk.

[14] The return and risk functions illustrated in figures 9 and 10 are based on equations (24) and (25) applied to the Japanese equity and currency markets.

[15] See equation (34) for the regression to estimate minimum variance hedge ratios for a given multiple currency portfolio.

[16] It must be emphasised that the portfolios presented in this paper may be seriously affected by outliers in the data and consequently the literal interpretation of these may be hazardous to your wealth. Methods of dealing with this problem have been suggested in the literature and in particular JORION (1985) and (1986) suggested a Bayes-Stein approach for estimating ex-ante expected returns for use in solving the portfolio problem. Such adjustments are outside the scope of this paper.

[17] The formal description and intuitive explanation of the Markowitzian approach is described in KNIGHT (1989).

[18] Risk aversion is a measure of an investor's attitude to risk taking. An investor with a lower (higher) than average risk aversion requires a lower (higher) return than average to induce him to take more risk. Clearly a high risk aversion implies a low risk tolerance. Throughout this paper both terms are used. See footnote [24] for a more formal explanation.

[19] The new BVG/LPP regulations restrict total foreign investments (bonds and stocks)to 30%. See ODIER/ SOLNIK/MIVELAZ (1991) for a discussion of the impact of the regulations on international asset allocation.

[20] All hedged returns in this paper have employed a hedge ratio of minus one and the forward rate implied in euro-currency one month deposit interest differentials. See equation (13) for the definition of interest rate parity. The transformation is described in equation (22).

[21] In a similar study using the German mark as the numeraire currency over the same period the minimum variance portfolio had the following allocations.

	Allocations	Hedge Ratios
France	2.98	-1
Germany	3.34	N/A
Japan	45.66	-0.61
Switzerland	32.06	0
UK	14.90	-1
	100%	

[22] Remember that alpha positive reflects the cost of hedging in return terms.

[23] The exact allocations for this portfolio with the associated hedge ratios are

	Allocations	Hedge Ratios
France	5.79	0
Germany	15.01	0
Japan	65.69	22.62
Switzerland	0	N/A
UK	9.68	3.13
US	3.83	0
	100%	

[24] Risk tolerance (*RT*) is an investor specific factor which represents his or her marginal rate of substitution of variance (risk) for return. Formally, risk tolerance is a function of the investor's degree of absolute risk aversion (*ARA*)

$RT \quad = 1/\lambda$

$\lambda \quad = - 1/2\ ARA$

Where *ARA* is the Arrow Pratt absolute risk aversion factor and λ represents a coefficient of risk aversion. See INGERSOLL (1987) pp. 37 and 38. See also equation (36) in the appendix.

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