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Arbitrage Conditions for Option Pricing on the SOFFEX

1. Introduction

On May 19, 1988, the Swiss Options and Financial Futures Exchange (SOFFEX) started its activities. Option contracts are presently traded on 13 underlying stocks and on the Swiss Market index (SMI)[1]. As it is the case for other similar markets , arbitrageurs are expected to play a fundamental role. Because of their activities, one can hypothesize that option prices must meet the boundary conditions derived by STOLL (1969), MERTON (1973b), KLEMKOSKY and RESNICK (1979) and COX and RUBINSTEIN (1985). If these conditions are not satisfied, arbitrages can be made.

As shown by PHILLIPS and SMITH (1980), arbitrageurs' transaction costs are not negligible. Hence, hedging strategies by efficient traders cannot disregard these costs. Clearly, market inefficiencies would result when persistent above normal hedging profits after transaction costs are detected. This paper examines the boundary conditions on call prices and the call-put parity relationships for the second quarter of activity on the SOFFEX market, the first three and a half months being considered as a starting period during which the market operators learned how to trade more efficiently on this new market. Because a complete set of data on bid and ask prices was not available, tests were conducted with intradaily observations of market prices. Thus, an arbitrage seen to be profitable under these circumstances would not necessarily remain profitable if bid-and-ask prices were used instead of market prices, particularly when the lack of liquidity on the market results in wide bid-and-ask spreads. Section 2 presents the arbitrage relationships tested in the paper. The market structure and the data used are described in Section 3. Testing methodology and strategies are explained in Section 4. Section 5 contains the empirical results of both ex-post and ex-ante tests with or without transaction costs. Finally, Section 6 offers some concluding remarks. This study investigates the impact, if any, of several variables such as option maturity, dividends and prices of underlying stocks on the frequency and size of option prices violations.

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2. Arbitrage Relationships

2.1 Lower Bounds for American Call Options

MERTON (1973a) proved several necessary conditions for rational pricing of options (calls and puts) in perfect markets. These conditions define boundaries for option values and do not rely on a specific process for the prices of the underlying stock nor on

a particular evaluation model for options. The lower boundary conditions were expanded and generalized by GALAI (1978) for American calls with no compensating adjustments if dividends are paid to the stockholders during the life of the option, which is the situation for the SOFFEX options.
The first lower bound for an American call option is that its value cannot be less than the larger of zero or the stock price minus the striking price, that is the value of the call if exercised immediately:

$$C(S, T, X) \geq \max\,[0, S-X], \qquad (1)$$

where C(S,T,X) is the market value of the American call option with exercise price X and time to maturity T and S is the current market price of the underlying stock. GALAI (1978) refers to the above condition as the "weak dominance condition". The second lower bound condition, the "strong European call dominance condition", assumes that the call will be held to maturity. If it is known with certainty that the stock will pay n dividends D_j at calendar times t_j over the life of the option, then:

$$C\,(S, T, X) \geq \max\left[0, S-Xe^{-rT} - \sum_{j=1}^{n} D_j\, e^{-rt_j}\right], \qquad (2)$$

where r is the default-free interest rate. A porfolio containing the call and several discount bounds with face values respectively equal to the exercise price and to each dividend paid during the remaining life of the option is preferred to a portfolio with a long position in the stock, since the call is only exercised when the stock price at the expiration date is greater than the exercise price.
A third condition, the "strong early exercising dominance condition", considers the fact that it can also be optimal to exercise an unprotected American call only just before the stock goes ex-dividend. The value of the call should never be less than the larger of 0 or the stock price minus the present value of the exercise price minus the present value of the dividends paid during the remaining life of the option:

$$(S, T, X) \geq \max \left\{ 0, \max_j \left[0, S-Xe^{-rt_j} - \sum_{i<j} D_i e^{-rt_i}\right] \right\} \qquad (3)$$

The rationality behind this bound is the same as for the preceeding bound, the only difference being the date of exercise.

2.2 Calls Convexity Condition

Call prices are convex in the striking price for identical maturity options on the same underlying stocks, as shown by MERTON (1973a) and GALAI (1979). For α defined by $X_2 = \alpha X_1 + (1-\alpha)\, X_3$ with $0 < \alpha < 1$ and $X_1 < X_2 < X_3$ being striking prices, we should have:

$$C(S, T, X_2) \leq \alpha\, C(S, T, X_1) + (1 - \alpha)\, C(S, T, X_3). \qquad (4)$$

Note that these call prices depend on the same stock price as simultaneity of the option prices must be observed.
The convexity of a call value as a function of its exercise price may be explained by the incidence of a one-franc increase in the striking price on the probability to exercise the call. When the striking price is very low relative to the stock price, a one-franc increase in the striking price is expected to have a greater effect in reducing this probability than when the striking price is high. For increasingly higher striking prices, each franc increase should have a smaller and smaller impact on the call price as the probability to exercise the call becomes lower and lower.

2.3 Put-Call Parity

Assuming that it is optimal to hold to maturity both the put and the call options to make up a hedge, KLEMKOSKY and RESNICK (1979) derived the following boundary conditions:

$$C(S, T, X) \leq P(S, T, X) + S - Xe^{-rT} - \sum_{j=1}^{n} D_j e^{-rt_j}$$

(long hedge condition) (5)

$$P(S, T, X) \leq C(S, T, X) - S + Xe^{-rT} + \sum_{j=1}^{n} D_j e^{-rt_j}$$

(short hedge condition) (6)

According to these authors, a sufficient condition for no premature exercise of the call is that the present value of the expected dividends is less than the present value of the return that could be realized from investing the exercise price at the risk-free rate:

$$\sum_{j=1}^{n} D_j e^{-rt_j} < X[1-e^{-rT}]. \quad (7)$$

As noted by KLEMKOSKY and RESNICK, this condition is equivalent to that of ROLL (1977) under perfect market assumptions [2]. In contrast, rational premature exercise of the put is always possible. If at inception,

$$C(S, T, X) < X\left[1-e^{-rT}\right] - \sum_{j=1}^{n} D_j e^{-rt_j} \quad (8)$$

it would be to the put holder's advantage to exercise the put immediately against the hedger. It is also possible that this inequality fails to hold at inception but holds later with an interim call price. Hence, it is impossible to establish a perfect short hedge since one is not able to assume that the put would never be prematurily exercised.

The above conditions (5) and (6) respectively define a lower and a higher bound for the put. However, it is possible to derive tighter bounds as shown for instance by COX and RUBINSTEIN (1985). These new bounds are given by the two following conditions:

$$C(S, T, X) \leq P(S, T, X) + S - Xe^{-rT}, \quad (5')$$

$$P(S, T, X) \leq C(S, T, X) - S + X + \sum_{j=1}^{n} D_j e^{-rt_j}. \quad (6')$$

The possible violations of call-put parity relationships are examined using the two sets of conditions (5) and (6) or (5') and (6'), as explained hereafter [3] (Section 4.3).

3. Market Structure and Data

3.1 Market Structure

Options traded on the SOFFEX over the studied period are dividend payout unprotected American options on stocks [4]. If dividends are paid to owners of the underlying stocks during the lives of options, no adjustments are made on the striking price. Each option contract represents rights to 5 shares of the underlying stock. At any time, options on the same underlying stock (options class) are available with 4 expiration months, the longest maturity an option can have being 6 months [5]. In its expiration month, an option normally expires on the Saturday immediately following the third Friday of the month.

For each option class and each maturity, there is a minimum of 3 striking prices. When a new maturity is proposed, 3 different striking prices are introduced based on the last closing price of the underlying stock on the Zurich Exchange. For an option series with a particular expiration date, when the closing price of the underlying stock on the Zurich Exchange is smaller (or greater) than the smallest (or greatest) striking price during two consecutive business days, a new smaller (or greater) striking price is introduced, except when the maturity of this option series is less than 3 weeks.

The SOFFEX trading is organized around two basic groups: brokers and market makers. Brokers act both as agents and as principals. They receive and execute customers' orders. They are also authorized to trade for their own account. Market makers are responsible for providing liquidity in options of underlying stocks assigned to them. It is their duty to maintain reasonable bid-and-ask prices throughout the hours of trading. They have no ability to accept orders from the public. Furthermore, in meeting their market responsabilities or during their hedging activity, market makers are faced with transaction costs, including the expenses of the bid-ask spread. Unfortunately, these latter costs are ignored in this study because informations concerning them were not included in our data base. However, one can expect that they are smaller than those incurred by other market operators.

When examining the efficiency of the SOFFEX, one must consider the above normal profits earned by market makers, who are the traders incurring the lowest trading costs. Supposing that market makers cannot obtain any above normal gains after transaction costs with hedges based on violations of boundary conditions or put-call parity relationships, hence it is a fortiori impossible for other traders with higher trading costs to realize abnormal gains on the same market.

The direct transaction costs for a market maker who can trade directly on the Swiss Stock Exchanges are SF 0.50 per option contract, plus SF 8 each time an option is exercised, in addition to taxes equivalent to 0.09 % of the underlying stock price at which the trade occurs [6].

3.2 Data

This study covers 68 trading days from August 29, 1988 to November 30, 1988. The data consist of three sets of 90192 intradaily observations made up respectively of call, put and underlying stock market prices that were in effect on the SOFFEX at the end of each 15 minutes intervals of time starting at 12:00 a.m. until 2:00 p.m. [7]. In this manner, option prices and stocks prices are matched together. Dividends used are actual dividends instead of estimates. The Eurofranc rate and the Swiss domestic monetary rate matching with the option maturities are both used for the risk-free rate [8]. It should be noted that, during the period studied, the market closed around 2:00 p.m. and trading on stocks was not systematically interrupted as is presently the case between 12:00 a.m. and 1:00 p.m.. Therefore, there is no evidence that the market was much less liquid between 12:00 a.m. and 2:00 p.m. than before noon, even if one can conjecture that the volume of each daily trade was important near the opening time of the SOFFEX.

4. Tests and Strategies

4.1 Ex-post and Ex-ante Tests

Ex-post tests of boundary conditions, convexity conditions or call-put parity relationships state that the price of an option is not less than the computed rational boundary. It is assumed that the observed violations and the corresponding hedging strategies are based on the same prices. For this reason, these ex-post tests are not valid for testing market efficiency since the observed prices at t are not necessarily the prices for the next transactions. Persistent violations can only indicate that prices of options and underlying stocks are not well synchronized or are not equilibrium prices.

When studying market efficiency, a relevant question is whether an arbitrageur obtains abnormal arbitrage profits by trading at time t+1 given the information at time t. For ex-ante tests, any violation registered at time t is taken as a signal for a transaction that is assumed to be executed at time t+1. There is no guarantee that a profit identified at t will imply an abnormal return at t+1 since the market prices generally change between t and t+1. Trading based on information at time t will result in a risky position at time t+1 [9]. Thus, hedging becomes a risky activity and, when a hedge is profitable, it is difficult to distinguish between the normal profit which represents a normal reward for this risk, and the abnormal gain which can be associated with market inefficiency.

4.2 Boundary and Convexity Conditions

For an immediate exercise, the tested hypothesis is:

$$\varepsilon_1 \equiv S - X - C - M \leq 0, \qquad (9)$$

where M is the transaction cost. To profit from a mispriced call, hence inducing $\varepsilon_1 > 0$ (type 1 violation), the strategy consists of buying the underpriced call, exercising it immediately and selling the stock thus acquired.
The tested hypothesis for the strong European condition is:

$$\varepsilon_2 \equiv S - Xe^{-rT} - De^{-rt} - C - M \leq 0, \qquad (10)$$

provided that the stock will be subject to no more than one dividend payment D at time t. If $\varepsilon_2 > 0$ (type 2 violation), the profit strategy consists of buying the call, shorting the stock and lending an amount equal to the sum of present values of the exercise price and the expected dividend, and holding this portfolio until expiration.
The tested hypothesis related to the strong dominance condition may be written as:

$$\varepsilon_3 \equiv S - Xe^{-rt} - C - M \leq 0, \qquad (11)$$

where $\varepsilon_3 > 0$ (type 3 violation). The profit strategy is analogous to the preceeding one except that if premature exercise is optimal, the hedge is terminated at t instead of T.
The call convexity test is:

$$\varepsilon_4 \equiv C(X_2) - \alpha C(X_1) - (1 - \alpha)\, C(X_3) - M \leq 0. \qquad (12)$$

When type 4 violations occur ($\varepsilon_4 > 0$), the butterfly spread for obtaining profits is to sell one contract of the call with the striking price X_2 and buy α contracts with the lowest exercise price X_1 and 1-α contracts with the highest exercise price X_3.

4.3 Call-Put Parity Relationships

When call prices are too high relative to put prices, an arbitrageur can set up a profitable long hedge according to KLEMKOSKY and RESNICK's condition (5) by selling a call, assuming a long position in the underlying stock, buying a put and financing in part the position by borrowing at the risk-free rate the amount $Xe^{-rT} + De^{-rt}$, where D is the dividend payment at t. To prevent such a profitable strategy, the following condition must hold:

$$\varepsilon_5 \equiv C - P - S + Xe^{-rT} + De^{-rt} - M \leq 0. \qquad (13)$$

Using the long hedge condition (5'), the amount borrowed is only Xe^{-rT}, which implies the following inequality (13') instead of (13):

$$\varepsilon'_5 \equiv C - P - S + Xe^{-rT} - M \leq 0. \qquad (13')$$

In case many type 5 violations of the above two conditions are observed (ε_5 or $\varepsilon'_5 > 0$), hedging profits are guaranteed if the condition for no premature exercice of the call holds at inception for (13) and (13').
The conversion of a call into a put is used if the put price is too high relative to the call price (type 6 violation), providing that the put will never be prematurely exercised. As stated by KLEMKOSKY and RESNICK, this short hedge consists of selling the put, purchasing the call, shorting the underlying stock and lending at the risk-free rate an amount, $Xe^{-rT} + De^{-rt}$. This strategy is unprofitable if:

$$\varepsilon_6 \equiv P - C + S - Xe^{-rT} - De^{-rt} - M \leq 0. \qquad (14)$$

With the boundary conditon (6'), an unprofitable short hedge is defined by :

$$\varepsilon'_6 \equiv P - C + S - X - De^{-rt} \leq 0. \qquad (14')$$

In contrast to a call, possible rational premature exercise of a put may not be determinable at inception. For this reason, it is impossible to establish a perfect short hedge and test options market efficiency using (14) or (14') except when it is verified that the put will never be prematurely exercised for all interim sets of values. Consequently, the short hedge profit calculated at inception is only conditional.

5. Results

5.1 Lower Boundary Conditions

5.1.1 Ex-post test

For each of the 90192 registered market call prices, ex-post hedges were constructed using conditions (9) and (10). There are no violations (type 1) of the immediate exercise condition. However, 2293 violations (type 2) of the European dominance condition are detected before trading costs. When dividends are paid to owners of underlying stocks during the remaining lives of call options (48840 observations), no ex-post violations (type 3) of the early exercising dominance condition occur. After trading costs, no profitable type 2 violations are detected. These results are summarized in table 1. The absence of type 1, 2 and 3 violations after costs and the presence of only a small number of type 2 violations before costs show that option prices are generally within their rational boundaries. Consequently, the hypothesis that the call prices used in hedges are synchronized with their underlying stock prices cannot be rejected although these prices are not necessarily fully synchronized, as shown by our data base.

The preeminence of type 2 violations was previously described for Canadian transaction data by HALPERN and TURNBULL (1985) who reported that 58% of their observed violations occured in the type 2 category. By expressing type 2 in terms of type 1 violations, they showed that there will tend to be more type 2 than type 1 violations, expecially for long maturity options. This explanation cannot fully account for results presented in table 2, since before trading costs, only 20,4% of the calculated ex-post type 2 violations occur for options with maturity of over 60 days.

In table 2 are examined the amounts and frequencies of type 2 before costs violations with respect to maturity, dividends and in-the-money category [10]. As there is no definition of deep-in-the-money, it was decided to consider a call as deep-in-the-money if the stock price was more than 20% above the

Table 1: Number and Average Amounts of Type 2 Violations Classified by Type of Test, Costs and Dividends.

Type of Test	Number of Violations			SF Average Amount of Violations		
Ex-post	Without Dividends	With Dividends	Total	Without Dividends	With Dividends	Total
Before costs	1070	1223	2293	6.9	1.6	4.1
After costs	0	0	0	0	0	0
Ex-ante						
Before costs						
Profits	870	1006	1876	7.2	1.6	4.2
Losses	60	62	122	-17.0	-10.6	-13.8
Total	**930**	**1068**	**1998**	**5.63**	**0.88**	**3.1**
After costs	0	0	0	0	0	0

Table 2: Number (followed by Percentage) of Ex-post Type 2 Before Costs Violations Classified by Dividends, Number of Days to Maturity, and In-the-Money Category.

	Without dividends							With dividends						
SF Amount of Violation	0-5		6-10		over 10		Total	0-5		6-10		over 10		Total
Number of days to maturity	DI	I	DI	I	DI	I		DI	I	DI	I	DI	I	
00-07	92 (8.6)	259 (24.2)	0	0	0	0	351 (32.8)	163 (13.4)	607 (49.6)	0	0	0	0	770 (63.0)
08-30	6 (0.6)	85 (7.9)	0	12 (1.1)	0	0	103 (9.6)	48 (3.9)	216 (17.7)	0	0	0	0	264 (21.6)
31-60	0	111 (10.4)	0	62 (5.8)	0	19 (1.8)	192 (18.0)	0	73 (6.0)	9 (0.7)	42 (3.4)	16 (1.4)	5 (0.4)	145 (11.9)
over 60	21 (2.0)	55 (5.1)	39 (3.6)	63 (5.9)	19 (1.8)	227 (21.2)	424 (39.6)	20 (1.6)	18 (1.5)	3 (0.2)	3 (0.2)	0	0	44 (3.5)
Total	119 (11.2)	510 (47.6)	39 (3.6)	137 (12.8)	19 (1.8)	246 (23.0)	1070 (100)	231 (18.9)	914 (74.8)	12 (0.9)	45 (3.6)	16 (1.4)	5 (0.4)	1223 (100)

Notes:
DI denotes Deep-in-the-money.
I denotes In-the-money

exercise price. The largest frequency of before costs violations is obtained when calls are in-the-money (83.4% when no dividends are paid and 78.8% when dividends are paid). In addition, hedging profits per contract are in majority less than or equal to SF 5 (58.8% when no dividends are paid and 93.7% when dividends are paid). Observed type 2 violations are more important for long maturity hedges with no dividends remaining than for short maturity hedges with dividends.
These results differ from those of GALAI (1978) who found that the number of violations increased for deep-in-the-money and short maturity options. However, given the absence of profitable violations after costs, it is of little interest to extrapolate guidelines for hedging based on prices, time to maturity and dividends.

5.1.2 Ex-ante Test

Without transaction costs and based on signals given by ex-post gains at time t, 1998 hedging strategies were constructed using t+1 prices. Profits are obtained for 1876 hedges before costs with an average profit of SF 4.2 per contract although losses occur in other cases (see table 1). However, there are no profitable hedges after transaction costs. Thus, it is clear that, for the studied period, a market maker using ex-post profits before instead of after transaction costs as signals for the construction of a hedge would have undoubtly incurred large losses. The absence of positive returns associated with ex-ante risky hedging strategies are in favour of the market efficiency hypothesis.
The impact of dividends, time to maturity and in-the-money category of options on the average magnitude and frequency of ex-ante type 2 viola-

tions is not reported in detail here since no marked differences between ex-ante and ex-post hedges were found. Moreover, such a comparison may mean very little due to the fact that after trading costs, all profits disappear.

5.2 Call Convexity Tests

Among the triplets of calls written on the same underlying stock and with identical maturity, 2661 type 4 violations (6.1% of the sample) were found before transaction costs, this number falling to 785 after transaction costs. In any case, when trading costs are included, the number of type 4 violations seems to be more important and profitable than violations detected for other boundary conditions. The ex-ante tests confirm these results since it is observed that before costs 1803 hedges generate profits, a number which turns out to be 563 after trading costs [11].

In type 4 violations, around 50% of calls are in-the-money but less than 10% are out-of-the money, and more than 60% have a life to maturity between 8 and 30 days. The only marked difference between hedges with or without dividends is the average magnitude of profits, which is higher when dividends are to be paid during the remaining lives of calls. Before costs, the higher percentage of violations is associated with arbitrage profits less than or equal to SF 5 per contract. After trading costs, profits are concentrated between SF 6 and SF 30 per contract.

Contrary to our previous results, these type 4 violations may appear to be a source of profitable ex-ante arbitrages, during the period covered by this study. An arbitrageur who had relied on ex-post violations after costs in order to build up hedges, would have obtained after costs profits in 82% of the hedging strategies [12], market prices being used. With respect to the convexity condition and assumptions concerning the evaluation of the transaction costs, the SOFFEX could have presented some minor inefficiencies, the ratio of after costs profitable ex-ante hedges to the total number of convexity conditions examined being rather small (1.3%). This differs from the findings of GALAI (1979) and BHATTACHARYA (1983) who reported practically no type 4 violations for the CBOE call options. However, it must also be emphasized that the use of

Table 3: Number (Followed by Percentage) and Average Magnitude of Convexity Violations Classified by Type of Test.

SF Amount of Violation	0-5	6-15	16-30	31-50	51-75	over 75	Total	SF Average Magnitude of Violation
Type of Test								
Ex-post								
Before costs	1270 (47.7)	588 (22.1)	398 (15.0)	200 (7.5)	116 (4.4)	89 (3.3)	2661 (100)	16.9
After costs	203 (25.9)	258 (32.9)	173 (22.0)	80 (10.2)	47 (6.0)	24 (3.0)	785 (100)	22.2
Ex-ante								
Before costs	749 (41.6)	414 (23.0)	302 (16.7)	163 (9.0)	101 (5.6)	74 (4.1)	1803 (100)	19.6
After costs	111 (19.7)	181 (32.1)	145 (25.8)	67 (11.9)	41 (7.3)	18 (3.2)	563 (100)	23.5

the bid-and-ask prices would have normally reduced the number of profitable hedges.

5.3 Call-Put Parity

All tests were conducted using conditions (13) and (13') for long hedges as well as (14) and (14') for short hedges. To show the impact of the conditions used, all the results obtained using KLEMKOSKY and RESNICK hedges are reported between parenthesis.

5.3.1 Ex-post Tests

Because of editorial constraints, no table is provided to detail the results obtained with these tests. Out of the 38251 (45422) potential profitable long hedges identified, 1305 (6372) were eliminated because the condition for no premature exercice of the call failed to hold at inception [13]. The average gain before trading costs for each of the 36946 (39050) remaining hedges is SF 63.5 (SF 72.2) per contract. The majority of these arbitrage profits before trading costs [23270 (23561) hedges representing 63% (60.3%) of the sample] are, however, less than or equal to SF 40 per contract. The price range does not seem to have a deterministic effect on type 5 violations, despite the finding that 70.4% (68.4%) of the violations occur for underlying stock prices less than or equal to SF 3500 with a surge reaching a maximum of 8896 (9603) hedges for prices between SF 1500 and SF 2000. For the sample studied, dividends do not explain the observed violations since 98.6% (93.2%) of the profitable long hedges do not include any dividends. Moreover, the percentage of violations is higher for options with time to maturity between 30 and 60 days, equaling 33.4% (32.3%). When trading costs are introduced, 14915 (16614) hedges still have profits after costs, the average gain per contract being SF 87.9 (SF 97.4). In addition, profits follow the same pattern as before trading costs with respect to dividends, option's remaining life and price range of the underlying stock prices.

The total number of short conversions was 14374 (44020) and 4882 (9549) short hedges were eliminated because it was in the advantage of the put holder to exercise it immediately at inception. Since our data covered only three months, it was impossible, as described by KLEMKOSKY and RESNICK (1980), to verify if this condition does or does not hold for the beginning of each interim week and each hedge until the expiration date of the options. In considering that the put will not be prematurily exercised to cause a destruction of the hedge, profits obtained from conversion strategies tend to be overevaluated for the 9492 (34471) remaining short hedges. The average gain per hedge before costs is SF 131.8 (SF 106.8).
Slightly more than 50% of the short hedging profits are less than or equal to SF 40 per hedge. Moreover, 62.8% (57.4%) of the profits occur when underlying stock prices are less than or equal to SF 3500 with a surge reaching 2689 (6223) profitable hedges in the SF 1500 - SF 2000 stock price range. In addition, the percentage of short hedging profits is 95.9% (94.8%) when no dividends are paid and 51.5% (44.3%) when the option's expiration time ranges between 14 and 60 days. After trading costs, 4390 (15782) short hedging returns remain positive with an average profit of SF 225.5 (SF 152.8) per contract. Moreover, violations follow the same pattern as before trading costs with respect to dividends, lives of the options and price range of the underlying stocks.
Thus, it is clear that by using the more accurate conditions (13') and (14'), instead of KLEMKOSKY and RESNICK's conditions, a smaller number of type 5 and 6 violations is produced, especially with short hedges. However, the average gains as well as the incidence of the stock price range, dividends and term to maturity on the number of violations are very similar in both sets of conditions.

5.3.2 Ex-ante Tests

When ex-ante arbitrages are based on positive expost hedging returns after trading costs, the 14915 (16614) ex-post long hedges provide 12749 (14556)

Table 4: Distribution of After Costs Ex-Post Profitable Long Hedges According to Ex-Ante Long Hedges.

SF Amount of Ex-Post Violation / SF Amount of Ex-Ante Profits or Losses	0-40		41-100		101-500		501-1000		over 1000		Total	
	NB	KR	NB	KR	NB	KR	NB	KR	NB	KR	NB	KR
under -120	3*	52	3	29	8	34	0	2	0	0	14	117
-120 -- 41	48	229	11	62	13	39	0	0	0	0	72	330
- 40 - 0	1478	1596	87	110	32	36	0	0	0	0	1597	1742
0 - 40	4236	4412	622	730	44	41	0	0	0	0	4902	5183
41 - 100	600	708	1705	1816	352	365	1	1	0	0	2658	2890
101 - 500	38	46	359	375	2867	3419	40	100	0	0	3304	3940
501 -1000	0	0	0	0	45	104	107	200	9	9	161	313
over 1000	0	0	0	0	0	0	12	12	29	29	41	41
Total	6403	7043	2787	3122	3361	4038	160	315	38	38	12749	14556

Notes:
NB (New Bounds) denotes hedges based on condition (13').
KR denotes Klemkosky and Resnick hedges
* 3 is the number of after costs ex-ante losses greater than SF 120 per contract for which the corresponding profitable after costs ex-post profits are less than or equal to SF 40 per contract. All other values in this table are computed in the same way

ex-ante long hedges from which 11066 (12367), or 86.8% (85%), are profitable and 1683 (2189), or 13.2% (15%), are unprofitable (see table 4). As is the case before costs (not reported here), unprofitable hedges are concentrated in small losses: 94.9% (79.6%) are less than or equal to SF 40 per contract and generally originate from small ex-post gains since 90.8% (85.7%) of these profits are less than or equal to SF 40 per contract.
As for ex-post hedges, gains before and after trading costs continue to follow the same pattern. The percentage of these profitable hedges before and after costs is, respectively, 70.6% and 64.5% (68.5% and 61.5%) for stock prices less than or equal to SF 3500, 98.7% and 97.3% (93.2% and 87.2%) in the SF 0 dividend class as well as 53.2% and 51% (54.8% and 49.8%) when the option's expiration time is comprised between 30 and 90 days (see table 5 for after costs long hedges). When ex-post profitable short hedges after costs serve as signals for trading, ex-ante after costs short hedges are profitable in 3282 cases or 90.6% of the sample (11270 or 83% of the sample) and unprof-itable in the remaining 341 cases or 9.4% of the sample (2316 or 17% of the sample). Around 77% of the ex-ante unprofitable hedges originate from ex-post gains less than or equal to SF 40 per contract (table 6).
Almost all profitable short hedges after costs are without dividends. Time to maturity tends to be a little longer for short hedges than for long ones (table 7).
Based on the present study, no clear relationship can be established between dividends, number of weeks to expiration, stock price range and the amount and frequency of type 5 and 6 violations, which is consistent with the previous works of KLEMKOSKY and RESNICK (1979, 1980).
The use of conditions (13') and (14') instead of (13) and (14) stated by KLEMKOSKY and RESNICK decreases the number of hedges and violations,

Table 5: Number of After Costs Ex-Ante Long Hedges Classified by Amount of Profits or Losses, Stock Price Range, Dividends and Time to Expiration.

Factors \ SF Amount of Profits or Losses	under 120		(120-41)		(40-0)		0-40		41-100		101-500		301-1000		over 1000		Total	
	NB	KR	NB	KR	NB	KR	NB	KR	NB	KR	NB	KR	NB	KR	NB	KR	NB	KR
Stock Price Range (SF)																		
250-500	0	0	0	0	99	103	276	276	52	52	4	4	0	0	0	0	431	435
1000-1500	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
1500-2000	0	3	3	33	467	537	1967	2220	336	533	132	177	0	0	0	0	2905	3503
2000-2500	0	4	1	7	64	69	212	212	144	144	85	85	0	0	0	0	506	521
2500-3000	3	12	3	45	274	295	755	755	578	578	486	486	8	8	0	0	2107	2179
3000-3500	1	5	1	65	443	453	1016	1016	409	409	403	403	0	0	0	0	2273	2351
over 3500	10	93	64	180	250	285	676	703	1139	1174	2194	2785	153	305	41	41	4527	5566
Total	14	117	72	330	1597	1742	4902	5183	2658	2890	3304	3940	161	313	41	41	12749	14556
Value of Dividend (SF)																		
0	14	78	72	266	1588	1645	4886	4872	2592	2580	3048	3047	161	161	41	41	12402	12690
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	12	1	54	1	269	20	229	69	115	0	0	0	0	91	679
72,5	0	39	0	52	8	43	15	42	46	81	187	778	0	152	0	0	256	1187
75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
108	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total	14	117	72	330	1597	1742	4902	5183	2658	2890	3304	3940	161	313	41	41	12749	14556
Time to Expiration (days)																		
0-7	0	0	0	1	15	15	54	54	10	10	0	0	0	0	0	0	79	80
8-14	0	2	4	7	41	45	248	248	130	130	75	75	0	0	0	0	498	507
15-21	1	4	2	4	119	117	419	412	266	266	315	315	27	27	12	12	1161	1157
22-30	2	9	5	10	177	178	690	690	525	525	694	694	43	43	14	14	2150	2163
31-60	0	9	21	67	510	535	2073	2084	732	767	518	629	0	0	0	0	3854	4091
61-90	5	33	21	154	475	512	929	944	429	467	762	1031	21	21	0	0	2642	3162
91-120	2	12	7	17	137	147	312	312	314	322	98	371	0	0	0	0	870	1181
120-150	1	35	12	42	88	120	143	215	201	300	637	694	70	70	15	15	1167	1491
151-180	3	13	0	28	35	73	34	224	51	103	205	131	0	152	0	0	328	724
Total	14	117	72	330	1597	1742	4902	5183	2658	2890	3304	3940	161	313	41	41	12749	14556

Notes : NB (New Bounds) denotes hedges based on condition (13'). KR denotes Klemkosky and Resnick hedges.

Table 6: Distribution of After Costs Ex-Post Profitable Short Hedges According to Ex-Ante Short Hedges.

SF Amount of Ex-Ante Profits or Losses \ SF Amount of Ex-Post Violation	0-40		41-100		101-500		501-1000		over 1000		Total	
	NB	KR	NB	KR	NB	KR	NB	KR	NB	KR	NB	KR
under - 120	3*	47	2	30	5	35	2	3	1	3	13	118
- 120 - - 41	22	322	9	91	12	62	2	2	1	1	46	478
- 40 - 0	236	1405	31	228	14	85	1	2	0	0	282	1720
0 - 40	557	2519	122	688	23	156	2	4	2	1	706	3368
41 - 100	121	681	351	1393	124	599	0	4	0	0	596	2677
101 - 500	29	151	130	597	1351	3517	47	102	1	3	1558	4370
50 - 1000	2	2	4	6	51	106	229	510	24	20	310	644
over 1000	0	1	0	0	6	8	27	24	79	178	112	211
Total	970	5128	649	3033	1586	4568	310	651	108	206	3623	13586

Notes:
NB (New Bounds) denotes hedges based on conditions (14').
KR denotes Klemkosky and Resnick Hedges.
* 3 is the number of after costs ex-ante losses greater than SF 120 per contract for which the corresponding profitable after costs ex-post profits are less than or equal to SF 40 per contract. All other values in this table are computed in the same way.

particularly that of ex-ante profitable short hedges. The number of violations and the amount of profits after costs may be explained respectively by the number of hedges built and by the stocks price ranges included in hedges. However, the percentage of after costs ex-ante violations is around 30% for long hedges and 33% for short hedges with both sets of conditions [(13) and (14) or (13') and (14')]. These percentages are higher than those reported by KLEMKOSKY and RESNICK (20% and 14% for long and short hedges respectively) but compare favorably with the percentage of long hedges violations found by TRAUTMANN (1989) for the Frankfurt Options Exchange.

During the first months of its activity, the SOFFEX seems to have been less efficient with the call-put parity relationships than with the other arbitrage conditions studied. However, the number of after costs profitable ex-ante hedges would have most likely diminished considering that arbitrages would have been effectively realized using bid-and-ask prices, bid-and-ask spreads being presumably important in a rather illiquid market. Moreover, as explained earlier, it is a priori difficult to determine what is an abnormal profit since hedging is a risky activity, the prices at which hedges are realized being unknown at decision time.

6. Conclusion

During the period covered by this study, no evidence was obtained from the ex-post tests of the lower boundary conditions for the rejection of the hypothesis stating that option prices are synchronized with those of the underlying stocks, even if it could not be asserted that synchronization was perfect. However, the number and the amount of observed call-put parity relationships violations lead to the suggestion that the SOFFEX market could have contained some inefficiencies in its first age. Unfortunately, the significance of this study is

Table 7: Number of After Costs Ex-Ante Short Hedges Classified by Amount of Profits or Losses, Stock Price Range, Dividends and Time to Expiration.

SF Amount of Profits or Losses / Factors	under 120		120-41		40-0		0-40		41-100		101-500		501-1000		over 1000		Total	
	NB	KR	NB	KR	NB	KR	NB	KR	NB	KR	NB	KR	NB	KR	NB	KR	NB	KR
Stock Price Range (SF)																		
250-500	0	0	0	0	17	113	33	160	0	0	0	0	0	0	0	0	50	273
1000-1500	0	0	0	1	0	0	0	1	1	0	10	11	0	0	0	0	11	13
1500-2000	2	2	2	30	121	423	192	1276	229	522	641	899	13	24	0	0	1200	3176
2000-2500	0	6	0	6	21	64	21	130	32	96	51	111	0	0	0	0	125	413
2500-3000	2	12	4	34	17	254	128	365	158	418	247	439	9	20	0	0	565	1542
3000-3500	0	1	7	81	51	334	232	658	88	582	113	352	0	5	0	0	491	2013
over 3500	9	97	33	326	55	532	100	778	88	1059	496	2558	288	595	112	211	1181	6156
Total	13	118	46	478	282	1720	706	3368	596	2677	1558	4370	310	644	112	211	3623	13586
Value of Dividends (SF)																		
0	13	78	46	421	277	1635	686	3240	530	2529	1330	3798	309	556	112	211	3303	12468
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	7	5	31	20	46	66	43	228	279	1	6	0	0	320	412
72.5	0	40	0	50	0	54	0	82	0	105	0	293	0	82	0	0	0	706
75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
108	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total	13	118	46	478	282	1720	706	3368	596	2677	1558	4370	310	644	112	211	3623	13586
Time to Expiration (days)																		
0-7	0	0	0	0	14	28	29	49	20	20	5	6	0	0	0	0	68	103
8-14	0	0	3	15	48	67	106	218	33	54	75	77	0	0	0	0	265	431
15-21	2	3	7	18	52	106	183	347	86	202	249	283	53	83	7	14	639	1053
22-30	1	4	12	31	70	145	210	304	164	379	551	642	134	162	75	110	1217	1857
31-60	2	14	5	151	44	403	48	681	164	299	399	651	17	49	11	11	690	2259
61-90	6	31	7	95	34	387	89	422	61	397	204	638	88	179	10	45	499	2194
91-120	1	13	5	33	7	240	8	555	5	478	4	796	0	0	0	0	30	2115
120-150	1	42	2	68	7	223	25	428	49	446	49	756	0	92	1	3	134	2058
151-180	0	11	5	70	6	121	8	284	14	402	22	521	18	79	8	28	81	1516
Total	13	118	46	478	282	1720	706	3368	596	2677	1558	4370	310	644	112	211	3623	13586

Notes: NB (New Bounds) denotes hedges based on condition (14'). KR denotes Klemkosky and Resnick Hedges limited because of the short sample period, the use of market prices instead of bid-and-ask prices and the inability to report all prices quoted during a trading day. Further research is presently conducted in order to examine the influence of the above factors on the results obtained in this paper.

Footnotes

[1] This article examines options trading in 12 underlying securities, contracts being traded on the BBC security only since November 17, 1988. An additional stock (Alusuisse) was recently introduced on May 19, 1989. Furthermore, options on the Swiss Market Index (SMI) are traded since December 7, 1988. The SMI is a value weighted market index constructed with a total of 24 securities including the 13 underlying stocks of the SOFFEX.

[2] The MERTON and ROLL (MR) sufficient condition for no premature exercise of a call is similar to inequality (7) if the discount factor for dividends applicable between the j^{th} ex-dividend date and the j^{th} dividend payment date in the MR condition is introduced under perfect capital market assumptions.

[3] The same conditions (7) and (8) are used in both cases to emphasize the link between the number of profitable violations and the set of conditions (5) and (6) or (5') and (6') used.

[4] Common Stocks and Certificates of Participation.

[5] The present month, the two following months and the next month in the calendar choosed among a January/April/July/October expiration cycle. For example, in March, trading occurs for options which expire at the end of March, April, May and July.

[6] Transaction costs (except those expressed by the bid-ask spread) are estimated for each arbitrage according to the nature of the transaction and the price of the underlying securities, contrary to KLEMKOSKY and RESNICK (1979, 1980) and BHATTACHARYA (1983) who used average costs.

[7] We only record the last market price quoted in the last 15 minutes for each option and underlying stock. In certain cases, because of the unavailability of underlying stock prices, the arithmetical mean of the last bid-and-ask underlying security prices, supplied by Telekurs, was calculated.

[8] Because results obtained with both rates are similar, only those calculated with the Eurofranc rate, in use by the market makers on the SOFFEX, are reported in this paper. Moreover, it is assumed that a year is made up of 360 days. Calculations computed considering 365 instead of 360 days did not yield significantly different results.

[9] It is assumed that a decision to make a transaction at time t is executed 15 minutes later with the market prices prevailing at that time (see footnote 7).

[10] 41667 calls are out-of-the-money but none of these calls are part of type 2 violations.

[11] According to condition (12), trading costs depend both upon the proportions α and $(1-\alpha)$ of the two calls bought and the terminal conditions for each hedge. Here, it is assumed that the proportions are both equal to 0.5 and the purchased calls are both exercised. The number of after costs profitable hedges found varies when these assumptions are modified.

[12] Using ex-post profitable violations as signals to build up hedges, the 2368 before costs possible hedges are split respectively into 1803 profitable and 565 unprofitable ex-ante hedges. The numbers of profitable and unprofitable ex-ante hedges turn out to be respectively 563 and 126, after costs.

[13] In these cases, earned profits can be computed assuming that long hedges are maintained until ex-dividend dates. For example, in the case of KLEMKOSKY and RESNICK long hedges, 44% of the 6372 hedges eliminated with condition (7) were in fact profitable with an average profit before costs amounting to SF 45.8.

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