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European Currency Options: The Effects of Volatility Changes

1. Introduction

In recent years, markets for foreign currency options have developed around the world. These new markets can be used to hedge foreign currency risks in a manner that is not easily available in forward or futures markets. For an example, consider a corporation that will receive a significant portion of its earnings in foreign currency from a foreign subsidiary or from foreign operations. The corporation's earnings, stated in the domestic currency, are exposed to exchange rate risk. The company can hedge this risk by selling an appropriate amount of the foreign currency in the forward market, but they sacrifice the gain from an increase in the value of the foreign currency. An alternative is to buy a put option on the foreign currency, and the corporation effectively purchases insurance against a decrease in the value of the foreign currency. Put options can be expensive, but call options on the same currency can be sold with the proceeds being applied to the purchase of the puts. The combination of buying a put and selling a call at the same strike price merely duplicates a short forward position, but we can modify this strategy by selling calls with higher strike prices (at a higher rate of domestic currency to foreign currency). This strategy requires a smaller initial investment, provides the put protection, and allows the corporation to keep some of the gains if there is an increase in the value of the foreign currency [1]. The option markets allow us to achieve risk-return tradeoffs that are not available in the forward or futures markets.

Traders in option markets must be able to value different options and most traders use option pricing models. For foreign currency options, the most popular model is the modified Black-Scholes (BS) formula which has been developed by GARMAN and KOHLHAGEN (1983), GRABBE (1983), and BIGER and HULL (1983). In this option pricing model, the volatility of exchange rate changes is assumed to be constant, but traders revise and update their volatility estimates every day. In this paper [2], we examine the effects of volatility changes on the prices of European currency options by using a random variance (RV) option pricing model that has been recently developed by HULL and WHITE (1987a) and SCOTT (1987). We compare the prices generated by the BS model and the RV model with actual prices of foreign currency options traded in Geneva. Results are presented on the effectiveness of hedging price and volatility risk and on the profitability of trading with the RV model. HULL and WHITE (1987b) have shown that hedging price and volatility risk can be important for financial institutions that write customized foreign currency options for their clients. European options represent a unique opportunity for studying the performance of alternative pricing models because most option pricing models are designed to price European options that can be exercised only at

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maturity. The options traded in the United States are primarily American options that can be exercised prior to maturity, and the early exercise feature on American foreign currency options has added value.

The paper is organized as follows: In the next section, the option pricing models that are relevant for this study are presented. The empirical implementation of the various models is discussed in section 3. Data and empirical estimates are presented in section 4. Section 5 contains the results for hedging currency risk. The paper concludes with a summary of the main findings.

2. The Option Pricing Models

We present first the modified BS model for foreign currency options. In the model, the spot exchange rate is assumed to be a random walk with a lognormal distribution:

$$\Delta \ln S = \mu \Delta t + \sigma \Delta z_1,$$

where S is the spot exchange rate, Δt is a small time increment, and Δz_1 is a normally distributed random variable with a mean of zero and a variance equal to Δt. The foreign and domestic interest rates, r_f and r_d, are assumed to be constant and σ, the volatility parameter, is also assumed to be constant. The modified BS formula for a European call on a foreign currency is

$$C(S,t) = S_t \exp(-r_f(T-t))N(d_1) - X\exp(-r_d(T-t))N(d_2),$$

where

$$d_1 = \frac{\ln(S_t/X) + (r_d - r_f + (1/2)\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

and N(x) is the standard normal distribution function. T represents maturity, (T-t) is time to maturity, and X is the strike price. The foreign interest rate enters the model because we earn the foreign interest rate whenever we take a long position in the foreign currency. Without the foreign interest rate, the model is identical to the usual BS model. Prices for European puts can be easily obtained via the put-call parity theorem:

$$P(s,t) = C(S,t) + \exp(-r_d(T-t))X - \exp(-r_f(T-t))S_t.$$

With the BS model, we are assuming that changes in foreign exchange rates are lognormally distributed, but recent empirical tests by BOLLERSLEV (1987) and WASSERFALLEN and ZIMMERMANN (1986) indicate rejection of this lognormal model. One explanation for the rejection of the lognormal model is the possibility that the variance of $\Delta \ln S$ changes randomly. To incorporate randomly changing variance rates, we apply the results of recent papers by HULL and WHITE (1987a) and SCOTT (1987). In the RV model, we have a second equation to describe random changes in volatility σ as follows:

$$\Delta \ln\sigma = \beta(\alpha - \ln\sigma)\Delta t + \gamma \Delta z_2.$$

To keep the model from becoming too complicated, we assume that the two interest rates are fixed and that volatility changes are uncorrelated with changes in the exchange rate. In the CHESNEY and SCOTT (1989) paper, we show that the price of a European call is given under these assumptions as

$$C(S,\sigma,t) = E(S_t \exp(-r_f(T-t))N(d_1) - X\exp(-r_d(T-t))N(d_2))$$

where

$$d_1 = \frac{\ln(S_t/X) + (r_d - r_f)(T-t) + (1/2)V}{\sqrt{V}}$$

$$d_2 = d_1 - \sqrt{V}$$

and $V = \int_t^T \sigma^2(s)\,ds$, which is the variance of $\Delta \ln s$ over the life of the option. The term inside the

brackets is the BS formula with V in place of $\sigma^2(T\text{-}t)$. V is a random variable and the RV price is the expected value of the BS formula taken over the possible values for the volatility of the exchange rate over the life of the option. Technically there is a risk adjustment that needs to be performed on the volatility process, but we find that the RV model does a better job of pricing European currency options without the risk adjustment.
Unlike the BS model, the RV model does not produce an exact formula for calculating option prices, but there are several numerical methods available. In this paper we use the method of Monte Carlo simulation. The random variable V is simulated by simulating σ over discrete time intervals from t to T. Each value of V is plugged into the formula and the procedure is repeated many times. The average value, or the sample mean, converges to the expected value as the number of simulations gets large. There are several techniques available for improving the accuracy of a Monte Carlo simulation and these are discussed, within the context of option pricing models, by BOYLE (1977). Another approach is to apply an analytic approximation. We have recently found that a mean-variance approximation produces a convenient formula that is very accurate.

3. Pricing European Currency Options with the BS and RV Models

An important consideration for both the BS model and the RV model is the estimation of parameter inputs. For the BS model, we need to estimate the volatility parameter. In the RV model, we need to estimate the current value of σ, as well as the values of the parameters in the volatility process. The degree of mean reversion, the rate at which volatility tends to move back to some long-run average, has an important effect on the pricing of options in the RV model. We consider first the parameter estimation for the volatility process. The continuous time volatility process, shown in the previous section, implies the following model for $\ln \sigma_t$ at discrete points in time:

$$\ln\sigma_t = \alpha(1-e^{-\beta}) + e^{-\beta}\ln\sigma_{t-1} + \varepsilon_t,$$

and the variance of ε_t is a function of β and γ. For estimation purposes we rewrite the model as follows:

$$\ln \sigma_t = a + \rho \ln \sigma_{t-1} + \varepsilon_t,$$

which is a familiar first order autoregressive process. The discrete time process is the one that is used in the Monte Carlo simulation for option prices. For this process, we need to estimate three parameters: a, ρ , and the variance of the error term. We use a method of moments estimator by calculating a set of sample moments from daily changes in the log of the spot rate. The details of the estimator are contained in the paper by CHESNEY and SCOTT (1989). Since we are pricing Swiss options on the \$, we use daily data on the Swiss franc-dollar exchange rate from November 1979 to December 1983. The sample size is approximately 2000 trading days and covers more than four years. The options that we price with these estimates are for the year 1984. The parameter estimates (with standard errors in parentheses) are as follows:

a	=	-.1045	(.0777),
ρ	=	.9790	(.0157), and
$Var(\varepsilon_t)$	=	.005246	(.004026).

Next, we need to consider estimation of σ, the current level of volatility, for both models. In the BS model with σ fixed, one can use data on exchange rate changes (Δ ln S) to compute a sample variance as an estimate for σ^2. It is well known that these historical volatility estimates do not perform well in pricing options, and we present some evidence in the next section to confirm this observation. Traders form their own current estimates and revise these each day. A common approach among finance researchers is to let the option market tell us the current level of volatility; these implied standard deviations (ISD's) are computed by finding the value of σ that gives the best fit between the BS model and actual option prices on a given day. Our approach is to use at-the-money call options and minimize the sum of squared errors between the model prices and actual prices. In the RV model, the variance is changing and sample variances (historical volatilities) are merely estimates of the average level of volatility over some previous period. To estimate the current level of σ for the RV model, we also use the ISD's by finding the σ that produces the best fit between the RV model and actual prices. The ISD calculation for the RV model requires substantial computing time on ordinary computers, and for this study we have used a Cray supercomputer. Calculation of ISD's with an analytical approximation for the RV model would require a substantially smaller amount of computing time.

Finally, we need to mention the number of simulations necessary to compute accurate RV prices via the Monte Carlo method. With the Monte Carlo method, one can calculate a sample variance for the simulations and compute 95% confidence intervals (± 2 standard errors). Using the antithetical variate method which is described in BOYLE, we can compute RV prices with only 1000 simulations and the largest 95% confidence intervals are only ± .03 centimes. In Table 1 for a representative set of currency options, we present RV prices with the standard errors for the simulations.

4. Data and Empirical Results

Both models are used to price calls and puts on the dollar-Swiss franc exchange rate and the model prices are compared with the bid-ask quotes for European calls and puts traded in Geneva. Our data set consists of prices on foreign currency options traded by Credit Suisse First Boston (CSFB) Futures Trading. We use the same data that was used by CHESNEY and LOUBERGÉ (1987). CSFB writes and buys calls and puts denominated in Swiss francs on the spot rate of the U.S. dollar. The options can be exercised only at maturity (the third Wednesday of March, June, September, or December), which means that they are of the European type. The option prices are quoted in Swiss centimes per dollar and the value of each contract is $50,000. The striking prices are quoted in Swiss francs per dollar, and the interval between two striking prices is five Swiss centimes per dollar.

Data for the year 1984 were collected from Finanz und Wirtschaft and we use three sets of data. The first set includes prices of call options and put options on U.S. dollars quoted in Geneva at 2:00 P.M. on Tuesdays and Fridays. CSFB Futures Trading communicates these prices to Finanz und Wirtschaft. Typically 18 different call and put prices are quoted. We have bid and ask prices corresponding to three striking prices (in-, at-, and out-of-the money) for the next three standardized maturities. The second data set is the spot price of the currency (U.S. dollar) quoted on the same days at the same time (2:00 P.M.) on the foreign exchange market. These prices are also communicated to Finanz und Wirtschaft by CSFB Futures Trading. The third data set includes the Eurodollar and Euro-Swiss franc rates observed on the same days at 11:00 A.M. These interest rates are official middle rates for maturities of one, two, three, six, and twelve months communicated to Finanz und Wirtschaft by CSFB. The relevant interest rates for our currency options are computed by interpolating between the two closest interest rates [3].

We compare the model prices to the bid-ask quotes from CSFB and we calculate the size of the deviations outside the bid-ask spread. Many of the theoretical prices fall within the bid-ask spread and the corresponding deviations are zero. We have 101 days of prices for 1984 and the total number of calls and puts is 1574. We calculate both the mean squared error and the mean absolute deviation for each model. The analysis includes the RV model with the mean reverting process discussed in the previous section and a RV model in which ln σ follows a random walk. The random walk for volatility has $a = 0$ and $\rho = 1$, and our historically based sample estimate for $Var(\varepsilon_t)$ is .1315, with a standard error of .2052. Our first test is a comparison of the RV model and the BS model, using

Table 1: Call Option Prices and Standard Errors in the Random Variance Option Pricing Model (Monte Carlo Simulation, 1000 Simulations).

Strike Price = 2.00 Swiss Franc/dollar
$r_d = 0.09$ per year
$r_f = 0.11$ per year

Spot Rate (S.Fr./Dollar)	Time to Maturity	Option Price (S.Fr./Dollar)	Standard Error
1.80	30	.0665	.0016
	60	.3386	.0051
	90	.6607	.0079
	120	.9829	.0096
	150	1.2934	.0104
	180	1.5880	0110
	210	1.8711	.0115
	240	2.1389	.0116
	270	2.3904	.0118
2.00	30	3.6383	.0041
	60	4.8876	.0075
	90	5.7347	.0101
	120	6.3964	.0117
	150	6.9416	.0127
	180	7.4025	.0134
	210	7.8113	.0142
	240	8.1726	.0146
	270	8.4923	.0149
2.20	30	19.5978	.0024
	60	19.6281	.0066
	90	19.6507	.0098
	120	19.7222	.0115
	150	19.7949	.0125
	180	19.8608	.0131
	210	19.9256	.0138
	240	19.9827	.0141
	270	20.0290	.0144

historical estimates for the fixed parameters in both models. In the BS model, σ is a fixed parameter and we use the sample standard deviation calculated from $\Delta \ln S$ for the last six months of 1983. Because it is possible that we do not have a good sample estimate of σ, we also examine the BS model using a constant σ, computed as the average of the ISD's which are calculated from the BS model and revised every trading day. Our second test is a comparison of the RV model with the BS model in which the ISD is revised every trading day. The BS model with revised estimates of volatility is the model that is regularly applied, and one can view it as a less expensive approximation for the RV model.

The results for all five models are contained in Table 2. First, we compare the RV model with the BS model using either an historical σ or a constant σ. These applications of the BS model perform very poorly: the mean squared error and the mean absolute deviation are several orders of magnitude grea-

ter. We also find that the RV model with a mean reverting volatility process performs better than the RV model in which ln σ is a random walk.
For our second test we compare the RV model and the BS model with an ISD that is revised daily. We find that the BS model with a changing ISD outperforms the RV model: for the RV model, the root mean squared error is 1.5 times greater and the mean absolute deviation is roughly twice as large, but the differences in Swiss centimes are not large. We have two possible interpretations for the superior performance of the BS model with ISD's revised daily. One, the BS model may serve as a good approximation for the true underlying model. The other explanation is that the market maker and the traders are using variations of the BS formula with daily revisions in the variance rate.

Table 2: Call Options and Put Options on the Swiss Franc/ Dollar Exchange Rate, 1984, (1574 Options).

Pricing Errors as Difference Between Model Prices and the Bid-Ask Spread.

	Mean Squared Error	Mean Absolute Deviation
Random Variance Model, (Mean-reverting ln σ process)	0.125	0.204
Random Variance Model, (Random walk for ln σ)	1.431	0.895
Black-Scholes Model (ISD revised daily)	0.056	0.104
Black-Scholes Model (Historical σ)	21.384	3.128
Black-Scholes Model (Constant σ)	24.725	3.151

Note:
Whenever the model price falls within the bid-ask spread, the pricing error is zero.

5. Hedging Price and Volatility Risk and Trading with the RV Model

In this section we examine the effectiveness of hedged trading strategies implied by the RV model. An additional insight gained from RV option pricing is the need to hedge against price (exchange rate) and volatility changes. To establish a riskless hedge in the RV model, one must hedge an option with a position in the spot rate and a position in another option. With delta hedging in the BS model, we hedge an option with a position in the spot rate only (or with another option). The RV model predicts that one can reduce risk exposure by also hedging against volatility changes with a second option. HULL and WHITE (1987b) have used the modified BS model for foreign currency options to examine the effectiveness of hedging against volatility changes and they find that one can reduce the variability of a hedged position by incorporating the effect of volatility changes in the BS formula.
As in HULL and WHITE, we refer to hedging both price and volatility risk as delta-sigma hedging. In our analysis we consider three different hedging models: (1) the BS delta hedge, (2) delta-sigma hedging with the BS model, and (3) delta-sigma hedging with the RV model. The last hedging model requires the partial derivatives with respect to S and σ in the RV model, and we compute these derivatives off the Monte Carlo simulation.
To test the hedging effectiveness of each model, we buy one medium maturity at-the-money call and take positions in the spot rate and the longest maturity at-the-money call. We then compute the net gain on the hedged position over a discrete time period. Because we earn the foreign interest rate on a long position in the foreign currency (or pay the foreign rate on a short position), the net gain for a hedged position in currency options is slightly more complicated than the net gain for a hedged position in stock options. The hedged position in the RV model is

$$C(S,\sigma,t,T_1) + w_1 S_t + w_2 C(S,\sigma,t,T_2),$$

and the hedge ratios are

$$w_1 = -\frac{\delta C(S,\sigma,t,T_1)}{\delta S} - w_2 \frac{\delta C(S,\sigma,t,T_2)}{\delta S}$$

$$w_2 = -\frac{\dfrac{\delta C(S,\sigma,t,T_1)}{\delta\sigma}}{\dfrac{\delta C(S,\sigma,t,T_2)}{\delta\sigma}}$$

The net gain on the zero investment hedge is

$$[C(t+1,T_1) - C(t,T_1)] + w_1[S_{t+1}(1+r_f) - S_t] + w_2[C(t+1,T_2) - C(t,T_2)] - r_d[C(t,T_1) + w_1 S_t + w_2 C(t,T_2)].$$

For delta hedging in the BS model, we set $w_2 = 0$ and $w_1 = \delta C(S,t,T_1)/\delta S$.

We take a hedged position on each of the trading days in our sample (a Tuesday or a Friday) and maintain that position until the next trading day in our sample. In all cases, the time interval is three or four days. We examine the hedging effectiveness from the perspective of the market maker and we use the mid-point of the bid-ask spread to measure price changes. Can we reduce the variability of the net gain on a hedged position by moving from delta hedging to delta-sigma hedging? To make the comparisons, we look at delta-sigma hedging with the RV model and the BS model. We find that the derivatives used for delta hedging are virtually the same for both models, but the two models produce very different hedge ratios for the second option. The derivatives with respect to σ are all greater with the BS model, but the hedge ratios for the second option are all greater in absolute value with the RV model. The ratios for the RV model vary above and below one, but the ratios for the BS model are all less than one. In all cases, we hedge with a second option that has the same strike price, but a longer time to maturity.

The hedging performance for the three strategies is presented in Table 3. The normalized gain is the net gain divided by the price of the call option that we are hedging. We report the average net gain, but note that the expected value is zero. To measure the variability for each hedging strategy we compute the standard deviation and the mean absolute deviation. In all cases the variability is less if we use delta-sigma hedging. This reduction in variability is between 20 and 30 percent, depending on how we measure variability. In three of the four measures, the variability of the BS delta-sigma hedging is lower than that of delta-sigma hedging with the RV model, but in all cases the numbers are close. The results in Table 3 are consistent with the results of HULL and WHITE on delta-sigma hedging with BS model and indicate that there is some volatility risk that can be reduced by using delta-sigma hedging.

Finally, we examine the profitability of trading with the RV model. The prices on European currency options in our sample conform well to the BS model with implied volatilities revised each day, and the RV model produces prices that differ significantly from the BS prices. This observation suggests that there may be some trading opportunities available. If volatility is changing randomly, a trader with a RV model may be able to identify mispriced options. To test this implication we use the RV model to form hedged positions with call options and the foreign currency. Our trading rule proceeds as follows. On each day price the options with RV model and look for model prices that fall outside the bid-ask spread. Buy or sell the call option that has the largest deviation outside the bid-ask spread. If the model price for the call is below the bid, sell it; if the model price is above the ask buy it. Then take an opposite position in a second call. If a second call is to be sold, use one that has a model price below the mid-point of the bid-ask spread. If a second call is to be bought, use one that has a model price above the mid-point of the bid-ask spread. Finally, take a position in the foreign currency to hedge exchange rate risk.

The positions are established with actual prices, and the RV model is used to calculate the derivatives for the hedge ratios. The net gain on this zero-invest-

Table 3: Hedging Results for 99 Options.

	Black-Scholes Delta	Black-Scholes Delta-Sigma	Random Variance Delta-Sigma
Normalized Gain			
Average Net Gain	0.03519	0.01377	0.00729
Standard Deviation	0.13601	0.09816	0.10211
Mean Absolute Dev.	0.09728	0.06818	0.06798
Gain			
Average Net Gain	0.10091	0.03036	0.00719
Standard Deviation	0.45231	0.32397	0.36645
Mean Absolute Dev.	0.35011	0.24336	0.25763

Note:
The standard deviations have been computed with E(Gain) = 0.

ment hedge is calculated by the same formula used to calculate net gains above for delta-sigma hedging. The mid-point of the bid-ask spread is used to compute these gains. The average bid-ask spread for calls in our sample is 0.59 Swiss centimes per dollar which is relatively large. Here we are computing potential net gains for a market maker or a trader who can trade inside the bid-ask spread. The hedged positions are formed on each trading day on which mispriced options are found. The RV model identifies mispriced options for all but two days. The average net gain is 0.2587 Swiss centimes per dollar and the sample standard deviation is 0.4778. The t statistic for the sample mean is 5.33 so that it is statistically significant, but the average net gain is more than offset by the bid-ask spread. These results indicate that there are no trading opportunities for a small investor who must buy at the ask and sell at the bid. The results, however, do indicate that there is mispricing in the market and that there are some profit opportunities for a market maker or a trader who can transact inside the bid-ask spread. Each call option contract is for $50'000 and typical prices are around 5 Swiss centimes per dollar, which represents a contract cost of 2500 Swiss francs. The average net gain is equal to 129.4 Swiss francs, or 5.2% of the typical contract cost. The cumulative gain, defined as the average net gain per contract times the number of mispriced contracts traded, is 12'548.5 Swiss francs.

A similar trading strategy has been examined for the BS model with standard delta hedging (buy or sell a call and hedge with a position in the foreign currency). With the BS model we find trading opportunities on only 79 out of 99 trading days in our sample. The average net gain is 0.2170 with a sample standard deviation of 0.4974, and the t statistic is 3.88 which is statistically significant. The average net gain of 0.2170 centimes per dollar is 108.5 Swiss francs per contract, or 4.3% of the typical contract price. The cumulative gain for this strategy is 8'573 Swiss francs, which is much less than the cumulative gain for the RV model.

6. Summary

We apply recent results on random variance option pricing to the pricing of foreign currency options and we use actual prices on European currency options from Geneva to compare the performance of this model with the familiar Black-Scholes model. We find that the actual prices on calls and puts conform more closely to the Black-Scholes model if we allow the variance rate to be revised every day. When we use a constant variance rate, we find that the Black-Scholes model performs very poorly. Even though we find that the Black-Scholes model outperforms the random variance model, we do find much evidence from both the option prices and the foreign exchange rate series to support the notion that volatility changes randomly.

We examine several different hedging strategies and find that a trader can significantly reduce the potential variability of a hedged position by hedging both price risk (exchange rate risk) and volatility risk. Traders can hedge the price risk for an option by taking an appropriate position in the foreign currency, but potential variability remains due to volatility risk. To hedge volatility risk one must take an offsetting position in another option. We find that both the Black-Scholes model and the random variance model work equally well in terms of reducing volatility risk. A hedged trading strategy that attempts to identify mispriced options with the random variance model produces profits that are statistically and economically significant, but these profits are more than offset by the bid-ask spread. There is some evidence of mispricing in this market, but the magnitude of this mispricing is not large enough for a small investor to earn abnormal profits.

Footnotes

[1] This strategy is actually used by an American firm.

[2] A more technical presentation can be found in CHESNEY and SCOTT (1989).

[3] The official middle rates are midpoints of bid-ask quotes for the Eurodollar and Euro-Swiss franc rates. Continuously compounded interest rates are used in the option pricing models. The interest rates are first converted to continuous time rates. When the option maturity falls between the maturity dates for the Eurocurrency deposits, we interpolate to get rates for the option maturity.

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