Cynthia M. Latta

Duration-based Strategies: Time for Implementation

The importance of duration mismatches in determining portfolio performance in the presence of unanticipated interest rate changes is widely recognized in theory but little honored in practice. There are several reasons for this discrepancy. One is a lack of understanding of the duration concept that has resulted in duration being first oversold, then undersold, as an aid to reducing the variability of portfolio returns. Another is the absence, at least until very recently, of compelling empirical evidence that the outcomes of duration-based strategies are superior to those of the most common alternative strategy, maturity matching. A third is the downtrend in interest rates over the past three years. It is difficult to convince managers of liability-sensitive portfolios – as are those of the vast majority of depository institutions – that their recent good performance is more the result of luck than skillful fund-raising and investing. A possible fourth reason – the vast information requirements of a duration-based interest rate risk management system – is actually the first one in different guise, since the data needed to implement a duration strategy are also essential to sophisticated application of any alternative strategy.

This article deals with the first two reasons why duration receives little more than lip-service from many asset-liability managers. The first section seeks to de-mystify duration; the second reviews the major empirical work on duration. Section III presents some new empirical evidence from tests conducted jointly by this author and others. The conclusion, section IV, considers some of the problems that remain to be solved before any reliable asset-liability management system can be put in place.

I. The Duration Concept

Theory leaves no doubt as to the usefulness of duration in managing interest-rate risk in certain situations [2, 3, 11, 13]. When conditions are right, a proper alignment of the durations of assets and liabilities renders net worth (or alternatively, the capital-to-asset ratio or the ratio of net economic income to assets[1]) nearly immune to interest rate changes. Moreover, this outcome can be achieved without seriously limiting the portfolio manager in his choice of assets and liabilities. Matching maturities, or repricing opportunities, which is the most commonly used approach to managing interest rate risk, can guarantee as good an outcome only if each asset flow, including interim interest payments, is matched with a liability flow. Although technically possible, the diverse demands of savers and borrowers make such detailed matching practically infeasible.

The right conditions, however, are restrictive. Duration works in theory because of the simplifying assumptions it makes about the way interest rate changes are related across the term structure. How well it works in practice is an empirical question. The challenge to proponents of duration is to find a set of assumptions that permits duration to work well enough in any interest rate environment to justify the cost of implementing the strategy derived from those assumptions.

The theoretical beauty and practical shortcomings of duration are easily understood. Let r_t be the per-period rate at which the market discounts payments certain to be received t periods hence. Then the price of a $ 1 payment is given by

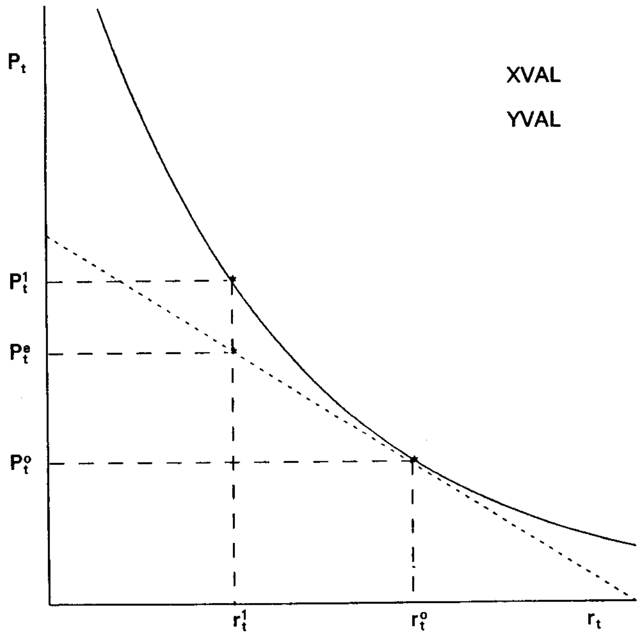

Figure 1: The relationship between the spot rate and price, given t (curved line).

$$P_t = (1+r_t)^{-t} \quad (1)$$

The relationship between the spot rate and price, given t, is shown by the curved line in Figure 1. At any point on the curve, the relationship between a change in the spot rate and a change in price is given by the derivative of P with respect to r:

$$\frac{dP_t}{dr_t} = -t\,(1+r_t)^{-t-1} \quad (2)$$

Equation (2) gives the rate of change in price at any point on the curve. It does not give the change in price associated with an actual change in the interest rate. For small changes in r_t, however, the slope times the change in r_t provides a good approximation:

$$\Delta P_t = -tP_t\,(1+r_t)^{-1}\,\Delta r_t \quad (3)$$

The relationship between (2) and (3) is shown in Figure 1, where the straight line gives the rate of change in P_t only at the point r_t. When the t-period interest rate falls from r_t^o to r_t', the actual change in price is $P_t^o - P_t'$. The change estimated from (3), however, is only $P_t^o - P_t^e$. This inherent failure of (3) to give the exact change in price is known as 'the convexity problem'.

For a portfolio of cash flows, the change in market value following a shift in the term structure is the sum of the changes in price of each individual cash flow as given in (3). For simplicity, consider a coupon bond that matures in m periods and has periodic cash flows of C_i, $i=1, 2, \ldots, m$. Let B be the price of the bond. Then

$$B = \sum_{i=1}^{m} C_i\,(1+r_i)^{-i} = \sum_{i=1}^{m} C_i\,P_i \quad (4)$$

and the approximate change in B following a shift in the term structure is, citing equation (3):

$$\Delta B = \sum_{i=1}^{m} C_i\,\Delta P_i = -\sum_{i=1}^{m} iC_i\,P_i\,\Delta r_i\,(1+r_i)^{-1} \quad (5)$$

Equation (5) as it stands is not very helpful. In order to know the change in the price of the bond, it is necessary to know first just how each of the r_i has changed. If $\Delta r_i/(1+r_i)$ were the same for all i, however, equation (5) would simplify to

$$\Delta B = -[\sum_{i=1}^{m} iC_i\,P_i]\,\dot{r} \quad (6)$$

where $\dot{r}$, the relative change in $(1+r_i)$, is the same for all i. Dividing both sides by B, the initial market price, yields what has come to be called duration:

$$\frac{\Delta B}{B} = -D\dot{r} \quad (7)$$

where the formula for D, duration, is given by

$$D = \sum_{i=1}^{m} iC_i\,P_i / \sum_{i=1}^{m} C_i\,P_i \quad (8)$$

If the summation in (5) is done for both an entire portfolio of assets and the corresponding liabilities,

$$\frac{\Delta A}{A} = -D_A\dot{r} \text{ and } \frac{\Delta L}{L} = -D_L\dot{r} \quad (9)$$

where A is the current market value of assets and L the same for liabilities. If $D_A A$ and $D_L L$ are equal, then assets and liabilities will change by the same amount when the term structure shifts and net worth will be immunized[2]. If $D_A A$ is greater than $D_L L$, then the change in assets will exceed that in liabilities, a result that may generate management bonuses when interest rates fall but dismissals when rates rise. At best, however, these relationships are only approximate. Even if the term structure shifts in the assumed manner, the equations hold exactly only for infinitesimal changes in the term structure. If the term structure does not shift in the orderly way assumed, there are two sources of error: the deviation of actual changes from assumed and the convexity of the price-interest rate relationship.

Duration statistics can be derived from other assumptions about the way interest rate changes are related across the term structure [8, 9]. They all have in common, however, the presumption that all rates move in lock step, i.e. once the change in one rate is known, changes in all the others are known also. Otherwise, it is not possible to summarize in a single statistic the interest sensitivity of a portfolio of cash flows due at different times in the future. If there is just one aberrant rate, a second statistic is needed to reflect the sensitivity of the portfolio to changes in that rate. Thus, assuming that matched durations will immunize against interest rate risk is tantamount to assuming that there is only one factor in the economy that affects interest rates. Perhaps it is monetary policy, which influences directly the federal funds rate, but whatever it is, once the change in any one rate is known, the changes in all others are given.

Although alternative assumptions about the way in which rate changes are related allow considerable flexibility in the way the term structure shifts, casual observation suggests that not one assumption holds for an extended time. Sometimes short rates move more than long, but at other times they do not. Most often, short and long rates move in the same direction, but occasionally they go in opposite directions. Duration-matching may do well in a period when the term structure is shifting according to a given pattern, but less well in a period when the pattern changes. With the duration statistic in [6], for instance, which assumes long rates always move more than short in absolute terms when the yield curve is upward-sloping and cannot change from upward- to downward-sloping, duration matching may fall far short of providing immunization if used over an interval in which short rates move more than long.

An obvious means of increasing the applicability of the duration model is to postulate the existence of at least two separate influences on interest rates, for instance, monetary policy and inflation expectations. Monetary policy might be the only factor affecting the federal funds rate but have no influence on the long-term bond rate, while inflation expectations might swing the long-term bond rate but have no effect on the funds rate. Changes in all other rates would reflect both factors, with the shorter rates being more dependent on monetary policy and the longer ones more influenced by inflation expectations. The result would be a two-factor model, where the $\frac{\Delta r_t}{1+r_t}$ in (3) would be replaced with

$$\frac{\Delta r_t}{1+r_t} = \dot{r}_t = \frac{\partial \dot{r}_t}{\partial f_1}\Delta f_1 + \frac{\partial \dot{r}_t}{\partial f_2}\Delta f_2 \qquad (10)$$

The variables f_1 and f_2 are the factors that affect the term structure. The model can easily be extended to three or more factors. The difficulty comes in specifying the factors and estimating the coefficients of the Δf_i.

For convenience, the f_i are usually identified in two-factor models with two 'reference' rates – typically a very short rate and a very long rate, which we will call r_s and r_l. Then $\dot{r}_t$ becomes an average of changes in the two independent rates.

Two different approaches suggest themselves for specifying the relationship of the remaining rates to the reference rates. One assumes a functional rule[3]. One common assumption is that the changes are proportional to maturities:

$$\frac{\partial \dot{r}_t}{\partial r_s} = \frac{t-s}{l-s} \text{ and } \frac{\partial \dot{r}_t}{\partial r_l} = \frac{l-t}{l-s} \qquad (11)$$

where s, l and t are the terms to maturity of the short, long, and t-period rates, respectively. This formulation leads to the following expression for the relative price change of a bond[4]:

$$\frac{\Delta B}{B} = -k\,[D_1 h_1 + D_2 h_2] \qquad (12)$$

where k, h and h_2 are the constants, D_1 is the same as D in (7) and D_2 is given by

$$\sum_{i=1}^{m} i^2 C_i P_i / \sum_{i=1}^{m} C_i P_i \qquad (13)$$

Net worth will be immunized in this model if $D_1 A = D_1 L$ and $D_2 A = D_2 L$ where, as before, A and L are the market values of the portfolio's assets and liabilities.

The other approach to estimating the responsiveness of the $\dot{r}_t$ to the reference rates is with regression analysis[5]. While this method reflects actual past experience, there is no reason to expect the coefficients to be stable over time. Moreover, since only a finite number of coefficients can be estimated, many of the coefficients required in an actual application must be interpolated. Thus, a functional rule is still required to determine changes at points not selected for estimation.

Even with multiple factors, immunization is possible only to the extent that actual interest rate changes conform to the model. Thus, which model is best is an empirical question. The 'best' model is the one that consistently produces the tightest cluster of actual results about the target. It is possible that there is no best model in this sense, since the one that performs best when the shape of the term structure is stable may not be the one that does best when it is changing. There may be only a second-best model that does reasonably well under all types of conditions. It is also possible that term structure shifts are so erratic that no model improves on results from simple maturity matching of major flows. This result is unlikely, however, since a maturity-based strategy assumes away the problem of interest rate risk by ignoring the effect of rate changes in market values while a duration-based strategy should always protect against some of the actual movement in interest rates.

II. Previous Empirical Work

Numerous tests of both one- and two-factor models have produced mixed results. The first, and most widely publicized, work was a 1971 study by LAWRENCE FISHER and ROMAN WEIL [11] that compared the results of a maturity-matching strategy with those of a duration-matching strategy. The maturity strategy involved buying a coupon bond with maturity equal to the holding period and reinvesting all coupon payments in the same bond. The duration strategy involved purchase of two bonds, one with maturity equal to the holding period and the other the longest available, in proportions that would make the duration of the portfolio equal to the holding period.

Using the duration statistic shown in (8) above, present value factors derived from the DURAND corporate bond yield curves[6] for 1925–1968, and an expected rate of return over m periods equal to the m-period spot rate at the beginning of the period, the authors found that for holding periods of five, ten, and twenty years, the returns from duration matching were closer to the expected return than the returns from maturity matching about 80% of the time. Moreover, moving from maturity- to duration-matching reduced the variability of the results by about 16% for the five-year holding periods, 29% for the 20-year periods and 62% for the 20-year periods. 'These reductions are so dramatic', the authors wrote, 'that we conclude that a properly chosen portfolio of long-term bonds is essentially riskless' [9, p. 423].

Subsequent studies using different data sets, time periods and alternatives often failed to replicate the extremely favorable results of FISHER and WEIL[7]. Although duration strategies usually did at least as well as the alternatives and sometimes better, unqualified support for duration proved elusive. Any generalization of the findings is difficult, however, because different studies used different standards of comparison in judging the effectiveness of duration-based strategies. Some of the studies that purported to prove the inferiority of duration matching actually showed only that a two-factor model outperformed a one-factor model, a result that would be expected a priori[6]. Moreover, few studies made any attempt to eliminate pricing errors from the bond data, a step that is necessary because recorded bond prices often reflect educated guesses rather than prices of actual trades. Without clean data, it is impossible to know whether a failure of duration to do as well as expected is the result of poor data or what has become known as 'stochastic process risk', the failure of interest rates to change in the assumed manner.

III. New Work

The combination of high expectations generated by the seminal FISHER-WEIL study and the mediocre results of later testing led to serious questioning about the value of implementing a duration-based risk management strategy. Believing, however, that much of the poor showing for duration resulted from improper test procedures, this author in conjunction with G. O. BIERWAG and GEORGE G. KAUFMAN (BKL) formulated a set of tests that would be free of the defects of earlier studies. The clean data requirement can be met in many ways. The BKL work used synthetic, noise-free bonds constructed from the zero-coupon equivalent of a par value market yield curve. The yield curve was obtained by fitting the yields to maturity of 18 U.S. on-the-run Treasury securities with maturities ranging from one month to 30 years to a polynomial in the log of maturity. Spot rates were then derived from the yield curve and a term structure fitted, again to a polynomial in the log of maturity.

The target return for the tests was the promised return on a zero-coupon security with a maturity equal to the holding period since this is the only bogey that is independent of the assumptions about the future behavior of the term structure. The test consisted of comparing the returns from duration-matched portfolios with the return on a single bond with maturity equal to the holding period and reinvestment of all coupon payments in the same bond. In the duration strategy, the portfolios were rebalanced, using the same bonds, every six months to keep the duration equal to the remaining holding period.

Using an equilibrium model of bond returns and the assumption that any two arbitrarily selected (reference) rates on the term structure can shift independently, the authors derived an immunizing condition valid under a wide range of assumptions regarding the stochastic process underlying shifts in the term structure. Adding the specific assumption that changes in individual points on the term structure are linear weighted averages of the changes in the two reference rates yielded the duration measures D_1 and D_2 as in (12) above. As shown there, if there is in fact only one independent rate, D_2 becomes irrelevant and the model collapses to a single-factor model.

Both the one- and two-factor models were tested. The tests covered the years 1975–1985, a period of considerable interest-rate volatility. Tests were run for the five-year periods starting in August and November of 1975 and 1985 and the eight 2½-year subperiods contained in the longer periods. Despite the overlap, there were no common observations because portfolios were rebalanced only every six months. Sixty portfolios of randomly selected bonds were used for the 5-year tests; 40 for the 2½-year tests.

The results were encouraging[8]. On the four five-year tests, the average absolute difference between the return promised by the zero-coupon security and the actual return from duration matching was only ¼ to ⅛ as large as the difference from maturity matching. For the 2½-year periods, duration turned in the better performance in 7 out of 8 time periods. The addition of a second factor cut the average absolute difference by ⅓ to ½ for the 5-year periods and by ⅕ to ½ for 7 of the 8 shorter periods. The second factor also sharply reduced the dispersion of results in 11 of the 12 periods tested.

The initial set of tests used two-bond portfolios for the single-factor model and three-bond portfolios for the two-factor model. Thus, it was possible either that the small number of bonds accounted for the overall good results or that the extra bond in the two-factor test, not the second factor itself, accounted for the better results of the two-factor tests. To check for these possibilities, both the one- and two-factor tests were repeated for the five-year periods using four- and five-bond portfolios. The portfolios contained the same bonds for both models, although the proportions of the bonds differed because of the different restrictions in the two tests. The two-factor model still outperformed the one-factor and there was no significant deterioration relative to the results for the two- and three-bond portfolios [5].

These favorable results were obtained with the arbitrary but convenient assumption that interest rates can serve as proxies for the true factors affecting interest rates and that shifts in points on the term structure are linear weighted averages of those rates. There are many possible alternative assumptions that could be used, however, some of which are more appealing than others. Consider, for instance, the assumption that the two independent rates are a very short, say overnight, rate and the 30-year rate. Then the standard method of weighting the two – see (11) above – says that the 15-year rate is affected equally by changes in the two reference rates. Experience, however, suggests that the 15-year rate behaves much more like the 30-year than the overnight rate. Hence, a more realistic assumption might be that the weights are proportional to the location of the rates on a logarithmic time scale rather than an absolute scale. Substituting logarithms of maturity for maturity in (11) does not affect the formula for D_1 but changes the i^2 in the formula for D_2 to ilog(i).

Substituting the logarithmic for the absolute weights did tend to improve the results, which are summarized in the Appendix. Since the first factor under this formulation is the same as in the original model, the crucial question is whether the revised D_2 measure produces better results in the two-factor model than the original D_2 measure. In the initial set of tests using three-bond portfolios, the average absolute error was smaller using the log formulation for the second factor than the squared time to payment for three of the four five-year periods. The standard deviation of the errors was smaller in three cases and the same in one. For the eight 2½-year periods, the log formulation produced larger average errors in two and larger standard deviations in three. When the five-bond portfolios were used for all the tests, the log formulation was clearly superior in each of the five-year periods. The 2½-year tests have not yet been run using five-bond portfolios.

IV. Conclusion

Recent work by this author and others strongly support the effectiveness of duration in hedging portfolios against unanticipated changes in interest rates. It also suggests that the effectiveness of hedging increases when the strategy is expanded to deal with the limitations of assuming all spot rates move in lock step. Earlier studies did not produce such unambiguous support, but they suffered several shortcomings of design and implementation.

Persuading asset-liability managers that duration-based strategies can be genuinely useful will not, necessarily, be easy. First, many managers expected more from duration than just another imperfect hedge. They should not have, but incomplete understanding of the concept caused the caveats to be ignored. Second, hedging against unanticipated changes in market values is of limited interest to institutions that do not use market value accounting. Net interest income (NII) in the short run is often of more immediate concern than the market value of net worth, and the most effective way of maintaining NII in the short run is to match cash flows of assets and liabilities insofar as possible. To the extent that flows of receipts and payments coincide, profitable interest-rate spreads, hence NII can be maintained. Positive income, however, does not preclude the simultaneous erosion of net worth. Thus, a careful manager will concern himself not only with generating a positive cash flow but also with hedging the value of his existing franchise against the possible untoward effects of unexpected interest rate fluctuations.

Implementing a duration strategy does require a large amount of information. Ideally, every future cash flow is specified, both as to time of payment and expected amount, i.e. amount adjusted for expected default. Present value factors are also required for discounting the payments, but these are readily derived from the Treasury yield curve. Given this information, computation of the statistics required by either a single- or multiple-factor model is a simple matter. The most difficult-to-obtain data, the estimates of probable cash flows, are also crucial to maturity-matching strategies. Al-

Appendix: Summary of test Results for Maturity- and Duration-Matching Strategies (Deviations of Actual from Promised Returns)

	Maturity	One Factor		Two Factors*			
	Deviation	Avg.	Std. Dev.	Avg.	Std. Dev.	Avg.	Std. Dev.
5-Year Periods				A		B	
8/75– 8/80	0.0732	0.0176	0.0091	0.0056	0.0039	0.0050	0.0022
11/75–11/80	0.0668	0.0083	0.0052	0.0036	0.0037	0.0042	0.0036
8/80– 8/85	0.1345	0.0336	0.0208	0.0171	0.0154	0.0106	0.0074
11/80–11/85	0.1876	0.0439	0.0232	0.0232	0.0141	0.0200	0.0097
2½-Year Periods							
8/75– 2/78	0.0138	0.0131	0.0089	0.0074	0.0045	0.0053	0.0034
2/78– 8/80	0.0058	0.0164	0.0128	0.0066	0.0069	0.0030	0.0035
11/75– 5/78	0.0116	0.0088	0.0052	0.0060	0.0027	0.0060	0.0032
5/78–11/80	0.0076	0.0051	0.0038	0.0026	0.0020	0.0026	0.0029
8/80– 2/83	0.0145	0.0085	0.0047	0.0097	0.0054	0.0101	0.0063
2/83–11/85	0.0119	0.0084	0.0054	0.0069	0.0040	0.0096	0.0038
11/80– 5/83	0.0297	0.0035	0.0110	0.0075	0.0048	0.0054	0.0041
5/83–11/85	0.0165	0.0154	0.0082	0.0120	0.0057	0.0104	0.0048

Deviations are absolute deviations from promised return in decimals over the entire period. The average is over 60 portfolios for the 5-year periods, 40 portfolios for the 2½-year periods. For the maturity strategy, there is only one deviation.

* Version A uses time to payment squared (t^2) in calculating D_2; version B uses time to payment times log of time to payment [t log (t)].

though much remains to be done on estimating cash flows in the presence of credit risk and options, the usefulness of duration measures in short-term trading means that considerable resources are being devoted to the problem. Thus, the real need is to persuade the stewards of net worth that use of duration can contribute to the achievement of long-run performance goals.

Footnotes

[1] Net economic income is net operating income plus the gain or loss from marking all accounts to market.

[2] For the conditions that immunize the capital-asset ratio and net income-asset ratio, see [4].

[3] This approach is used in [1] and [13].

[4] Substituting (11) into (10) and (10) into (3) and simplifying,

$$\Delta P_t = -tP_t \left(\frac{t-s}{1-s} \dot{r}_s + \frac{1-t}{1-s} \dot{r}_1 \right)$$

$$= -\frac{1}{1-s} [t^2 P_t (\dot{r}_s - \dot{r}_1) + tP_t (1\dot{r}_1 - s\dot{r}_s)]$$

Let $h_1 = 1\dot{r}_1 - s\dot{r}_s$, $h_2 = \dot{r}_s - \dot{r}_1$, and $k = 1/(1-s)$. Since h_1, h_2, and k are the same regardless of t, the change in the price of a bond may be written as

$$\Delta B = \sum_{i=1}^{m} C_i \Delta P_i = -k [(\sum_{i=1}^{m} i C_i P_i) h_1 + (\sum_{i=1}^{m} i^2 C_i P_i) h_2]$$

In proportional terms,

$$\frac{\Delta B}{B} = -k [(\sum_{i=1}^{m} i C_i P_i / \sum_{i=1}^{m} C_i P_i) h_1 + (\sum_{i=1}^{m} i^2 C_i P_1 / \sum_{i=1}^{m} C_i P_i) h_2]$$

$$\text{or } \frac{WB}{B} = -k (D_1 h_1 + D_2 h_2)$$

If $\dot{r}_s = \dot{r}_1 = \dot{r}$, then $h_2 = 0$, $h_1 = \dot{r}\,(1/k)$ and the right-hand side of (14b) collapses to $-D_1 \dot{r}$, making $\Delta B/B$ the same as in (7).

[5] This approach is used in [16]. Brennan and Schwartz [9] use regression analysis to estimate a stochastic process which is then combined with a risk aversion parameter and an equilibrium bond pricing model and solved for points on the terms structure.

[6] Construction of these curves is described in David Durand, *Basic Yields of Corporate Bonds, 1900–1942*, Technical Paper No. 3 (Cambridge, MA: National Bureau of Economic Research, 1942).

[7] For a summary of the findings, see [15].

[8] See [12] and [14] for such models and [6] for a critique.

[9] For a more complete discussion of the tests and results, see [5].

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