Testing the Arbitrage Conditions for Option Pricing -A Survey

1. The Importance of Testing Arbitrage Conditions

The boundary conditions for pricing call and put options were derived and presented by MERTON (1973a) and STOLL (1969). Any option model should yield rational prices consistent with the more general distribution-free boundary conditions. A violation of a boundary condition implies that arbitrage profits may be available for a trader who can simultaneously and costlessly transact the stock, and a riskless bond.

In this paper the tests of the boundary conditions will be surveyed. If these conditions cannot be empirically validated then there is little hope of validating any specific and more restrictive pricing model. If, for a given data set, violations of the boundary conditions are found, these violations will reappear when a specific pricing model is tested with the same data. The market cannot be shown to be inefficient for a subset of weak conditions and, at the same time, efficient for compatible but stronger assumptions.

In order to illustrate the issue of arbitrage profit opportunities, let us look at actual data as reported in the Wall Street Journal of May 12, 1983, (see Table 1). The stock market during the previous three weeks had advanced significantly and therefore many options were deep-in-the money, and as such were potential arbitrage candidates. The quoted closing price for a Boeing (BA) May 30 call was 7 1/8 on May 11, 1983. The closing stock price, that day was 37 1/4. If one can trade at these prices, a simple arbitrage condition is violated; the price of the call should be at or above its exercise (or intrinsic) value, that is: S - X = 37 1/4 - 30 = 7 1/4. But the call price was only 7 1/8. For Control Data Corporation (CDA) for example the simple arbitrage rule is violated for CDA May 30, 35, 40 and 45 calls.

It is appropriate at this point to advise the reader that the chances of making arbitrage profits the next day by using the closing prices quoted in Wall Street Journal are rather slim. This conclusion is drawn for many reasons, discussed in detail later in this paper. One additional reason is the Wall Street Journal (and most other newspapers) shows the last trade price for the day, which may reflect market conditions much earlier in the day. Assume, for example at 10:34 a.m. XXX Jan 30 calls traded at 9 1/4, and no other trades occured later in the day. The newspaper will show a price of 9 1/4 for XXX Jan 30 calls. Let us assume that when the market closed, XXX was 39 3/4 last. The newspaper's quotes will be 39 3/4 and 9 1/4 respectively, for the stock and option. They indicate an arbitrage opportunity. However, it may be the case that at 3:00 p.m. the market for XXX Jan 30 calls was 9 3/4 bid - 9 7/8 asked. These closing bid and ask prices do not offer any arbitrage profit. This information is not being reported in the newspaper.

Therefore an important question is the extent to which apparent violations are real and can actually be exploited. Can trades be executed at these prices to reap riskless profits? If so, such apparent violations conflict with the hypothesis that options markets are efficient.

2. Terminology

Before discussing the results of specific tests a few terms should be defined and explained:

(a) "*Efficient market*" - A market (or a set of markets) will be termed efficient if no trader can consistently make above-normal, risk-adjusted profits after transactions costs and taxes. In our context, the efficiency of the option markets will be questioned, given the prices of the underlying stock. It should be emphasized that all the above conditions must be met before it can be decided whether any given market (or set of markets) is efficient. For example, if profits can be made before transaction costs, but they disappear after incurring these costs, the market cannot be regarded as inefficient. When testing for market efficiency we ouhgt to state from whose point of view the analysis is performed. A market can show no extraordinary profits for an outside ("off-floor") trader yet it can have profit opportunities in abundance for a market maker. (By their actions, the on-floor market makers can establish an efficient market for the outside traders.)

(b) "*Synchronous markets*" - Synchronous markets are markets in which trading in related assets can take place simultaneously and quoted prices reflect this simultaneity. This is especially important in option markets because of the arbitrage relation between synchronized prices of the underlying asset an its options. Synchronization, therefore, has two facets: trading synchronization and data synchronization. The former stands for contemporaneous trading in two related securities. This is not a necessary condition for market efficiency, though it may be required for "locking in" arbitrage profits. However, trading synchronization might not be sufficient for proving the market is synchronized since transaction reporting is rarely synchronized. Each market uses different procedures and encounters different reporting delays at various times. The method of transaction reporting must be such that the data accurately presents the time of the transaction and the time the information is made available to market participants.

Nonsynchronous markets are herein described as markets where simultaneously quoted prices reflect transactions that took place at different times. More specifically, if data on a class of options and the underlying stock are used and the price quotes for both are not published simultaneously, based on parallel trading, the markets will appear to be nonsynchronous, even though they may still be efficient.

An arbitrage can only be riskless if price dissemination, inter-market communication of orders, and execution reports are instantaneous. Anything less implies some form of risk in time.

(c) "*Above-normal profits*" - For a completely riskless strategy, above-normal profits means profits in excess of the risk-free rate of interest (say, on a government security with the same maturity as the strategy). Uncertain yield is considered above-normal if, after adjusting for risk, the average risk-adjusted realized rate of return is in excess of the risk-free rate. It should be noted that the expected return for a given level of risk is also determined by an equilibrium model, either the capital-assets pricing model (CAPM) or, more recently, by the arbitrage-pricing model (APM). These models estimate the sensitivity of the stock's rate of return to changes in the rate of return on the market index (CAPM) and to changes in additional economic factors (APM).

(d) "*Ex-post Tests*" - Based on prices known at time t, a trading strategy is devised and a position is established based on these same prices. This is as if the trader can roll the price tape backwards and establish a position at the previously available prices. The position will be liquidated one period later at time t + 1 at prices that are unknown at time t.

The ex-post test can be used to locate deviations from market efficiency and/or model validity in the case of full information about simultaneous prices of options and their underlying stock. A model may be accurate, and ex-post, given all relevant prices, it may distinguish between overpriced and underpriced assets. Still, it may turn out that when using such a model to devise a profitable strategy, it will fail and the markets will appear efficient. This occur because prices observed at t are not necessarily the prices for the next transactions. A trader looking at the electronically disseminated market prices, may actually observes prices of the previous transactions. If a strategy to buy or sell options or stocks is based on previous prices, there is no assurance the trader will be able to execute the orders at these prices.

If bid and ask quotes are exhibited, a better practical strategy can be devised. But still, the quotes are for a limited number of contracts and are valid for a short time interval.

(e) ''*Ex-ante tests*'' - Based on prices known at time t, a trading strategy is devised, but the position is established at time t + 1 at prices that are unknown at time t. The position will be liquidated at prices available at time t + 2. The ex-ante test may sometimes more accurately represent the alternatives open to the trader and then it is a suitable test for market efficiency.

Concepts (a) through (e) will be further explained and their usefulness in empirical studies will be exhibited in the following sections. They should give a unified framework for classifying the reported empirical studies and their results.

Table 1 will help us demonstrate the terms just defined.

Table 1 : Hypothetical Transactions Data for XXX May 30 Calls and its Underlying Stock.

	XXX Stock Prices			XXX May 30 Call Pric.		
Time	Bid	Ask	Transact.	Bid	Ask	Transact.
10:08:31	35 1/8	35 1/4	35 1/8	4 3/4	5	
10:08:52	34 7/8	35 1/8		4 3/4	5	5
10:09:25	34 7/8	35 1/8		4 7/8	5 1/8	5 1/8
10:09:27	34 3/4	35	35	4 3/4	4 7/8	

At 10:08:52 the table shows the XXX May 30 call one month before expiration traded for 5, while the underlying stock XXX traded at 10:08:31 for 35 1/8. Based on these prices the option is obviously underpriced and ex-post an arbitrage profit could have been realized. But observing these prices at 10:08:52 can a trader enter into stock and option transactions at the same prices at 10:08:53? Placing a market order to buy the presumably underpriced option and sell the relatively ''overpriced'' stock, the transactions may be consummated at 5 1/8 and 35 respectively. These latter prices do not exhibit any arbitrage opportunity.

The initial observed violation may be due to market nonsynchronization, since even within 21 seconds from 10:08:31 to 10:08:52 prices may change. One simple reason for frequent price changes may be due to alternating orders to buy or sell the asset at the ask and bid prices respectively.

Actually, there is a typically 30 to 90 seconds delay in reporting last sales prices. The delays may be longer in periods of heavy trading. Option market makers are aware of the fact that the data they use is 30 - 90 second stale, and they try to incorporate this into their pricing decisions. Researchers, however, may find it extremely difficult to incorporate a variable delay time in their study of market efficiency.

3. Boundaries for the Rational Pricing of Options

By using the ''dominant asset'' argument, MERTON (1973 a) proves several necessary conditions for rational pricing of options in perfect capital markets [1]. The conditions are quite general as they do not rely on a specific pricing model; they give boundaries for option values and for relationships between options which differ by one parameter (or more). The conditions are derived for American and European calls (and puts) [2], and mainly for the case where no dividends are expected to be paid on the underlying shares, or for options with a clause that protects them fully against the adjustment in

the stock price on the ex-dividend day.
In the empirical work of pricing of CBOE options we will face two types of American options. The first will be an American call option that is not adjusted for dividend payment - there are no adjustments in the striking price or the number of shares on which the optionholder will have a claim - and dividends are expected to be paid before the maturity of the option [3]. The second is like the first, in the terms of the contract, except that now no dividends are expected to be paid until the expiration date. Therefore, the latter should be priced like an American call option when no dividends are expected.
The condition for the American option, when no dividends are expected (denoted by $C_A = C_A(S, \tau, X)$, where τ is the length of time to maturity from the current period, S is the current market price of the stock and X is the striking price), is to be found in detail, with proofs, in MERTON (1973a). Here we will summarize MERTON's conditions and will add some more for the unprotected American call option (denoted by $C_{AN} = C_{AN}(S, \tau, X, D)$, where D is the dividend payment). The assumptions needed for the proofs are perfect capital markets, no economies of scale in buying securities, and independence of the distribution of rates of return on the stock from the dividend policy and from the stock's price level. We also assume that the ex-dividend day is known with certainty, and so is the amount of the declared dividend. For simplicity of exposition, it is assumed that, at most, one ex-dividend day is expected, during the remaining life of the option.

General Conditions for Pricing C_A

The conditions for pricing C_A are [4] [5]:

(a) $C_A(S, \tau, X) \geq Max(O, S-XB(\tau))$

Where $B(\tau)$ is the market value of a riskless bond that will mature after τ periods with face value of one dollar (and $\tau = T-t$, where T is the maturity date and t is the current date) [6].
A call option with longer life is more valuable than a call with shorter life:

(b) $C_A(\tau_1) \leq C_A(\tau_2)$ for $\tau_1 < \tau_2.$

The value of a call is a decreasing function for the striking price:

(c) $C_A(X_1) \geq C_A(X_2)$ for $X_1 < X_2$.

For a dividend-protected American call, it does not pay to exercise the option prematurely, and therefore its value is equal to an identical European call:

(d) $C_A = C_E$

where C_E is the European option with the same terms as the American option (only that the former cannot be exercised before maturity). Additional conditions are:

(e) C_A is a convex function of X.
Or, for $X_2 = \lambda X_1 + (1-\lambda)X_3$, and $0 < \lambda < 1$, we should find
$C(X_2) \leq \lambda C(X_1) + (1-\lambda) C(X_3)$.

(f) $-B(\tau)(X_2-X_1) \leq C(X_2) - C(X_1) \leq 0$

(g) C_A is homogeneous of degree one in S and X.

(h) C_A is a convex function of S.
Or, for $S_2 = \lambda S_1 + (1-\lambda)S_3$ and $0 < \lambda < 1$ we should find $C(S_2) \leq \lambda C(S_1) + (1-\lambda) C(S_3)$.

General Conditions for Pricing C_{AN}

For CBOE type of option, if dividends are paid to owners of the underlying stock, no compensating adjustments are made in the terms of the option. Hence, the optionholder has no claim on the dividends paid to the stockholders during the life of the option. He can have a claim on a dividend by exercising his option before the ex-dividend day.

For an unprotected option, whose value we denoted by C_{AN}, we will expect that

(d') $C_E = C_A \geq C_{AN}$

or, the value of an American option adjusted for dividends is not smaller than the value of the unadjusted American option. It is possible that it is more profitable to exercise the option at t_D-1, the day before the ex-dividend day [7], and thus to disregard the additional period of $\tau_D(=T-t_D)$ to the contractual expiration of the option. Several necessary conditions for rational pricing of C_{AN} are listed below.
Conditions (b) and (c) are valid for C_{AN}

(b') $C_{AN}(\tau_2) \geq C_{AN}(\tau_1)$ for $\tau_2 > \tau_1$.

(c') $C_{AN}(X_2) \leq C_{AN}(X_1)$ for $X_2 > X_1$.

From (b') we see that $C_{AN}(S,\tau,X,D) \geq C_{AN}(S,\tau',X)$, where $\tau = T-t$, $\tau' = t_D-t$, and $\tau > \tau'$. The right-hand side of the inequality is an American call option with τ' periods to maturity and no dividends are expected to be paid during the τ' periods. Therefore $C_{AN}(S,\tau,X,D) \geq C_A(S,\tau',X)$ and combining it with (a) we get

(a') $C_{AN}(S,\tau,X,D) \geq MAX(0,S-Xe^{-r\tau'})$

When D is known with certainty, it can be shown that:

(a'') $C_A(\tau) - De^{-r\tau'} \leq C_{AN}(\tau) \leq C_A(\tau)$.

It is assumed for simplicity of exposition that only one dividend payment, D, is expected during the life of the option.

By combining this condition with condition (a), we get:

(a''') $\text{Max}[0,S-X,S-Xe^{-r\tau}-De^{-r\tau'}] \leq C_{AN}(S,\tau,X,D)$.

Condition (a'') can be easily expanded for the multi-dividends case (assuming certainty with respect to dividend payments). Then it will become:

$$(a'')\quad C_A(\tau) - \sum_{i=1}^{n} D_i^{-r\tau} \leq C_{AN}(\tau) \leq C_A(\tau)$$

for $t_i \leq T$ (i= 1, 2,..., n), where t_i's are the days at which dividends are known to be paid, and $\tau_i = t_i - t$ and $\tau = T - t$. Condition (a''') will then be expanded to:

$$(a''')\quad \text{Max}[0, S - X, S - Xe^{-r\tau} - \sum_{i=1}^{n} D_i e^{-r\tau_i}] \leq C_{AN}(S,T,X,\underline{D})$$

where $\underline{D}$ is a known vector of dividends to be paid at $\underline{t} = t_1,...t_i,...t_n$.
Condition (a''') is not the strictest condition possible for the lower boundary of $C_{AN}(S,\tau, X, 0)$. From (a'), we get $C_{AN}(S, \tau, X, D) \geq \text{Max}(0, S - Xe^{-r\tau'})$. This condition should be combined with condition (a''') to get what seems to be the strictest general lower boundary.
(a*) $\text{Max}[0, S - Xe^{-r\tau'}, S - Xe^{-r\tau} - De^{-r\tau'}] \leq C_{AN}(S, \tau, X, D)$.

Its general form for the case that all dividends D_i are known, and so are the ex-dividend days t_i's(i = 1,...,n) is:

$$(a^*)\quad \text{Max}\left\{0, \text{Max}_i\left[S\text{-}Xe^{-r\tau_i}-\sum_{j<1} D_{t_j} e^{-r\tau_j}\right], S\text{-}Xe^{-r\tau}-\sum_{i=1}^{n} D_{t_i} e^{-r\tau_i}\right\} \leq C_{AN}(S, \tau, X, \underline{D}).$$

The convexity of C_{AN} with respect to its striking price K can be proven.
For $K_2 = \lambda K_1 + (1-\lambda) K_3$ $(0<\lambda<1)$ it should be true that

(e') $C_{AN}(X_2) \leq \lambda C_{AN}(X_1) + (1-\lambda) C_{AN}(X_3)$

The relationship between put and call prices was formally established by STOLL (1969) for European options:

(i) $C_E - P_E = S - Xe^{-r\tau}$

The parity condition is based on arbitrage considerations.

MERTON (1973 b) showed that for American options the parity condition does not hold as an equality, even when the options are dividend protected since $P_A \geq P_E$, and hence:

(i') $C_A - P_A \leq S - Xe^{-r\tau}$

For CBOE type options KLEMKOSKY and RESNICK (1979) derive the boundary condition for the difference $C_{AN} - P_{AN}$:

(i'') $C_{AN} - P_{AN} \leq S - Xe^{-r\tau} - \sum_{i=1}^{n} D_{t_i} e^{-r\tau_i}$

4. The Results of Empirical Tests for Call Options

A study by GALAI published in 1978 argues that ex-post arbitrage profits based on violations of boundary conditions can only indicate whether the relevant markets are unsynchronized. To test for market inefficiency, i.e., revealing and exploiting above normal profits by means of arbitrage, an ex-ante test should be carried out. When a dealer observes a deviation from a boundary condition he must place orders in two different markets to exploit what seems to be a profit opportunity. There is no real guarantee that the prices at the next available transaction will still be favorable for the arbitrageur.

Two classes of empirical tests, one ex-post, the other ex-ante, were performed by GALAI in order to test the synchronization and efficiency hypotheses. The data used were the daily closing prices of the Chicago Board Options Exchange during the Exchange's first six months of operation (April 1973 to October 1973). The components of the boundary condition for unprotected American call options on dividend paying stocks were subjected to empirical tests in order to identify the factors that might explain observed deviations from the "rational" conditions.

The first type of deviation, the "weak dominance condition" implies that the current market value of the call, C, should be greater than the immediate exercise value of the option S-X:

$$\varepsilon_1 = (S-X) - C \leq 0 \quad (1)$$

where S is the current market price of the stock, and X is the striking price. The second type of deviation, the "strong European call dominance condition" takes into consideration a tighter boundary condition for an unprotected call by subtracting from the current stock price the present value of the exercise price and the present value of dividends scheduled to be paid over the life of the option, assuming that the call will be held to maturity

$$\varepsilon_2 = S - Xe^{-r\tau} - \sum_{i=1}^{n} D_i e^{-r\tau_i} - C \leq 0 \quad (2)$$

Where D_{t_i} is the dividend payment at time t_i, n is the number of ex-dividend days during the life of the option, r is the default free interest rate, and τ_i is the time left to maturity from time t_i. The third component, to be termed the "strong early exercising dominance condition", takes into consideration the possibility of exercising the unprotected American call just before the stock goes ex-dividend.

$$\varepsilon_3 = \mathrm{Max}\left[S - Xe^{-r\tau_i} - \sum_{j<i} D_{t_j} e^{-r\tau_j} \right] - C \leq 0 \quad (3)$$

The results of the tests are summarized in Table 2 [8]: (see next page)

Table 2 [9]: Number of violations of each condition $\varepsilon_k \leq 0, k = 1,2,3$, and the average magnitude of the violation for CBOE data (Apr. 1973 - Oct. 1973)

	r = bill rate		r = 10 percent	
Type of violation	Frequency	Average $	Frequency	Average $
ε_1	281	35.0	281	35.0
ε_2	482	36.3	567	38.1
ε_3	107	41.7	115	46.0

ε_1 Weak dominance condition
ε_2 Strong European call dominance condition
ε_3 Strong early exercising dominance condition

Out of 7445 observations of in-the-money options, 281 violations of the weak dominance condition were recorded, of which 274 occurred within two months of expiration and 174 within the last month. The average violation over the 281 violations was $35 per contract. When the T-Bill rate was used to discount the dividends and the exercise price, 482 violations of the ''strong European call dominance condition'' were recorded. There were 107 violations of the third type of deviation.

The ex-ante test concentrated on the ''strong European call dominance condition'' and whenever it was ex-post violated, a position was established to exploit the deviation, but at the next days closing prices. Hence, the violation was used as a signal to enter into a now risky option position. While ex-post the average violation was $36.3, on an ex-ante basis, profit fell to $12 and in many cases the apparent profit disappeared.

The major results of GALAI's study can be summarized as follws:

1. The closer the option is to maturity and the larger is the stock price (given the exercise price), the greater the number of observed boundary violations. Relatively more violations of the boundary conditions are observed for short-maturity, deep-in-the money options.
2. The hypothesis that the stock and options markets are sufficiently synchronized and that simultaneous closing prices are within the theoretical boundaries is rejected. The frequent deviations could not have been explained by either the assumption of perfect knowledge of the firm's dividend policy or by possible inaccuracy of data. Tests that were run on hourly data yield similar results.
3. The ex-ante test showed that on average positive profits could have been exploited by a market participant who followed the arbitrage trading rules. The magnitude of profits, however, was small relative to the dispersion of yields, and there was non-neligible probability of incurring a loss.

The results actually show that a regular investor, who executes his trading through a broker, has very small chances, if at all, to make arbitrage profits on the CBOE. What may seem as a sure arbitrage condition may only reflect absence of data or market synchronization. Market-makers, by following major deviation from ex-post arbitrage conditions, stand a good chance of making above-normal profits.

Another general condition applying to call options is the convexity rule. MERTON (1973) shows that the European call price is a declining convex function of the striking price. In GALAI (1979) the convexity is shown to hold for American dividend-unprotected options and is subjected to an empirical test. During April to October 1973, on the basis of approximately one thousand relevant CBOE observations, 24 violations of the convexity revealed that deviations occurred when closing prices were considered [10], while during the day prices behaved as expected. The lesson is, therefore, that reported closing prices may mislead the researchers, as well as amateur investor, and if this is true for general boundary conditions it may be even more so for any specific pricing model.

Another possible explanation of the deviations is transaction costs. PHILLIPS and SMITH (1980) show that the transaction cost involved in exploiting any deviation from the boundary conditions is between $35 to $40. A major component of the cost of arbitraging is the difference between the buying price (ask), and the selling price (bid). Evidence from market-makers however, indicates that their transaction costs are much smaller than PHILLIPS and SMITH's estimates as they generally trade at a price inside the bid-ask spread. Still, the most efficient traders in the stock and option markets are also subject to transaction costs. Therefore, the empirical results are not necessarily inconsistent with market synchronization and efficiency. But, as discussed above, pricing models should take the same transaction costs explicitly into account; if not, they may indicate the existence of non-realizable profits.

In a recent study by BHATTACHARYA (1983), transactions data for CBOE options on 58 underlying stocks for 196 trading days ending June 2, 1977 were used. In testing the boundary condition the imputed bid and ask prices for the options and the stocks were considered as well as the simultaneity of prices, the depth of the market, the executing lag and the transaction costs.

Out of 86137 observations, 1120 ex-post violations of the weak boundary condition were recorded when transaction costs were ignored. Almost all ex-post violations (97 percent) were for options with less than three months to expiration and 42 percent were recorded for options with less than a week to expiration. The average violation was 12 dollars per contract. In fact $12.50 per contract is the minimum differential allowed between bid and ask prices on any underlying stock or any option trading for a price over $3.

When an ex-ante test, centered on the ex-post violations, was performed, the average profit was reduced to 4.9 dollars per contract (with 16 percent of executed transactions unprofitable). BHATTACHARYA assumes that an arbitrageur, by owning seats on both the stock and options exchange, incurs a smaller transaction cost than a market-maker who only owns a seat on the options exchange. Then, by assuming that the transaction costs to an arbitrageur are 5.5 dollars per contract and for a market-maker on the CBOE they are 16.75 dollars per contract, both including the bid-ask spread, the profit on the arbitrage activity based on the weak boundary conditions became a loss.

A similar picture repeats itself when the other components of the boundary conditions are being tested. For the ''strong European call dominance condition'', 7.6 percent of the relevant sample violated the ex-post condition. By examining a subset containing only the first violations per option series per day 804 violations were found with an average profit of 10 dollars per contract. Ex-ante test showed an average profit of 5 dollars per contract before transaction costs, and an average loss after transaction costs. The convexity test performed on transaction data indicated only one violation out of a relevant population of 1006 observations.

BHATTACHARYA concludes that while small and infrequent violations of the boundaries were recorded the returns resutling from executing the would-be arbitrage opportunities were on average positive only when transaction costs are ignored. BHATTACHARYA admits in his paper that his tests are biased in favor of market efficiency by the method he used to estimate the bid and ask prices of both the options and especially the underlying stocks. Still it may be the case that with the expansion of trading on the Exchange, the more traders competing and additional experience accumulated, arbitrage profits are giving to be harder to find and exploit on the organized options exchanges.

Boundary conditions were also tested for options traded in two foreign exchanges: the Toronto Stock Exchange (TSE) in Canada, and the Frankfurt Stock Exchange (FSE) in Germany. A similar methodology to the one used in testing the boundary conditions for CBOE options was employed. The results, in general, indicate more substantial violations of the conditions for both the ex-post and ex-ante tests, compared to the American markets.

A study by HALPERN and TURNBULL (1985) used Canadian transaction data over the period 1978-1979. Out of a total of 315'202 option transactions, 178'022 were recorded for in-the-money options. Table 3 summarizes the frequency and size of the derivations from the boundary conditions.

Table 3 [11]: Number of Violations of Each Condition $\varepsilon_k \leq 0$, K=1, 2, 3 and the Average Magnitude of the Violation Per Contract for the TSE Data for 1978 and 1979.

	1978		1979	
Type of Violation	Frequency	Average Size Canadian $	Frequency	Average Size Canadian $
ε_1	2346	29.33	10440	35.93
ε_2	5839	37.50	27152	36.34
ε_3	1543	26.57	9901	30.83

In general, they find the markets to be non-synchronous. The average magnitude of the violations ε_1 and ε_2 are similar to those reported by GALAI for CBOE options but their frequency is greater. They find a positive relationship between the maturity of the option and the average dollar size of the violation. They also find the average size of the violation to increase with the number of ex-dividend dates before the option expires. Surprisingly, HALPERN and TURNBULL show that both the frequency and magnitude of the violation have tended to increase from 1978 to 1979. These results, they claim, are consistent with the inability of the markets to adjust to the large increase in volume of trading and in the number of the traded underlying securities.

Finally, after conducting ex-ante tests and taking transaction costs into account, HALPERN and TURNBULL find that the Toronto Stock Exchange was inefficient in trading options during the period 1978-1979.

Stock options traded on the Frankfurt Options Exchange are dividend protected, as the exercise prices of the options are reduced by the amount of the dividend on the ex-dividend day. Taking this factor into account [12], TRAUTMANN (1985) found 891 violations of the sign of ε_2 out of a sample of 63,391 observations, covering the period April 1983 to September 1984. The average violation, net of the transaction costs paid by a floor broker, was almost DM 60 ($20-30) per contract for 50 shares.

5. Testing Put-Call Parity

For European options, the value of the put is a simple function of the call price and other observable variables. If the value of the put deviates from the relationship described by put-call parity, an arbitrage situation arises. A European put in an efficient market can be valued by the following function.

$$P = C - S + Xe^{-rt} \qquad (4)$$

STOLL (1969) tested the put-call parity model by transforming equation (4) into a least square regression model of the relative form

$$\frac{C}{S} = \alpha + \beta\left(\frac{P}{S}\right) + \gamma \frac{(r)}{1+r} \qquad (5)$$

and the absolute form

$$C = \alpha + \beta P + \gamma \frac{(r)}{1+r} S \qquad (6)$$

where r is the risk-free interest rate and α, ß and γ are the estimated coefficients. STOLL's data set included weekly submissions during 1967 to the Securities and Exchange Commission by the Put and Call Brokers and Dealers Association of representative prices for 10 companies with relatively great amounts of activity. In his regression analysis STOLL found deviations from expectations; the intercept was higher and the slope lower than expected.

GOULD and GALAI (1974) follow MERTON's (1973b) adjustment of the put-call parity conditions for American options. In addition, they introduce tax and transaction cost considerations in forming the put-call parity conditions. It is shown that the condition is not dependent on the tax rate, but transaction costs increase the upper boundary and reduce the lower boundary of the difference C-P. GOULD and GALAI claim that regression technique is not an appropriate methodology for testing the existence of arbitrage profits. If, in the absence of transaction costs, the boundary condition for at-the-money options is

$$(C - P)/S \leq r/(1 + r) \qquad (7)$$

then any deviation revealed in the market may indicate market inefficiency. The regression technique is based on averaging, while for market inefficiency only outliers or violations of the boundaries are of interest. Using STOLL's data set, expanded to include 1968 and 1969, they show a rather surprising number of violations of (4) that represent potential profit opportunities. These apparent returns would disappear for a non-member of the Exchange who has to incur greater transaction costs. These results are supported by the analysis of an additional data set on actual transactions in straddles and calls. GOULD and GALAI conclude that transaction's costs appear to play a nontrivial role in the explanation of observed premiums on puts and calls, at least in the period 1967-1969 (p.122). Even after transaction costs, it seems that a member of the NYSE could have filtered the high profit opportunities. The market, thus, was somewhat inefficient from the point of view of the members of the Exchange.

Trading listed in standardized puts on option exchanges commenced on June 1977, four years after trading in listed call options began. Since these options are not adjusted for the dividend payments, the put-call parity should be restated for the listed options. KLEMKOSKY and RESNICK (1979) derive the parity conditions and subject them to empirical testing based on CBOE puts. (See condtion (i'') in section 3.)

Transactions data for one day each month during the period July 1977 to June 1978 for 15 stocks and their options were employed in testing inequality. This data set made it possible to construct a nearly simultaneous position in the call, put, and the underlying stock. Only one position was constructed each day per option class, and altogether 606 positions were considered. After eliminating 66 positions due to violating the sufficient condition for no premature exercising of the call option, they found 234 profitable hedges. After introducing transaction costs of \$20 for a member of the exchange and \$60 for the non-member investor, it was found that ''paper'' profits were still available for the former, but almost completely eliminated for the latter. Similar to GOULD and GALAI, most of the deviations from parity were due to low to middle price stocks. With the exceptions of short term options, KLEMKOSKY and RESNICK did not reveal any significant relationship between time to maturity and the proportion of deviations from the parity conditions.

KLEMKOSKY and RESNICK also employed least squares regression based on the terms in inequality (i"). The results for the full sample show simultaneous put and call prices to be thoroughly consistent with the put-call parity. Running the regressions on the profitable hedges and unprofitable hedges separately, led them to conclude that overpriced calls and overpriced puts were the dominant factors in their respective hedges. The conclusion, is true by definition.

In KLEMKOSKY and RESNICK (1980) the previous work was extended by performing an ex-ante analysis of the boundary conditions. Hedges were constructed 5 and 15 minutes after they had been initially identified as having an ex-post return in excess of \$20 per hedge. They found that the majority of the five and fifteen minute lagged observations would be economically profitable ex-ante. However, the overall tendency for the ex-ante profitability was to be less than the ex-post profitability. By also including the average bid-ask spread in each position, they conclude that price correction appears to take place rapidly enough to eliminate most of the economic profits for an arbitraging member firm (p.372).

In both studies of KLEMKOSKY and RESNICK, short conversions, consisting of the purchase of a call, a short position in the underlying stock and lending at the riskless rate of interest to replicate the cash flow of a put, were also tested. However, for American puts no perfect short hedge can be established because of the put always positive probability of premature exercise. The results of these tests, under the assumption of no early exercise of the put (except as can initially be determined) were usually consistent with the long conversion results as summarized above. For the ex-ante analysis, positive net returns were earned on average from short conversions. Since these conversions are not riskless the positive returns may be regarded as a compensation for the risk associated with the investment.

A test of the put-call parity for options traded in Germany on the Frankfurt Option Exchange was carried out by TRAUTMANN (1985). He found that out of 15,507 observations for the period April 5, 1983 to September 28, 1984, 8,714 revealed a deviation from the parity condition, exceeding the explicit transaction costs of an efficient trader.

The importance of the tests of the put-call parity condition is similar to that of testing the boundary condition for the call option; any rational valuation model must be consistent with the weaker boundary conditions. If violations are revealed for boundary conditions, they should be expected to reappear, and usually by larger magnitude in specific tests of valuation models. Hence the same adjustments in the boundary conditions that are suggested for the general conditions should also affect to the models.

6. Conclusion

In summary, the evidence so far is consistent with rejecting the hypothesis of market synchronization. However, mis-synchronization does not necessarily mean inefficiency. The results with respect to market efficiency strongly indicate that an outsider to the Exchange cannot consistently make arbitrage profits. The results for the market-maker are less conclusive. If a market-maker can trade within the bid-ask spread, he may be able to make ''arbitrage'' profits. Though they are not sure profits they may amount to more on average than the risk-adjusted normal profits.

It should be re-emphasized that if the null hypothesis of market efficiency (or synchronization) is rejected based on testing the boundary conditions, it must also be rejected when a pricing model is used on the same data base. The market cannot be shown to be inefficient (or non-synchronous) for weak conditions and, at the same time, efficient for compatible but stronger assumptions.

Footnotes

[1] In perfect capital markets, there are no transaction costs, taxes, or commissions; the acces to available information is costless; and competition is atomistic. In such markets the borrowing interest rate is equal to the lending interest rate, and full use of the proceeds from a short sell are allowed.

[2] An American option can be exercised any time during the life of the option while a European option can be exercised only at the maturity day.

[3] For simplicity of expression, we will assume that dividends are paid on the ex-dividend day.

[4] In general, $C_A = C_A(S,\tau,X)$ or $C_{AN} = C_{AN}(S,\tau,X,D)$, but for simplicity of notation, when the effects of a change in, say, X are explored -- assuming all other factors to be the same - we will describe the premium as $C_A = C(X)$ or $C_{AN} = C(X)$.

[5] See MERTON (1973a), pp. 143-150. All the conditions are derived under the assumption of perfect capital markets.

[6] Under the assumptions of the analysis, it must be that $B(\tau) = e^{-r\tau}$.

[7] Later in the discussion, we assume that it is possible to exercise the option at t_D, just a second before the ex-dividend adjustment is carried out. It is done to maintain simplicity and clarity of notations.

[8] In order to test the sensitivity of the tests to the interest-rate, a flat rate of 10 percent was also used and the results are also presented in Table.

[9] The table is taken from GALAI (1978, p. 196).

[10] The reader should be aware to the fact that reported closing prices may describe transaction prices earlier in the day.

[11] The table is adapted from Table VI in HALPERN and TURNBULL (1981, p.493).

[12] GESKE, ROLL and SHASTRI show in their study (1984) that for the OTC options and the Frankfurt options early exercise is not rational and ε_3 is, therefore, not a boundary.

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