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Forward Foreign Exchange

1. Introduction

The foreign exchange market is by far the largest financial market in the world. The daily volume on this market is estimated to exceed $ 1 trillion. The primary role of the foreign exchange market is to facilitate international trade and investment. An important segment of the market is the *forward market,* where currencies are traded for future delivery. Forward exchange contracts are valuable for avoiding risks arising from exchange rate movements when trading or investing internationally. Alternatively, forward exchange contracts can serve as vehicles for speculation on currency movements. When a decision to hedge or speculate needs to be made, it is useful to be familiar with the thrust of existing theoretical knowledge and empirical evidence on the behaviour of forward foreign exchange rates. In this article we attempt to supply some of the relevant background. In Section 2 the market for forward foreign exchange is described. Section 3 presents two key relationships involving forward rates and in Section 4 the pricing of forward foreign exchange is discussed in detail. In Section 5 we pay attention to the problem of forecasting exchange rates and Section 6 contains some conclusions.

2. The Market for Forward Foreign Exchange

The forward exchange rate is the rate that is contracted today for the delivery of a currency at a specified day in the future. Payment is not made until delivery. The forward rate generally differs from the *spot* exchange rate, which can be thought of as the rate for immediate delivery of foreign currency. (More precisely, a spot contract involves delivery after one or two business days.) Active forward markets exist for the U.S. dollar, the pound sterling, the Canadian dollar, the Japanese yen, the German mark, the Swiss, French and Belgian francs, the Dutch guilder and the Italian lira. Forward rates are usually quoted for one, three, six and twelve months delivery.

The foreign exchange market is an *interbank market.* Commercial banks are market makers. Market makers stand ready to buy or sell any of the major currencies. Returns to market making are provided by the difference between the price at which the market maker sells *(ask price)* and the price at which he buys *(bid price).* The market making banks trade currencies mainly with other banks (either directly or through foreign exchange brokers) and corporations. Foreign exchange brokers do not deal on their own account; they specialise in matching net supplier and demander banks and receive a commission on all trades. Central banks sometimes intervene in spot or forward foreign exchange markets, attempting to influence exchange rates.

Forward rates can be quoted in two ways. Either an actual rate is quoted (the *outright* rate) or dealers quote the forward rate as a *discount* from, or *premium* on, the spot rate. The percentage forward premium at time t, FP (t), is computed as

$$FP(t) = (12/N)\,[F)(t) - S(t)]\,/\,S(t) \times 100\% \qquad (1)$$

where F(t) is the forward rate at time t, S(t) is the spot rate and N is the forward contract length in months.

3. Two Key Relationships

Forward exchange rates, spot rates and interest rates are not unrelated. Considerations of arbitrage and international capital market equilibrium in general impose a certain discipline on our thinking about these variables.

An important relationship in international finance is the *interest rate parity theorem:*

$$F(t)/S(t) = [1 + R_1(t)] / [1 + R_2(t)] \quad (2)$$

Here F(t) and S(t) are forward and spot exchange rates quoted at time t in units of country 1 currency per unit of country 2 currency, R_1 is the interest rate in country 1 and R_2 the interest rate in country 2. The forward rate and the two interest rates pertain to contracts with equal maturities.

The interest parity equation tells us that the position of the forward rate relative to the spot rate exactly reflects the two countries' relative interest rates. The currency of the country with a lower interest rate should be at a forward premium in terms of the currency of the higher rate country. If the interest parity relationship would not hold, riskless arbitrage profits could be obtained through a combination of borrowing in one currency, lending in the other and a forward sale of the proceeds. This covered interest arbitrage incentive tends to enforce interest parity.

Three important reasons for interest parity not to hold exactly are the following:

a) transactions costs in the foreign exchange market and the market for borrowing and lending;

b) the existence of impediments to international capital flows;

c) even if there are no existing capital controls, there may be political risk, which was defined by ALIBER (1973) as 'the probability that the authority of the state will be interposed between investors in one country and investment opportunities in other countries'.

Eurocurrency markets are not subject to capital controls and the like. One would, therefore, expect interest parity to hold more closely between securities issued in Euromarkets ('external pairs') as compared with securities issued in countries' domestic capital markets ('traditional pairs'). FRENKEL and LEVICH (1977) present some empirical evidence on the interest parity relationship, allowing explicitly for transactions costs. On the basis of data on transactions costs they construct a 'neutral band' within which we can find observations that do not strictly conform to interest parity without giving rise to arbitrage opportunities. Within this 'neutral band' transactions costs would dominate any potential arbitrage profits. FRENKEL and LEVICH analyse traditional pairs by examining weekly data on arbitrage of 90-day U.S. and U.K. Treasury bills as well as between 90-day U.S. and Canadian bills. External pairs are analysed by examining weekly data on arbitrage between 90-day Eurodollar and Eurosterling securities. The empirical results for July 1973 to May 1975 indicate that 81–100 percent of the observations on traditional pairs and 99–100 percent of the observations on external pairs fall within the neutral band. Indeed, the data show that interest parity holds more closely for the case of Euromarkets.

It is useful to step back for a moment and ask ourselves what the percentages reported by FRENKEL and LEVICH mean for the case of Euromarkets. When speaking with foreign exchange dealers, one realises that forward rates in Euromarkets are quoted simply from the interest parity relationship: dealers use the current spot rate and interest rates and interest parity to determine the 'right' forward rate. In the light of this practice, FRENKEL and LEVICH'S results for Euromarkets are not surprising. In fact, one could argue somewhat tautologically that any observed deviation from interest parity in Euromarkets must be due to some imperfection or mismatch in the data.

A second important relationship involving forward exchange rates is *forward parity.* Forward parity maintains that the (natural logarithm of the) current forward rate observed at time t for a contract maturing at t+1 is equal to the market's rational expectation of the (log of the) future spot rate which will be observed at t+1:

$$f(t, t+1) = E[s(t+1) \mid t] \quad (3)$$

Here f(t, t+1) is the log of the forward rate at t for a one-period contract, s(t) is the log of the spot rate at t and $E[. \mid t]$ denotes an expectation on the basis of information that is available to market participants at time t. The forward parity relationship implies that the forward rate is an optimal, unbiased predictor of the future spot rate (observed at the maturity date of the forward contract). Some evidence on unbiasedness is presented in Table 1.

Table 1: Summary Statistics on f (t, t+1) – s (t+1) for One-Month Contracts Involving Major Currencies

	Dollar/ pound	Dollar/ mark	Dollar/ yen
Mean*	0.224	0.308	0.148
Standard deviation*	2.569	3.130	2.944

* All statistics are scaled by 10^2.

In Table 1 means and standard deviations of f(t, t+1) – s(t+1) are reported for the U.S. dollar/pound sterling, U.S. dollar/German mark and U.S. dollar/Japanese yen exchange rates. The statistics were calculated for thirty-day forward contracts. Forward rates and subsequently observed spot rates were taken from the Harris Bank Database supported by the Center for Studies in International Finance at the University of Chicago. The rates are Friday closes, sampled at four-week intervals. There are 148 observations covering the period from 6 June 1973 through 13 July 1984.

From the table it is overwhelmingly clear that mean prediction errors of the forward rate are much smaller than the associated standard deviations. At this point, we are therefore unable to reject the hypothesis that the forward rate is an unbiased predictor of the future spot. This does not mean that the forward rate is necessarily the *optimal* predictor of the future spot rate. In fact, we will discuss evidence to the contrary in subsequent sections. In this section we have simply introduced forward parity as a benchmark relationship between the forward rate and the expected future spot rate which, at first sight, does not appear to be contradicted by the data.

4. The Pricing of Forward Foreign Exchange

In recent years academics in international finance have given a lot of thought to the forward parity relationship, both at the theoretical and empirical levels. In this section we will sketch some of the theoretical considerations and discuss the empirical evidence.

The forward parity relationship implies that the expected rate of return to speculation in the forward foreign exchange market, conditioned on available information, is zero. A number of authors have noted theoretical problems with this zero-expected-return proposition, since it ignores some intertemporal allocation and risk considerations. Equilibrium models of international asset pricing which discuss such considerations include those presented by Adler and Dumas (1983), Fama and Farber (1979), Hodrick and Srivastava (1984) and Stulz (1981). A common element of many models of international asset pricing is that we should expect to find the following equilibrium relationship between forward rates and expected spot rates:

$$f(t, t+1) = E[s(t+1) \mid t] + P(t) \quad (4)$$

where P(t) is a *time-varying premium* term resulting from intertemporal allocation and risk considerations (and not to be confused with the forward premium which was discussed in Section 2). Equation (4) informs us that the forward rate is, in general, not an optimal predictor of the ensuing spot. More sophisticated tests than the one presented in Table 1 do indeed allow us to reject the hypothesis that the forward rate is an unbiased predictor. Hansen and Hodrick (1980) and Hsieh (1984), for instance, show that forward rate forecasts can be improved upon on the basis of publicly available information.

There exists a growing body of empirical research which confirms the existence of time-varying premia in the pricing of forward foreign exchange. Conditional on the hypothesis that the foreign exchange market is efficient or rational, the existence of time-varying premia has been documented by Fama (1984), Hansen and Hodrick (1980, 1983), Hodrick and Srivastava (1984, 1986), Hsieh (1984), Korajczyk (1985) and Wolff (1987a).

Methodologies to measure time-varying premia are usually centered around regression equations. That is, f(t, t+1) – s(t+1) or a similar variable is regressed onto a set of variables which are in market participants' information sets at the time when the forward rate is quoted. Fama (1984) applied regression analysis to estimate empirically the degree of variation of the premium over time and to investigate the degree of covariation of the premium with the expected future spot rate. His findings indicate that most of the variation in forward rates is due to variation in premia and that the premium and expected future spot rate components of forward rates are negatively correlated.

Regression-based approaches depend heavily on the econometrician's choice of the set of variables onto which f(t, t+1) – s(t+1) is to be regressed. This set of variables is fairly arbitrary and usually dictated by availability of data. In WOLFF (1985, 1987a) this problem of arbitrariness of regressors is discussed and an alternative methodology is proposed. Starting point for this methodology is equation (4). Subtracting s(t+1) from both sides of equation (4) and defining $v(t+1) = E[s(t+1) \mid t] - s(t+1)$, we obtain

$$f(t, t+1) - s(t+1) = P(t) + v(t+1) \qquad (5)$$

where v(t+1) is an uncorrelated, zero mean variable. Equation (5) tells us that the forecast error f(t, t+1) – s(t+1) can be interpreted to consist of a *signal,* the premium term P(t), to which a *noise* term, v(t+1), is added. Given this interpretation in terms of signal and noise, it is then conceptually straightforward to apply signal-extraction techniques from the engineering literature (although the details are quite involved). Thus, in WOLFF (1985, 1987a) signal-extraction techniques are applied to f(t, t+1) – s(t+1) in order to recover time series of premia for various currencies. Figure 1 is an example of such a time series for the case of the U.S. dollar/German mark exchange rate. The data used in the computations correspond to those discussed in connection with Table 1.

The one-month premia in Figure 1 are reported in *percent per month.* From the figure we see that the premium for the case of the dollar/mark rate reached a magnitude of up to about 5–6 percent per month in absolute value. As is to be expected on the basis of theoretical models of international asset pricing, premia can be positive as well as negative; see, e.g., the formulas in ADLER and DUMAS (1983).

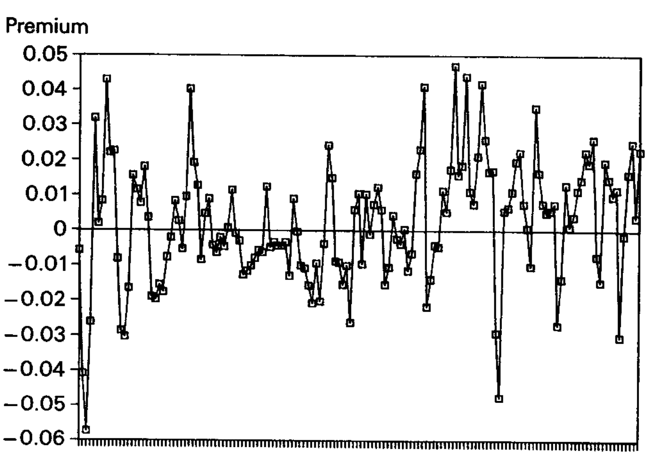

Figure 1: Estimated One-Month Premia for the Dollar/Mark Exchange Rate (6 April 1973 – 15 June 1984).

The time series of premia in Figure 1 – and those for other exchange rates studied by WOLFF (1987a) – exhibits positive serial correlation. That is, premia show a certain degree of *persistence* over time: if today's premium is positive, the probability that next month's premium will also be positive is greater than fifty percent. From the figure we also note that the premium has been quite volatile, a conclusion which is supported by formal volatility estimates reported in WOLFF (1987a).

So far, we have discussed the presence of a premium in the relationship between forward rates and expected future spot rates in purely statistical terms. Given that we have documented its presence, we should then ask ourselves what this premium really is. ADLER and DUMAS (1983) argue convincingly that the premium has two components. One is a *risk premium* component. It is the result of risk aversion on the part of market participants and can be thought of as the *systematic* risk (or *nondiversifiable* risk) of forward speculation. In some international asset pricing models this risk premium is a function of the covariance of the exchange rate with the real return on a world market portfolio, along the lines of standard domestic capital asset pricing models. The second component is somewhat less intuitive; it results from the stochastic (random) nature of inflation in these models; for details, see ADLER and DUMAS (1983).

What does all this mean for economic agents who speculate in forward foreign exchange markets? The presence of the premium term in equation (4) above implies that the expected return to forward speculation is nonzero. That is, one can construct trading rules that will, on average, be profitable. To the extent that the premium is simply a reward for the risk involved in speculation, however, such expected profits do not represent economically meaningful gains (on a risk-adjusted basis).

5. Forecasting Future Spot Exchange Rates

The existence of premia in the pricing of forward foreign exchange has implications for optimal prediction of future spot exchange rates.

In general, forward rates will not be the best available predictors. In principle one can use premium estimates to try to 'correct' forward rates. This has been attempted by WOLFF (1986a), who shows that forward rate forecasts can indeed be improved upon along these lines.

An even better forecasting alternative, however, appears to be available. Recent research indicates that it is very hard indeed to beat the *random walk* forecasting rule for exchange rates between currencies of major industrial countries; see MEESE and ROGOFF (1983a, b) and WOLFF (1985, 1986b, c, 1987b). The random walk model for the spot exchange rate is defined as

$$s(t+1) = s(t) + e(t+1) \quad (6)$$

where e(t+1) is an error term which is on average zero. An implication of the random walk model is that our best prediction of next period's spot rate is simply today's spot rate:

$$E[s(t+1) \mid t] = s(t) \quad (7)$$

since $E[e(t+1) \mid t] = 0$. Note that the 'next period' can be tomorrow, next month or next year. In all cases, according to the random walk forecasting rule, the current spot rate is the best available predictor. The random walk model is an approximate description of the experience in the 1970's and 1980's for currencies of major industrial countries. When dealing with currencies of high-inflation countries, the random walk model is obviously less appropriate, since highly predictable trends are present in the movements of these currencies.

The random walk forecasting rule, indeed, has been shown to perform better than the forward rate as a predictor of the future spot; see MEESE and ROGOFF (1983a), FAMA (1984) and WOLFF (1985, 1986a). An example of this superior forecasting performance is given in Table 2 for the one-month forecasting horizon.

In Table 2 root mean square forecast errors (RMSE's) resulting from the forward rate and the random walk forecasting rule are compared. The RMSE is simply a measure of forecasting accuracy. It is defined as:

$$RMSE = \left[\frac{1}{k}\,\Sigma\,(\text{forecast} - \text{actual})^2\right]^{1/2} \quad (8)$$

where k is the number of observations. The smaller the RMSE, the better the performance of a predictor. From Table 2 we see that

Table 2: A Comparison of Root Mean Square Forecast Errors (RMSE's) of the One-Month Forward Rate and the Random Walk Forecasting Rule

Country[1]	RMSE [Forward][2]	RMSE [Random Walk][2]
Belgium	3.22[3]	3.12
Canada	1.16	1.13
France	3.10	3.04
Italy	2.95	2.89
Japan	3.15	3.05
Netherlands	3.09	3.01
Switzerland	3.83	3.77
United Kingdom	2.65	2.60
West Germany	3.14	3.08

[1] All exchange rates are measured relative to the U.S. dollar, in logarithmic terms.
[2] The RMSE's are approximately in percentage terms.
[3] These statistics are calculated from FAMA (1984, Table 1) and pertain to the period 31 August 1973 – 10 December 1982 (122 monthly observations).

RMSE's are in all cases smaller for the random walk forecasting rule than for the forward rate.

When interpreting this result in the light of equation (4), it is actually quite surprising. According to equation (4), a minimum RMSE predictor of the future spot rate can be obtained by 'correcting' forward rate for the presence of a premium. Against this background, it is not at all clear why the random walk forecasting rule is about the best available predictor. Why would the current spot rate approximate such a 'corrected' forward rate?

6. Conclusion

This article provides an overview of the current state of knowledge with regard to forward foreign exchange rates. Important conclusions are that interest parity holds, that forward parity does not hold and that the random walk forecasting rule provides about the best available forecast of the future spot exchange rate. Evidence is presented and discussed to support these conclusions and some implications for trading in forward foreign exchange markets are explored.

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