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Portfolio Insurance with Options and Futures on the SMI

1. Introduction

The term "portfolio insurance" denotes investment strategies designed to protect a portfolio against losses without destroying the upward potential inherent in risky investments. Insurance strategies generally involve a well-defined mixture between risky and riskless investments. As with any other form of insurance, a premium has to be paid in order to get the desired protection. Portfolio insurance is widely used in the United States and other countries, especially by large institutional investors such as pension funds [1].

Portfolio insurance has been the topic of three articles published in previous issues of this journal. The basic ideas are explained in SCHWARTZ (1986/87). WYDLER (1988) contains various examples demonstrating the mechanics of this strategy using stock index futures. BENNINGA (1990) compares the performance achieved with different techniques using simulated data. Description of portfolio insurance are furthermore offered by RUBINSTEIN (1985) and ZIMMERMANN (1988, ch.7).

In the following, a number of portfolio insurance strategies are explained and illustrated with actual Swiss data for options and futures on the Swiss Market Index, SMI. Throughout, the risky investments are taken to be stocks. Risk-free opportunities would for example consist of Euromarket deposits or, more generally, bonds with the appropriate duration. The paper shows that portfolio insurance can be successfully implemented in Switzerland. It should therefore provide an attractive alternative to existing management procedures for Swiss investors.

The remainder of the paper is organized as follows. In the next section, relatively simple strategies, like stop-loss and constant proportion portfolio insurance, are briefly presented for comparison. Options and futures on the SMI, which are the most important instruments for the implementation of portfolio insurance, are discussed in section 3. Section 4 deals with portfolio insurance using options. In practice, the options required for the implementation of portfolio insurance are often not traded on organized exchanges. The alternative, discussed in section 5, is to create the options synthetically by holding a portfolio of bonds and stocks. Various possibilities of implementing insurance techniques using stock index futures are presented in section 6. Some concluding remarks complete the paper. An appendix provides a more technical presentation of the issues.

2. Simple Strategies

A well-known simple procedure for achieving the goals of portfolio insurance is a stop-loss strategy.

* The comments by Heinz Zimmermann are gratefully acknowledged.

The investor has to specify the length of the planning period and the minimal permissible value of the portfolio at the end of the planning period, called the floor. The floor, discounted to the present at the risk-free rate of interest, has to be lower than the current value of the portfolio.
The whole portfolio is then invested in the risky asset, usually a portfolio of stocks. The stocks are sold and the proceeds invested in the riskless asset if the portfolio value reaches the appropriately discounted floor over the planning period. After such a trigger point occurs, the investor foregoes all subsequently occurring profit opportunities in the stock market. The reason is that he has to remain completely invested in the riskless asset in order to guarantee that the floor is reached at the end of the planning period. This explains why the stop-loss strategy is widely viewed to be inefficient.
An equally simple but more flexible procedure is known as "Constant Proportion Portfolio Insurance" [2]. The crucial magnitude here is the cushion defined as the difference between the current value of the portfolio and the specified floor. A multiple of the cushion, also determined by the portfolio holder, is then invested in risky assets, usually stocks. The rest is held in the riskless asset. Risky investments go to zero with the cushion. The investor may change both the floor and the multiple over time. A more detailed presentation of this technique is offered in the appendix.

3. Options and Futures on the SMI

Diversified stock portfolios, riskless investments as well as options and futures on stock market indices are the major investment vehicles in more sophisticated strategies. A number of them will be illustrated in detail below with the help of examples based on actual data. Specifically, insurance strategies starting on April 28 1989 and ending on July 21 1989 will be simulated. Typically, portfolio insurance strategies have a substantially longer horizon (from one to five years). However, SMI futures data are not available to simulate portfolio insurance strategies with a long horizon. In this section, the instruments are briefly characterized.
The stock portfolio is approximated by the Swiss Market Index, SMI, which is a capitalization-weighted index of 24 stocks of large Swiss companies. In the following, it will be useful to work with the concept of an index share or SMI share. Each index point is assumed to be worth one Swiss franc. Consequently, the price of an index share is equal to the value of the index and therefore to the value of a portfolio of stocks with weights identical to those used in the construction of the index. The evolution of the SMI in weekly intervals over the period analyzed in this study is shown in table 1. The index falls over the first four weeks and then generally increases during the remainder of the period. The data therefore allow the simulation of insurance strategies in bear as well as bull markets.
Investments in the riskless asset, denoted as bonds, are required to implement portfolio insurance. In agreement with actual market conditions, the riskless rate is assumed to be 6.5% on an annualized basis over the whole period used in the examples.
Portfolio insurance based on options generally uses European put options expiring at the end of the predetermined planning period. This way, the investor acquires the right to sell his shares at a price known in advance, called the exercise price. The option expires worthless if the share price at maturity is higher than the exercise price. Table 1 contains prices for European put options on one SMI share computed using the Black-Scholes formula. The chosen exercise price of 1'710.28 is the one subsequently used in the simulations. It's determination will be explained later. The values of model parameters, listed below the table, reflect market conditions during the period examined. The time remaining to maturity of the option is also shown in table 1. Dividends are neglected because virtually no dividend payments occur over the time period considered.
Model instead of actual prices from SOFFEX are used in the following examples because the traded contract is an American put option which can be exercised any time until maturity. American puts

Table 1: Prices and Returns.

End of week	SMI	Put option on SMI [a] t(P) [b]	Price [c]	Delta [d]	SMI futures contract [e] t(F) [f]	Model price [g]	Market price [h]
0 (April 28 1989)	1'577.0	85	133.28	-0.7255	176	1'627.2	1'589.0
1	1'554.6	78	149.76	-0.7852	169	1'602.1	1'589.0
2	1'531.0	71	168.92	-0.8439	162	1'575.8	1'566.0
3	1'524.5	64	174.73	-0.8711	155	1'567.2	1'564.0
4	1'494.2	57	202.66	-0.9280	148	1'534.1	1'532.0
5	1'570.7	50	135.43	-0.8264	141	1'610.4	1'618.0
6	1'628.5	43	89.90	-0.7041	134	1'667.8	1'659.0
7	1'625.0	36	90.65	-0.7402	127	1'662.2	1'656.0
8	1'662.5	29	62.27	-0.6410	120	1'698.4	1'697.0
9	1'638.9	22	76.02	-0.7654	113	1'672.2	1'680.0
10	1'687.7	15	38.77	-0.5900	106	1'719.9	1'720.0
11	1'709.1	8	20.61	-0.4844	99	1'739.5	1'728.0
12 (July 21 1989)	1'747.5	0	0.00	0.0000	92	1'776.4	1'773.0

Notes:

a European put option on one index share as subsequently selected in table 2. Exercise price: 1'710.28.

b Number of days to maturity of the option.

c Black-Scholes model price for European put option on one index share. On SOFFEX, one index option is written on five index shares. Model parameters: Risk-free interest rate: 6.5% p.a., volatility: 21.04% p.a., dividends neglected. For week 12, the exercise value is given.

d Change of option price in S.Fr. if SMI changes by one point, e.g. [N(d1)-1] (see appendix).

e Futures written on one index share. The contracts traded by Bank Leu contained 25 index shares.

f Number of days to maturity of the futures contract.

g Model price equals SMI multiplied by $e^{0.065t(F)/365}$. Dividends are neglected.

h Actual price as quoted by Bank Leu.

are more expensive than European puts because of this early exercise option. In general, portfolio insurance is not feasible with American puts when the probability of early exercise is significant. Furthermore, contract size, striking prices and maturity dates offered by SOFFEX are likely to be different from the ones required by the strategies presented in the next sections [3].

As discussed below, the actual implementation of portfolio insurance is generally based on synthetically created options using a pricing model. According to the Black-Scholes formula, the price of the put is a positive function of the exercise price, the volatility of the return on the underlying asset and time to maturity. It varies inversely with the stock price and the riskless rate of interest.

The delta of the put is equal to the change in the option premium if the price of the underlying stock varies by one Swiss franc. Delta values implied by the Black-Scholes model are included in table 1. It will become clear in section 5 that knowledge of delta makes is possible to create synthetic options through stocks and bonds. The value of delta is a function of model parameters. Specifically, a higher stock price implies a lower absolute value of delta.

Insurance strategies are often implemented with the help of stock index futures instead of positions in a diversified portfolio of stocks. An index future enables investors to buy or sell positions in the index for future delivery at prices fixed at the inception of the contract. No cash payments are involved at that point. Funds can therefore be invested in other forms over the life of the futures contract. The use of futures allows transactions costs to be reduced because only one contract is needed to invest in a diversified portfolio of stocks, whereas holdings in a large number of individual shares would otherwise be necessary.

In the simulations, SMI futures traded by Bank Leu from January to October 1989 are used [4]. The contract chosen is the one expiring in October 1989. Maturity is therefore not the same as for the put option introduced above which ends on July 21 1989. Table 1 exhibits time to maturity as well as model and actual prices for futures on one SMI share over the period chosen for the simulations. Model prices are determined as the value of the index multiplied by one plus the risk-free interest rate until the maturity of the contract [5]. The results in table 1 indicate that the actual price was about 2.4% below the model price at the start of the period. Afterwards, differences are quite small. STULZ, STUCKI and WASSERFALLEN (1989) show that actual transactions costs preclude arbitrage profits. It will however be interesting to see whether the difference between theoretical and actual futures prices affects the results of portfolio insurance strategies.

4. Insurance Strategies with Options

An insurance strategy using options involves the combination of a risky investment, in this case stocks, with a purchase of put options on the underlying investment [6]. The options permit the investor to sell the stocks at the prespecified exercise price. A minimum value of the portfolio can thereby be guaranteed.

The strategy requires the specification of a target for the minimum value of the portfolio, called the floor, to be reached at the end of an assumed planning period. Consequently, European put options are chosen which expire at the end of the planning period. The total exercise value of the options must be equal to the floor. Furthermore, the sum of the investment in stocks and the premia paid for the options must be equal to the value of wealth when the strategy is started. The exercise price of the options can be determined using these two conditions. A more detailed but also more technical explanation is provided in the appendix.

The selection of the put option used in the numerical examples is shown in table 2. The risky investment consists of a portfolio of Swiss stocks, approximated through the SMI. The planning period extends over the twelve weeks from April 28 1989 until July 21 1989. The desired floor is set equal to the initial value of wealth, assumed to be 1'000'000 Swiss Francs. Table 2 contains examples for five different exercise prices. The option selected for the insurance program has an exercise price of 1'710.28. Given the value of the SMI on April 28 1989, 1'577.0, the put is therefore in the money by 133.28. The assumed initial wealth of 1'000'000 divided by the exercise price of 1'710.28 determines the required number of 584.7 index shares. The chosen portfolio therefore consists of 584.7 index shares at a total cost of 584.7 * 1'577.0 = 922'072 and the same number of put options requiring a total premium of 584.7 * 133.281 = 77'928. The options with other exercise prices in table 2 do not fulfill the conditions discussed above, namely that initial wealth, 1'000'000, is equal to the initial value of the portfolio, 922'072 + 77'928, and that total exercise value of the put options, 584.7 * 1'710.28, is equal to the desired floor, 1'000'000.

The evolution of wealth in weekly intervals is shown in table 3. Over the first four weeks, the value of the uninsured portfolio drops by 5.3% from 1'000'000 to 947'495. Due to the subsequently rising stock market, a terminal wealth of 1'108'117 is reached for a total return of 10.8%.

The results for the insured portfolio, consisting of 584.7 SMI shares and 584.7 put options with exercise price 1'710.28, are contained in the second part of table 3 and in figure 1. The put options are valued

Table 2: Determining the Put Option.

The value of the SMI on April 28 1989 is 1'577.0. The desired floor is equal to the initial value of wealth, e.g. 1'000'000.

Exercise price	Price of put option [a]	Number of shares [b]	Wealth [c]
1'577.00	52.227	634.12	1'033'125
1'600.00	63.313	625.00	1'025'196
1'700.00	125.698	588.24	1'001'595
1'710.28 [d]	**133.281**	**584.70**	**1'000'000**
1'750.00	164.257	571.43	995'006

Notes:

a Black-Scholes model price for European put option on one index share. On SOFFEX, one index option contains five index shares. Model parameters: Number of days to maturity: 85, risk-free interest rate: 6.5% p.a., volatility: 21.04% p.a., dividends are neglected.

b One share is equal to the value of the index. The number of shares is determined as the desired floor, 1'000'000, divided by the exercise price, implying that the total exercise price is equal to the floor.

c Wealth is defined as the number of shares multiplied by the sum of the value of the index, 1'577.0, and the price of the put option.

d The put option with exercise price 1'710.28 is selected for the insurance program because wealth is equal to the initial value of the portfolio, 1'000'000, and total exercise value is equal to the desired floor, 1'000'000.

Table 3: Evolution of Wealth.

End of week	Uninsured portfolio [a]			Insured portfolio [b]		
	Value	Weekly return [c] %	Total return [d] %	Value	Weekly return [c] %	Total return [d] %
0 (April 28 1989)	1'000'000			1'000'000		
1	985'796	-1.4	-1.4	996'533	-0.3	-0.3
2	970'831	-1.5	-2.9	993'943	-0.3	-0.6
3	966'709	-0.4	-3.3	993'540	0.0	-0.6
4	947'495	-2.0	-5.3	992'154	-0.1	-0.8
5	996'006	5.1	-0.4	997'574	0.5	-0.2
6	1'032'657	3.7	3.3	1'004'748	0.7	0.5
7	1'030'438	-0.2	3.0	1'003'141	-0.2	0.3
8	1'054'217	2.3	5.4	1'008'473	0.5	0.8
9	1'039'252	-1.4	3.9	1'002'714	-0.6	0.3
10	1'070'197	3.0	7.0	1'009'467	0.7	0.9
11	1'083'766	1.3	8.4	1'011'361	0.2	1.1
12 (July 21 1989)	1'108'117	2.2	10.8	1'021'763	1.0	2.2

Notes:

a Total wealth invested in SMI index shares.

b According to table 1, the portfolio consists of 584.7 index shares and the same number of European put options with exercise price of 1'710.28.

c Return relative to the previous week in percent per week.

d Return relative to week 0 in percent over respective period.

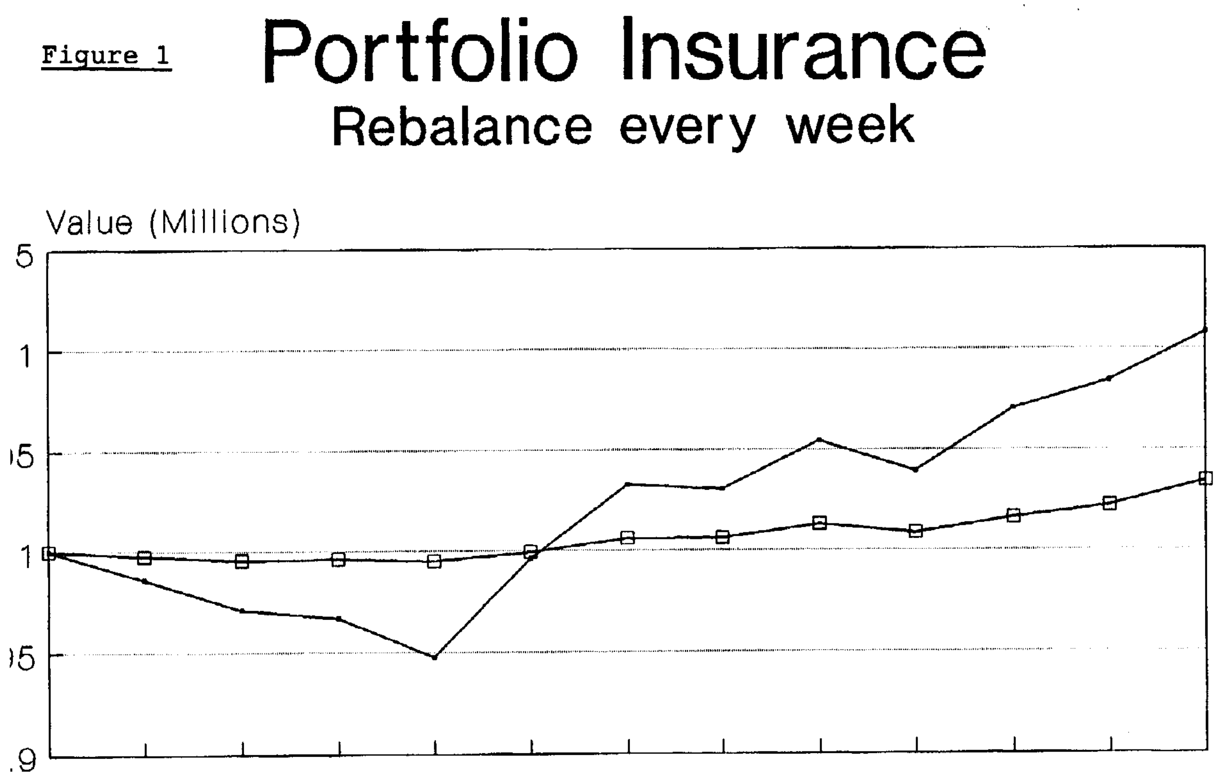
weekly using the Black-Scholes model. The protection achieved through insurance is demonstrated at the beginning of the period where wealth decreases much less than in the uninsured case. The costs of insurance do however reduce the upward potential. The rising stock market only leads to a terminal value of 1'021'763 or a total return over the twelve weeks of 2.2%. This number slightly overstates the performance because transaction costs on the option market are not taken into account.

In practice, it is generally not possible to implement portfolio insurance as demonstrated above. The reason is that the required put options are not available. Traded options are usually of the American instead of European type. The early exercise privilege is not needed but has a positive price leading to unnecessarily high costs of the insurance program. Furthermore, exercise prices, maturity dates and contract sizes of traded options are most likely inadequate. For these reasons, insurance strategies are usually carried out by creating put options synthetically. This topic is discussed in the next section.

5. Implementation with Synthetic Options

It is possible to replicate the payoff of options by holding appropriate positions in stocks and bonds. The required amounts have to be determined through an option pricing model. In the following examples the Black-Scholes formula for European put op-

Figure 1: Portfolio Insurance (Rebalance every week).



tions paying no dividends is applied. A put option on one index share is simulated by selling stocks short and lending at the riskless interest rate. The short position in stocks is equal to the delta of the put option, shown in table 1, multiplied by the current stock price. A long position in stocks forms the other part of the portfolio. The result is a portfolio consisting of positive amounts of stocks and bonds. For details the reader is again referred to the appendix.

In principle, the proportions invested in stocks and bonds have to be adjusted continously over time according to the evolution of the so-called hedge ratio implied by the option pricing model. The hedge ratio is given by one minus the delta of the put. Practically, positions are changed in discrete time intervals or when the price of stocks has moved by a prespecified percentage. Otherwise, transactions costs would lead to a significant deterioration of overall performance.

Examples of portfolio insurance through the simulation of put options are provided in tables 4 to 6. The planning period and prices are those used in the previous section. Transactions costs are not taken into account.

Table 4 shows how the portfolio evolves if rebalancing takes place at the end of every week. The relevant information to calculate the composition of the portfolio is included in tables 1 and 2, respectively. The initial stock position is 253'108 or 25.3% of starting wealth, assumed to be 1'000'000. To obtain this initial position, remember that in the previous section wealth was invested in puts and index shares. To replicate one put with positions in stocks and bonds, delta index shares must be short-sold, i.e. -0.7255, and the proceeds of the short-sale plus the price of the put must be invested in bonds. However, the investor already owns shares. Therefore, there is no need to short-sell because shares can simply be sold out of the existing portfolio. In the previous section, 584.7 puts had to be bought. Consequently, that number of puts must be replicated. To do so, a short sale of 0.7255 * 584.7 = 424.2 index shares must occur. Since 584.7 index shares are already owned, the final stock position is (1-0.7255) * 584.7 = 160.5 index shares. The value of the stock investment is therefore 160.5 * 1'577.0 = 253'108. The remainder of the initial wealth, 1'000'000 - 253'108 = 746'892, is invested in bonds.

At the end of the first week, the stock position declines to 249'513 because the SMI drops from 1'577.0 to 1'554.6 (see table 1). The value of the bonds grows at the riskless interest rate to 747'822. The last two columns of table 4 reveal that the stock position should be lowered to 195'247 and the bond position consequently increased to 802'088 at the beginning of the second week. The amount invested in stocks is determined as above. The delta of the put changes to -0.7852 and the new value of the SMI is 1'554.6. The number of index shares remains at 584.7 for the whole period. Consequently, the amount invested in stocks is (1-0.7852) * 1'554.6 * 584.7 = 195'247. Note that the strategy is self-financing, that is funds are neither added to nor taken out of the portfolio. This procedure is repeated at the end of every week.

Generally, the stock position is increased whenever stock prices grow faster than the riskless rate of interest. Table 5 contains the proportions of stocks and bonds in the portfolio. The stock position is lowered to 6.3% at the end of week 4 and then steadily increased to more than 50% because of the raising stock market.

Table 4: Portfolio Insurance with Stocks and Bonds (Weekly Rebalance).

End of week	Position at end of week [a] Stocks [b]	Bonds [c]	Wealth [d]	Position after rebalance [e] Stocks [f]	Bonds [g]
0 (April 28 1989)			1'000'000	253'108	746'892
1	249'513	747'822	997'335	195'247	802'088
2	192'283	803'088	995'371	139'737	855'634
3	139'143	856'700	995'843	114'898	880'945
4	112'614	882'043	994'657	62'903	931'754
5	66'124	932'915	999'039	159'432	839'607
6	165'299	840'653	1'005'952	281'751	724'201
7	281'145	725'104	1'006'249	246'845	759'404
8	252'542	760'350	1'012'892	348'970	663'922
9	344'016	664'749	1'008'765	224'808	783'957
10	231'502	784'933	1'016'435	404'586	611'849
11	409'716	612'612	1'022'328	515'243	507'085
12 (July 21 1989)	526'820	507'717	1'034'537		

Notes:

a European put option selected in table 1 is simulated through positions in stocks and bonds using the Black-Scholes model without dividends.

b SMI shares.

c Risk-free investment at 6.5% p.a. Interest factor for one week is $e^{0.065*7/365} = 1.00125$.

d Sum of stocks and bonds.

e Portfolio is restructured every week.

f Stock position after rebalance is equal to (1 + delta put) * number of SMI shares required for insurance program (584.7) * current stock price. The necessary information is contained in tables 1 and 2.

g Wealth minus stock position after rebalance.

Table 5: Proportions invested in Stocks and Bonds (%)[a].

End of week	Position at end of week Stocks	Bonds	Wealth	Position after rebalance Stocks	Bonds
0 (April 28 1989)			100.0	25.3	74.7
1	25.0	75.0	100.0	19.6	80.4
2	19.3	80.7	100.0	14.0	86.0
3	14.0	86.0	100.0	11.5	88.5
4	11.3	88.7	100.0	6.3	93.7
5	6.6	93.4	100.0	16.0	84.0
6	16.4	83.6	100.0	28.0	72.0
7	27.9	72.1	100.0	24.5	75.5
8	24.9	75.1	100.0	34.5	65.5
9	34.1	65.9	100.0	22.3	77.7
10	22.8	77.2	100.0	39.8	60.2
11	40.0	60.0	100.0	50.4	49.6
12 (July 21 1989)	50.9	49.1			

Notes:
a Proportions of portfolio shown in table 4.

Following this procedure, wealth reaches a value of 1'034'537 at the end of the chosen period. A comparison of tables 3 and 4 reveals that the evolution of wealth is not exactly the same if put options are actually bought instead of simulated. The reason is that the portfolio is not continously adjusted according to changes in the delta of the put.

The outcome of rebalancing the portfolio whenever the SMI has moved by plus or minus 3% is shown in table 6. In this case, readjustment occurs less frequently but with a similar result compared to weekly adjustments.

Strategies based on synthetic options have essentially three disadvantages. First, while the option pricing model generally performs well, it is not perfect. In particular, it assumes that stock prices change smoothly. If stock prices change dramatically in a very short period of time, the delta of the put becomes a poor approximation of the change in

Table 6: Portfolio Insurance with Stocks and Bonds (Rebalance whenever SMI has moved by + 3% or - 3%).

End of week	Change SMI(%) [a]	Position at end of week [b] Stocks [c]	Bonds [d]	Wealth [e]	Position after rebalance [f] Stocks [g]	Bonds [h]
0 (April 28 1989)		253'108	746'892	1'000'000		
3	-3.3	244'682	749'684	994'366	114'898	879'468
5	+3.0	118'380	881'661	1'000'041	159'432	840'609
6	+3.7	165'299	841'656	1'006'955	281'751	725'204
10	+3.6	291'993	728'820	1'020'813	404'586	616'227
12 (July 21 1989)	+3.5	418'922	617'763	1'036'685		

Notes:
a Relative change of SMI in since previous week listed in table.
b European put option selected in table 1 is simulated through positions in stocks and bonds using the Black-Scholes model without dividends.
c SMI index shares.
d Risk-free investment at 6.5% p.a. Interest factor for one week is $e^{0.065*7/365} = 1.00125$.
e Sum of stocks and bonds.
f Portfolio is rebalanced whenever the SMI changes by + 3% or - 3%.
g Stock position after rebalance is equal to (1 + delta put) * number of SMI shares required for insurance program (584.7) * current stock price. The necessary information is contained in tables 1 and 2.
h Wealth minus stock position after rebalance.

the value of the put when the stock price changes and hence the replication strategy does not perform well.

The second disadvantage has to do with the general functioning of financial markets. The investor will not be able to adjust the portfolio fast enough and with little effect on security prices if markets are not highly liquid. The stock market crash in October 1987 is the most prominent example in this respect. On Monday, October 19 1987, portfolio insurers incurred large losses because they were no longer able to execute their programs. Ironically, this situation occurred exactly when insurance was most badly needed.

The third disadvantage is associated with transaction costs which can be quite high if frequent and large adjustments are required. Stock index futures may therefore be attractive because they allow to take positions in a diversified portfolio of stocks by transacting in a single instrument. Several examples of using stock index futures are provided in the next section.

6. Implementation with Stock Index Futures

As described in STULZ, STUCKI and WASSERFALLEN (1989), the payoffs of long and short positions in stocks can be replicated by, respectively, purchasing or selling stock index futures contracts. With futures contracts, no cash payments must be made at the inception of the contract except for margin deposits that we assume to be made in the form of securities. Consequently, index futures enable an investor holding a portfolio of common stocks to change his exposure to stock market risk without changing his holdings of common stocks. Similarly, with index futures, an investor who holds bonds can acquire an exposure in stocks without purchasing stocks by simply taking a long position in index futures. Because typically futures contracts are cheap to trade relative to the underlying assets, investors benefit from changing their exposure to stocks using index futures rather than using the cash market.

The stock and bond investments required by the portfolio insurance policy in table 4 can be achieved for any investor whose wealth is fully invested in the cash market by taking appropriate positions in index futures contracts. In tables 7 to 9, three examples of portfolio insurance policies are shown that differ according to the proportion of initial wealth invested in stocks. In general, futures must be sold if a larger proportion of wealth is held in shares than implied by the insurance program in order to appropriately decrease the exposure in the stock market. Futures are bought if the opposite situation prevails. This strategy results in exactly the same evolution of wealth as in table 4 if actual futures prices are equal to the theoretical prices discussed in section 3 and listed in table 1. In the appendix, the techniques used are presented in more detail.

The problem of basis risk becomes important if actual futures prices differ from model prices. Table 1 indicates that quite substantial deviations are observed for futures on the SMI traded by Bank Leu. Market prices are below model prices for the majority of weeks analyzed in this paper. The implication is that the insurance strategy studied in tables 3 and 4 should perform better if the insurance program requires a long position in futures and vice versa. The results presented in tables 7 to 9 demonstrate that these implications are actually born out. Table 7 gives the results of using futures on the SMI if the same number of SMI shares is held as in the strategy using put options, namely 584.7. Note that considerably more shares are held than shown in table 4. At the beginning of the insurance program, at time 0, the stock position is for example 584.7 * 1'577.0 = 922'072. The resulting bond position at 0 is 1'000'000 - 922'072 = 77'928, because futures require no payments at the inception of the contract. Futures must be sold in this case. The number of futures contracts sold is equal to the delta of the put option times the number of shares required by the insurance program, 584.7, multiplied by an interest factor. The interest factor is given by $e^{-0.065*t(Fj+1)/365}$, where t(Fj+1) is the number of days from the beginning of next week to the maturity of the futures

contract. Note that the interest factor is related to the end of the rebalancing interval and not to its beginning.
This comes from the fact that the gain from holding one SMI index share over the next week is equal to the gain from holding a number of SMI index futures equal to the interest factor. Hence, to replicate the payoffs from one SMI cash share over a period of time, one needs to hold a number of futures index shares equal to the interest factor at the end of the period. At the beginning of the insurance program, at time 0, the number of futures contracts sold is therefore determined from table 1 as -0.7255 * 584.7 * 0.9704 = -411.64. The amount of Swiss Francs sold in the futures market, indicated by a minus sign in table 7, is given by the number of contracts multiplied by the market price of futures shown in table 1. At 0, 411.64 * 1'589.0 = 654'096 Swiss Francs are therefore sold in the futures market.
At the end of the first week, the stock position has decreased to 584.7 * 1'554.6 = 908'975. Theoreti-

Table 7: Portfolio Insurance with Stocks, Stock Index Futures and Bonds.

Stock position is 584.7 SMI shares, as in table 3

End of week	Position at end of week [a]			Wealth [e]	Position after rebalance [f]		
	Stocks [b]	Futures [c] (change)	Bonds [d]		Stocks [g]	Futures [h]	Bonds [i]
0 (April 28 1989)				1'000'000	922'072	-654'096	77'928
1	908'975	0	78'025	987'000	908'975	-708'805	78'025
2	895'176	+10'260	78'123	983'559	895'176	-751'696	88'383
3	891'375	+ 960	88'493	980'828	891'375	-775'885	89'453
4	873'659	+15'875	89'565	979'099	873'659	-810'658	105'440
5	918'388	- 45'507	105'572	978'453	918'388	-763'356	60'065
6	952'184	- 19'343	60'140	992'981	952'184	-667'698	40'797
7	950'138	+ 1'207	40'848	992'193	950'138	-701'581	42'055
8	972'064	- 17'370	42'108	996'802	972'064	-623'359	24'738
9	958'265	+ 6'245	24'769	989'279	958'265	-737'789	31'014
10	986'798	- 17'566	31'053	1'000'285	986'798	-582'977	13'487
11	999'311	- 2'712	13'504	1'010'103	999'311	-481'438	10'792
12 (July 21 1989)	1'021'763	- 12'537	10'805	1'020'031			

Notes:
a The insured portfolio in table 3 is simulated through positions in stocks, stock index futures and bonds.
b The stock position is 584.7 SMI shares over the whole period, as in table 3.
c Change in value of futures position based on market prices from table 1. Settlement is assumed weekly.
d Risk-free investment at 6.5% p.a. Interest factor for one week is $e^{0.065*7/365} = 1.00125$.
e Sum of stocks, change in futures and bonds.
f The portfolio is restructured every week. Total wealth is invested in stocks and bonds because futures require no payment at inception of contract.
g Number of SMI shares is the same as under b.
h The amount of futures sold (minus sign) is equal to delta put * number of SMI shares required for insurance program (584.7) * interest factor * market price of futures. The interest factor is $e^{-0.065*t(Fj+1)/365}$, where t(Fj+1) is the number of days from the beginning of next week to the maturity of the futures contract. The information is contained in tables 1 and 2.
i Bonds before rebalancing plus change in value of futures position over previous week as shown under c.

cally, the futures price should also decrease, leading to a partially offsetting gain for the seller of the contracts. This gain is then invested at the riskless interest rate for the next period. The resulting bond position for the next period is therefore composed of two parts. First, the amount invested at the beginning of the period plus interest and second the gain realized on the futures market. Equivalently, eventual losses on the futures contracts decrease the bond position for the next period. In the examples, the market price for futures does not change over the first week, meaning that basis risk resulted in a loss. Over subsequent weeks, the theoretical relationship between spot and futures prices is however fulfilled more closely.

The procedures just described are repeated at the end of every week in order to adjust the structure of the portfolio. Most importantly, the changes in the delta of the put, as shown in table 1, are taken into account. At the start of the second week, -0.7852 * 584.7 * 0.9716 = -446.07 contracts are therefore sold, resulting in 446.07 * 1'589.0 = 708'805 Swiss Francs. As in table 4, the exposure in the stock market is reduced by selling more futures if the SMI falls and vice versa. A lower value of terminal wealth, namely 1'020'031, is reached compared to table 4 because futures have on average to be sold at a discount relative to model prices.

The example shown in table 8 simulates the situation of an investor holding exclusively bonds at the start of the insurance program. In this case, futures on the SMI are bought in order to generate the necessary exposure in the stock market. The number of futures contracts needed to replicate the portfolio in tables 3 and 4 is given by (1 + delta put) times the number of SMI shares required for the insurance program (584.7) multiplied with the same interest factor as in table 7. Based on the information in table 1, (1-0.7255) * 584.7 * 0.9704 = 155.75 contracts are bought, yielding an amount of 155.75 * 1589.0 = 247'487 Swiss Francs.

Total wealth remains invested in bonds throughout the insurance program. As above, the gains and losses realized on the futures position lead to adjustments of the amount invested in bonds. During the second week for example, the investor loses 2'807 on his long position in futures of 122.03 contracts, because the futures price has declined from 1'589.0 to 1'566.0, resulting in the same decrease of wealth and bonds at the end of week 2. On July 21 1989, final wealth is 1'040'101 compared to 1'034'537 in table 4. The investor therefore gains from the fact that market prices of futures are generally somewhat lower than theoretical prices.

Table 9 shows the situation where total wealth at time 0, 1'000'000, is invested in stocks at the start of the insurance program. Given the price of 1577.0, 634.12 SMI shares are therefore held. It is shown in the appendix, that the number of futures contracts sold is determined as [(1 + delta put) * number of SMI shares required for insurance program (584.7) - number of SMI shares held (634.12)] * interest factor. The interest factor is again the same as in tables 7 and 8. Initially, [(1-0.7255) * 584.7 - 634.12] * 0.9704 = -459.60 contracts are sold, resulting in an amount of -730'304 Swiss Francs. This is slightly more than in table 7. The reason is that the investor holds more SMI shares and thus has to reduce his exposure in the stock market to a larger extent. Over the first four weeks, gains are made on the short position in SMI futures which are invested in bonds. Afterwards, losses are gradually accumulated because the stock market is rising. Therefore, funds have to be borrowed in order to finance these losses [7]. The final wealth position of 1'018'332 is about the same as in table 7.

In Switzerland, the currently lacking futures market is the most important obstacle for the practical implementation of portfolio insurance. The introduction of a futures contract on the SMI planned by SOFFEX should therefore be highly welcomed by investors.

7. Conclusions

Several portfolio insurance strategies are analyzed which have recently become popular, especially in the United States. Such techniques may also be attractive for institutional investors in Switzerland. Pension funds, which are confronted with large

Table 8: Portfolio Insurance with Stocks, Stock Index Futures and Bonds.

Stock position is equal to zero

End of week	Position at end of week [a] Stocks [b]	Futures [c] (change)	Bonds [d]	Wealth [e]	Position after rebalance [f] Stocks [g]	Futures [h]	Bonds [i]
0 (April 28 1989)				1'000'000	0	+247'487	1'000'000
1	0	0	1'001'250	1'001'250	0	+193'906	1'001'250
2	0	- 2'807	1'002'502	999'695	0	+139'045	999'695
3	0	- 178	1'000'945	1'000'767	0	+114'813	1'000'767
4	0	- 2'349	1'002'018	999'669	0	+ 62'889	999'669
5	0	+ 3'530	1'000'919	1'004'449	0	+160'360	1'004'449
6	0	+ 4'064	1'005'705	1'009'769	0	+280'603	1'009'769
7	0	- 507	1'011'031	1'010'524	0	+246'247	1'010'524
8	0	+ 6'096	1'011'787	1'017'883	0	+349'124	1'017'883
9	0	- 3'497	1'019'155	1'015'658	0	+226'145	1'015'658
10	0	+ 5'384	1'016'928	1'022'312	0	+405'112	1'022'312
11	0	+ 1'884	1'023'590	1'025'474	0	+512'456	1'025'474
12 (July 21 1989)	0	+13'345	1'026'756	1'040'101			

Notes:

a The insured portfolio in table 3 is simulated through positions in stocks, stock index futures and bonds.
b The stock position is zero over the whole period.
c Change in value of futures position based on market prices from table 1. Settlement is assumed weekly.
d Risk-free investment at 6.5 % p.a. Interest factor for one week is $e^{0.065*7/365} = 1.00125$.
e Sum of stocks, change in futures and bonds.
f The portfolio is restructured every week. Total wealth is invested in bonds because futures require no payment at inception of contract.
g Number of SMI shares is the same as under b.
h The amount of futures bought (plus sign) is equal to (1 + delta put) * number of SMI shares required for insurance program (584.7) * interest factor * market price of futures. The interest factor is $e^{-0.065*t(Fj+1)/365}$, where t(Fj+1) is the number of days from the beginning of next week to the maturity of the futures contract. The information is contained in tables 1 and 2.
i Bonds before rebalancing plus change in value of futures position over previous week as shown under c.

liabilities to be fulfilled in the future, may wish to ascertain a minimum value of their wealth without giving up the possibilities of large gains offered by the stock market.

The numerical examples based on actual Swiss data demonstrate that the implementation of these strategies is feasible in principle. The most promising procedure however involves stock index futures. A liquid market in this instrument is therefore a prerequisite for the practical use of portfolio insurance. For that reason, the planned introduction of SMI futures on SOFFEX should be highly welcomed by many investors.

Appendix

In this appendix, the portfolio insurance strategies discussed in the text are presented in a more formal way. Constant proportion portfolio insurance is treated first, followed by strategies using options.

Table 9: Portfolio Insurance with Stocks, Stock Index Futures and Bonds.

Stock position is equal to initial wealth of 1'000'000, or 634.12 SMI shares

End of week	Position at end of week [a] Stocks [b]	Futures [c] (change)	Bonds [d]	Wealth [e]	Position after rebalance [f] Stocks [g]	Futures [h]	Bonds [i]
0 (April 28 1989)				1'000'000	1'000'000	-730'304	0
1	985'796	0	0	985'796	985'796	-785'093	0
2	970'831	+11'364	0	982'195	970'831	-826'973	+11'364
3	966'709	+ 1'056	11'378	979'143	966'709	-851'160	+12'434
4	947'495	+17'415	12'450	977'360	947'495	-884'485	+29'865
5	996'006	- 49'651	29'902	976'257	996'006	-841'441	- 19'749
6	1'032'657	- 21'322	-19'774	991'561	1'032'657	-747'844	- 41'096
7	1'030'438	+ 1'352	-41'147	990'643	1'030'438	-781'698	- 39'795
8	1'054'217	- 19'354	-39'845	995'018	1'054'217	-705'562	- 59'199
9	1'039'252	+ 7'068	-59'273	987'047	1'039'252	-819'269	- 52'205
10	1'070'197	- 19'506	-52'270	998'421	1'070'197	-666'483	- 71'776
11	1'083'766	- 3'100	-71'866	1'008'800	1'083'766	-565'453	- 74'966
12 (July 21 1989)	1'108'117	- 14'725	-75'060	1'018'332			

Notes:

a The insured portfolio in table 3 is simulated through positions in stocks, stock index futures and bonds.

b The stock position is equal to 634.12 SMI shares over the whole period. This is equal to total initial wealth (1'000'000) invested in stocks at 0.

c Change in value of futures position based on market prices from table 1. Settlement is assumed weekly.

d Risk-free investment at 6.5% p.a. Interest factor for one week is $e^{0.065*7/365} = 1.00125$. Plus and minus signs denote lending and borrowing respectively.

e Sum of stocks, change in futures and bonds.

f The portfolio is restructured every week. Total wealth is invested in stocks and bonds because futures require no payment at inception of contract.

g Number of SMI shares is the same as under b.

h The amount of futures sold (minus sign) is equal to [(1 + delta put) * number of SMI shares required for insurance program (584.7) - number of SMI shares held (634.12)] * interest factor * market price of futures. The interest factor is $e^{-0.065*t(Fj+1)/365}$, where t(Fj+1) is the number of days from the beginning of next week to the maturity of the futures contract. The information is contained in tables 1 and 2.

i Bonds before rebalancing plus change in value of futures position over previous week as shown under c. Plus and minus signs denote lending and borrowing respectively.

A.1 Constant Proportion Portfolio Insurance

The portfolio of value V at the beginning of the insurance program (= time 0), V_0, is invested in a diversified portfolio of stocks and bonds. The bonds are assumed to be riskless. Therefore,

$$V_0 = H_0 + B_0 \qquad (A.1)$$

where H is the total value of shares and B the amount of bonds.

A cushion, C, is defined as the difference between the current value of the portfolio, V, and its exogenously determined minimal value, called the floor, Fl. Formally

$$C_0 = V_0 - Fl \qquad (A.2)$$

The investment in stocks is a multiple of the cushion, e.g.

$$H_0 = m_0 C_0 \quad (A.3)$$

Both the floor and the multiple can be changed over time. If the portfolio value reaches the floor, total wealth is invested in bonds. Borrowing and short-selling constraints can be introduced in addition. Due to transaction costs, it is not optimal to change the stock position continously. In practice, a tolerance level with respect to the change in H or equivalently in C needs to be defined. Transaction costs can also be lowered if the strategy is implemented with stock index futures instead of a portfolio of individual shares.

A.2 Portfolio Insurance with Options

The basic methodology of using options in the context of a portfolio insurance program is discussed first, followed by the implementation with European call options, synthetic put options and stock index futures under A.2.2, A.2.3 and A.2.4, respectively.

A.2.1 Basic Methodology

For strategies using options, the investor has to define a minimal value of the portfolio, the floor Fl, and a planning period of length t, extending from the present, denoted by 0, to maturity, T. The floor is the minimal acceptable wealth at maturity. Wealth, V, is invested in the risky asset, consisting of a diversified portfolio of stocks, and European put options on that portfolio with maturity T. The stock portfolio is expressed in index shares. In this paper, the index is the SMI. Current wealth is therefore

$$V_0 = n\, S_0 + n\, P_0 \quad (A.4)$$

where n is the number of index shares with price S_0 equal to the value of the index at 0. P is the price of the put option on one index share with exercise price K.
The floor is defined as a proportion f of initial wealth. In the text, f is assumed to be one. The goal of the strategy is

$$Fl = f\, V_0 \quad (A.5)$$

Equation (A.4) implies that the value of the portfolio at the end of the planning period is

$$V_T = n\, S_T + \text{Max}\,(n\, K - n\, S_T, 0) \quad (A.6)$$

If S_T is larger or equal to K, the puts expire worthless and V_T is equal to n S_T. If S_T is below K, V_T equals n K. Hence, for V_T to be at least Fl, it must be that K = Fl/n. It follows that

$$Fl = n\, K = f\, n\, (S_0 + P_0) \quad (A.7)$$

Using the Black-Scholes formula for P_0, equation (A.7) can be solved for K. Note that the price of the put depends positively on its exercise price. Equation (A.7) is used to determine n and K through numerical procedures or trial and error, as shown in table 2. For a solution to exist, f must be smaller than e^{rt}, where r is the risk-free interest rate and e = 2.7183. The reason is that an insurance strategy cannot earn the continously compounded risk-free interest rate for certain without giving up all the upward potential in the stock market.
Equation (A.4) implies that wealth evolves from period to period, denoted by j, according to

$$V_j = n\, (S_j + P_j) \quad (A.8)$$

Portfolio insurance can be implemented in a number of alternative ways, using call options, stocks, bonds and stock index futures. The respective examples presented in the text are discussed below in a more technical way under A.2.2 to A.2.4.

A.2.2 Implementation with Call Options

The first alternative is to replace the European put with a European call option with the same exercise price and the same maturity. Put-call parity gives

$$P_0 = C_0 - S_0 + K\, e^{-rt} \qquad (A.9)$$

Replacing P_0 by equation (A.4) implies that initial wealth will be invested in call options and bonds, B_0, as follows

$$V_0 = n\, C_0 + B_0 \qquad (A.10)$$

with $B_0 = n\, K\, e^{-rt}$.

A.2.3 Implementation with Synthetic Put Options

The second alternative is to create the put option synthetically through a combination of stocks and bonds. The respective investments are implied by an option pricing model. In the present case, the Black-Scholes formula for European puts without dividends is used. The put price is thus given by

$$P_0 = [N(d1)_0 - 1]\, S_0 + [1 - N(d2)_0]\, e^{-rt} K \qquad (A.11)$$

N(x) is the probability that a value of x or less is drawn from a standard normal distribution and

$$d1 = [\ln(S_0/K) + (r + 0.5\,\sigma^2)]\, t \,/\, \sigma\, t^{1/2}$$

$$d2 = d1 - \sigma\, t^{1/2}$$

ln is the natural logarithm and σ the annualized standard deviation of the continously compounded return on the underlying asset, in this case the SMI. Values for the delta of the put, given by [N(d1) - 1], are included in table 1. The delta is always negative. The hedge ratio N(d1) as well as N(d2) change over time as a function of the parameters of the option pricing model. Note further that N(d1) is equal to [1 + delta put].

The two parts of the sum in equation (A.11) are the investments in stocks and bonds respectively which are necessary to replicate the put option. Introducing equation (A.11) in equation (A.4) yields the desired portfolio composition

$$V_0 = n\, S_0 + n\, P_0 = n\, N(d1)_0\, S_0 + B_0 \qquad (A.12)$$

where $B_0 = n\,[1 - N(d2)_0]\, e^{-rt} K$ can be determined as a residual.
At the end of the first period, wealth, V_1, is equal to

$$V_1 = n\, N(d1)_0\, S_1 + e^{rt(A)/365}\, B_0 \qquad (A.13)$$

where t(A) is the length of the interval before rebalancing occurs, measured in days. In the examples, t(A) is seven days.
The new structure of the portfolio at the beginning of the next period has to be

$$V_1 = n\, N(d1)_1\, S_1 + B_1 \qquad (A.14)$$

where B_1 is again determined as a residual.

Equations (A.13) and (A.14) imply that the portfolio has to be rebalanced according to

$$n\, S_1\, [N(d1)_1 - N(d1)_0] = -\,[B_1 - e^{rt(A)/365}\, B_0] \qquad (A.15)$$

Rebalancing is approximately self-financing if t(A) is not too large. In the examples, the above procedure is repeated every week.

A.2.4 Implementation with Stock Index Futures

Insurance programs are usually implemented with a diversified portfolio of stocks. The strategy outlined in the previous section therefore requires frequent trading in shares of many companies whenever the portfolio is rebalanced according to equation (A.15). Transactions costs can be considerably lowered by leaving the initial stock position intact and by carrying out the insurance program with

stock index futures. That way, a diversified portfolio of shares can be bought or sold for future delivery with only one transaction. The required futures position is determined as shown below.
The goal of the strategy is to replicate the portfolio in equation (A.12). Total wealth can be invested in stocks and bonds because futures require no cash payments at the inception of the contract. Therefore

$$V_0 = k\,S_0 + D_0 \quad (A.16)$$

with the initial bond position, D_0, being determined as a residual equal to $(V_0 - k\,S_0)$. The number of shares, k, remains unchanged until maturity of the insurance strategy, T. Wealth at the end of the first interval before rebalancing occurs is

$$V_1 = k\,S_1 + e^{rt(A)/365}\,D_0 + x_0\,(F_1 - F_0) \quad (A.17)$$

F is the futures price and x is the number of futures contracts. x is positive for purchases and negative for sales.
Futures prices are determined through the no arbitrage condition (dividend payments are neglected)

$$F_j = e^{rt(Fj)/365}\,S_j \quad (A.18)$$

where t(Fj) is the number of days to maturity of the futures contract at the beginning of period j. Consequently

$$\begin{aligned} F_1 - F_0 &= e^{rt(F1)/365}\,S_1 - e^{rt(F0)/365}\,S_0 \\ &= e^{rt(F1)/365}\,(S_1 - e^{rt(A)/365}\,S_0) \quad (A.19) \end{aligned}$$

Combining equation (A.14) and (A.17), the replication of the insurance strategy implies that

$$\begin{aligned} &k\,S_1 + e^{rt(A)/365}\,D_0 + x_0\,(F_1 - F_0) \\ &= n\,N(d1)_0\,S_1 + e^{rt(A)}\,B_0 \quad (A.20) \end{aligned}$$

Solving equation (A.20) for x_0 yields the necessary number of futures contracts to be bought or sold initially.

$$x_0 = [(n\,N(d1)_0 - k)\,S_1 + (B_0 - D_0)\,e^{rt(A)/365}] / [(S_1 - e^{rt(A)/365}\,S_0)\,e^{rt(F1)}] \quad (A.21)$$

D_0 can be replaced by $V_0 - k\,S_0$. Afterwards, equation (A.12) is substituted for V_0 and the resulting expression is simplified. The solution for x_0 becomes

$$x_0 = [n\,N(d1)_0 - k]\,e^{-rt(F1)/365} \quad (A.22)$$

Note that the interest factor, $e^{-rt(F1)/365}$, is related to the end of the rebalancing period and not to its beginning. Obviously, $t(F_1)$ is equal to $t(F_0)$ if the adjustment of the portfolio occurs continously over time.

The futures position in monetary units at time 0 then is

$$\begin{aligned} x_0\,F_0 &= [n\,N(d1)_0 - k]\,e^{-rt(F1)/365}\,F_0 \\ &= [n\,N(d1)_0 - k]\,e^{rt(A)}\,S_0 \quad (A.23) \end{aligned}$$

The profits or losses resulting from the futures position become part of the investment in bonds for the next period, implying that the investment in bonds after rebalancing the portfolio is

$$D_1 = e^{rt(A)}\,D_0 + x_0\,(F_1 - F_0) \quad (A.24)$$

The procedure given by equations (A.16) to (A.24) is subsequently used each period in order to determine the evolution of wealth and its components. The examples given in tables 7 to 9 are based on values for k equal to n, 0 and V_0/S_0 respectively.
An additional problem posed by futures is basis risk, meaning that theoretical futures prices, as given by equation (A.18), diverge from actual ones, denoted by G. The actual gains and losses from the futures position are therefore $x_0\,(G_1 - G_0)$. Basis risk is $x_0\,(G_1 - G_0) - x_0\,(F_1 - F_0)$ which can alternatively be written as $x_0\,(G_1 - F_1) - x_0\,(G_0 - F_0)$. A reliable forecast of $G_1 - F_1$ makes it possible to reduce basis risk. For example, if $G_1 - F_1$ is expected to be zero, the part $x_0\,(G_0 - F_0)$ can be eliminated by investing in $x_0\,F_0/G_0$ instead of x_0 futures contracts. In the examples, this adjustment is not made. For week j, the

futures position is therefore $x_j G_j$. Empirically, the resulting differences are however small.

Footnotes

[1] An institutional investor can implement portfolio insurance directly or can contract with an agent to insure his portfolio. The stock market crash of 1987 reduced dramatically the amount of funds under contract with agents, partly because of the poor performance of some of these agents.

[2] BLACK and JONES (1987) provide a more detailed description.

[3] The contract size of traded options on the SMI is five index shares. Striking prices are fixed in intervals of 50 index points. Maturities up to six months in the future are available. SMI options are described in detail by DUBACHER and ZIMMERMANN (1989).

[4] The index futures traded by Bank Leu contained 25 index shares. A detailed analysis of SMI futures, including the contract used in this study, is provided by STULZ, STUCKI and WASSERFALLEN (1989).

[5] Dividends on the shares included in the index are again neglected.

[6] Exactly the same strategy can be implemented by combining bonds with call options on the SMI. The appendix provides the details of this approach.

[7] Any difference between the lending and the borrowing rate is ignored.

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