

Equity Pricing and Stock Market Anomalies

1. Financial theory and anomalous empirical regularities

You are asked to determine which of two portfolios of securities has the higher expected return. You do not know the composition of the two portfolios and you are allowed to ask a single question. Which question should you ask?

If you believe in certain fundamental principles of modern finance, you should ask which of the two portfolios has the higher risk since modern finance suggests that there is a positive relation between risk and return. If you buy the high-risk, high-expected-return portfolio and hold it over a sufficiently long period of time (say a few years), that portfolio should earn a higher actual return than the low-risk, low-expected-return portfolio¹.

Unfortunately, reality is not that simple. We need to be more specific about what we mean by 'risk'. What is the risk of a portfolio and how is it measured? What is the relation between a portfolio's expected return and its risk? In the following sections we review the concept of risk in the context of the Capital Asset Pricing Model (CAPM). This model, developed in the early sixties (SHARPE, 1964; LINTNER, 1965), provides a simple theoretical relationship between the expected return and the risk of a financial asset. But the purpose of this article is not to review the modern theory of finance. Our objective is to examine the *validity* of the CAPM in light of several recent *empirical* findings which appear to be at odds with some of the fundamental principles of finance. Indeed, since the early eighties, a growing number of empirical studies based on U.S., European and Japanese stock

market data have reported several anomalous empirical regularities also known as 'stock market anomalies' (HAWAWINI, 1984; KEIM, 1986). These *recurrent* empirical phenomena are not consistent with asset pricing according to the CAPM.

For example, recall the question which we asked in the opening paragraph: which of two portfolios has the higher expected return and hence the higher probability of achieving the higher average return in the near future? According to modern finance, it is the portfolio with the higher risk. But a substantial body of empirical evidence suggests that the portfolio containing the securities of firms with the smaller market capitalization (number of shares outstanding multiplied by the share price) should, on average, outperform the portfolio with the larger market capitalization. In other words, firm size may be a stronger indicator of expected return than risk. Of course, firm size may be interrelated with risk and we would intuitively expect small firms to be 'riskier' than large firms. Nevertheless the evidence shows that small firms outperform, on average, large firms even if one adjusts returns for differences in risk. In other words, portfolios of small firms will, on average, outperform portfolios of large firms even after controlling for differences in risk between the two portfolios. This so-called size effect, and other stock market anomalies, will be examined in more detail in this article. But first, we briefly review what risk is, how it is measured and how it is related to expected return according to the CAPM.

2. What is risk?

Securities are risky because their price fluctuations are unpredictable. And the wider the

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range of a security's price fluctuations, the higher its risk. To measure the risk of a security, we need to know the range of the possible outcomes and the likelihood of their occurrence, i.e., the probability distribution of its returns. Consider the following simple example. A security currently priced at 100 will be worth, a month from now, either 105 or 115 with an equal probability. In other words, its monthly return will be either 5 percent or 15 percent with each outcome equally likely. From the knowledge of this distribution of returns we can deduce two important characteristics of that security. Its *expected return* is 10 percent² and the *standard deviation of its return* is 5 percent. The latter is a measure of the dispersion of returns around the mean value of 10 percent³. If the distribution of returns is approximately symmetrical, the standard deviation of returns or its square, the variance, is a good measure of the extent to which the price of a security fluctuates and hence a good measure of the total risk of a security.

One of the main building blocks of the CAPM is the principle of risk decomposition. The total risk of a security can be broken down into two independent components: a market-related component and a firm-specific component. The former is a measure of the extent to which the price of a security fluctuates in response to the general market movements. The latter is a measure of the extent to which the price of a security fluctuates in response to information unique to the firm which issued the security. We can write:

$$\text{TOTAL RISK} = [\text{MARKET RISK}] + [\text{FIRM-SPECIFIC RISK}]$$

where the variance is the measure of the total risk of a security. In our numerical example we have a variance of returns equal to 0.25 percent, the square of 5 percent.

It can be shown that the market risk of a security is proportional to the variance of the market as a whole:

$$\left[\frac{\text{MARKET RISK OF SECURITY } i}{\text{THE MARKET}} \right] = (\beta_i)^2 \left[\frac{\text{VARIANCE OF THE MARKET}}{\text{THE MARKET}} \right]$$

where the proportionality factor β_i is called the beta coefficient of security i or its *systematic risk*. It is a measure of the sensitivity of security i 's return to the returns of the market. A security with a beta coefficient equal to one has as much market risk as the market as a whole. But

a security with a beta coefficient greater than one has more market risk than the market as a whole. And a security with a beta coefficient smaller than one has less market risk than the market as a whole. In other words, high-beta stocks have higher market risk than low-beta stocks⁴. To return to our example, suppose that the monthly variance of the market is 0.10 percent and that security i has a beta coefficient of 0.80. Its market risk is $(0.80) (0.10 \text{ percent}) = 0.08 \text{ percent}$. And since its total risk is 0.25 percent it follows that security i has a firm-specific or residual risk of 0.17 percent. Note that 32 percent $(0.08 \text{ percent divided by } 0.25 \text{ percent})$ of the total risk of security i is generated by the market movement (market risk) and the remaining 68 percent of its total risk is generated by information unique to firm i (firm-specific risk)⁵.

3. The CAPM: A link between expected return and market risk

Modern financial theory builds on two basic facts. First individuals dislike risk, i.e., they are risk-averse. Second, security prices do not move in perfect unison, i.e., their returns are less than perfectly correlated. As a result, increasing the number of securities in a portfolio will generally reduce the total risk of that portfolio. As a result, investors will tend to hold well-diversified portfolios. What then is the relevant measure of the risk of a security when that security is part of a well-diversified portfolio? It can be shown that only the market risk of a security is relevant in a portfolio context because the firm-specific risk is diversified away. Indeed, an investor holding a well-diversified portfolio of securities only bears the market risk of the securities making up the portfolio. The firm-specific component of the risk of the securities will offset one another and approach zero as the size of the portfolio increases.

Since the firm-specific risk of a security can be eliminated by investors by simply holding the security as part of a portfolio, the firm-specific risk of a security is irrelevant in the pricing of that security. In other words, since investors do not have to bear firm-specific risk, investors should not be compensated for it. They should only be remunerated for bearing

market risk because market risk cannot be diversified away. Thus, the expected return of a security (the remuneration for holding that security) must be related only to the market risk of that security. Securities with high-beta coefficients (a measure of their market risk) must earn higher expected returns than securities with low-beta coefficients.

The Capital Asset Pricing Model (CAPM) gives the theoretical relationship that must exist in equilibrium between an asset's expected return ($E[R_i]$) and its beta coefficient (β_i):

$$E(R_i) = R_F + [E(R_m) - R_F] \cdot \beta_i$$

According to the CAPM, the expected return on a risky asset is equal to the return on a risk-free asset (R_F) plus a risk premium which is proportional to the beta coefficient of that asset. The proportionality factor (also called the market price of one unit of risk) is the difference between the expected return on the market as a whole (indicated by the subscript m) and the risk-free rate of return. If the monthly risk-free rate is 1 percent and the expected monthly return on the market is 3 percent, then:

$$E(R_i) = 0.01 + 0.02 \beta_i,$$

and the expected return of an asset with a beta coefficient of 1.5 is 4 percent according to the CAPM, the sum of 1 percent risk-free rate and a risk premium of $(0.02)(1.5) = 3$ percent.

4. Early evidence on the validity of the CAPM

How can we test the validity of the CAPM? Note that the model is based on expected returns which are not observable. But if the relation between security returns remains relatively stable through time, then historical returns can be used to estimate the unobservable expected returns. Thus, to verify the empirical validity of the model we can examine the historical relation between portfolio returns and their corresponding estimated beta coefficients⁶. If that relation is linear with an intercept equal to the risk-free rate and a slope equal to the excess return on a broad market index then the CAPM is a valid model of stock price behavior. We will have an additional test of the validity of the CAPM if we can show that firm-specific risk is unrelated to average returns. Recall that, according to the CAPM, firm-specific risk can be

diversified away and, hence, should not be priced in the market. In other words, an asset's firm-specific risk should not be related to that asset's average return.

One of the first rigorous tests of the CAPM was performed by FAMA and MACBETH (1973) on portfolios of stocks traded on the New York Stock Exchange (NYSE) from 1935 to June 1968. They estimated the relationship between average returns (\bar{R}_i) and beta (β_i), beta squared (β_i^2), and firm-specific risk. They found the following relationship:

$$\bar{R}_i = 0.0020 + 0.0114\beta_i - 0.0026\beta_i^2 + 0.0516$$

[firm-specific risk]

(0.55) (1.85) (-0.86) (1.11)

where the number in parentheses below the estimated average coefficients are t-statistics⁷. They indicate whether these coefficients are significantly different from zero. In this case, when the absolute value of the t-statistics exceeds 1.70, we conclude that there is a 95 percent chance that the estimated coefficient is different from zero. Otherwise, the estimated coefficient is not significantly different from zero. Referring to the estimated relationship we can see that average returns are positively related to beta (t-statistics > 1.70) but unrelated to beta-squared or firm-specific risk (t-statistics < 1.70). The conclusion is obvious. The CAPM, when tested over a long period of time on the NYSE, cannot be rejected. Market risk is the only factor which has a significant relationship with the average return. Firm-specific risk is irrelevant for security pricing and so is beta squared⁸. Also, FAMA and MACBETH found that the intercept is not significantly different from the average risk-free rate over the test period, a result consistent with the CAPM.

Similar, but somewhat weaker, conclusions were reached when the CAPM was first tested on European data. In general, the price behavior of European common stocks was found to be consistent with the CAPM in the United Kingdom (GUY, 1976), Germany (GUY, 1977), France (HAWAWINI et al., 1983), Spain (PALACIOS, 1973) and Belgium (HAWAWINI et al., 1982). These and other studies are reviewed in HAWAWINI (1984).

Despite the empirical evidence supporting the CAPM, it has not been universally accepted. Criticisms range from its simplicity to the

problem of how one should define 'the market' (ROLL, 1977). Indeed, the CAPM is a single period model and it assumes that all market participants have a common view of the future, to mention but two of its more contentious properties. And defining 'the market' is not an easy task. In the U.S., it is possible to pretend that all NYSE stocks constitute the market, but in the relatively small European markets, clearly some kind of world market portfolio would be a better proxy given the actual investment practices of European investors. These criticisms notwithstanding, the CAPM is a useful and important model of asset pricing: its simplicity is its major strength. It focuses our attention on what appears to matter most for security pricing: market risk.

5. The size effect

According to the CAPM, the only determinant of the expected return on a firm's common stock is its market risk. This implies that other characteristics of firms, such as their size, should not be related to average returns. In other words, firms with relatively small market

capitalizations should not have average returns that differ systematically from those of firms with relatively large market capitalization.

But the evidence indicates that size affects average returns and that small firms earn, on average, higher returns than large firms. As we pointed out earlier, this phenomenon cannot be attributed to risk differentials between small and large firms. Small firms tend to outperform their larger counterparts on even a risk-adjusted basis. Some of the evidence for the United States, the United Kingdom, Belgium and Finland is summarized in Table 1. The United Kingdom has the largest equity market in Europe and the world's third largest market (after the U.S. and Japan). The Belgian and Finnish markets are among the smallest in Europe.

First note the difference in *size* between the largest and smallest portfolios in the samples drawn from the four countries. The largest portfolio is 124 times larger than the smallest portfolio in the United States, 182 times larger in the United Kingdom, 188 times larger in Belgium and 113 times larger in Finland. The annualized size premium (the difference between the annual returns on the smallest and the largest portfolios) is equal to 10.08 percent in the Unit-

Table 1: Evidence of a size premium in four countries

Country ¹	U.S.	U.K.	Belgium	Finland
Test period	1926–1979	1958–1982	1971–1983	1970–1981
Market value of largest portfolio of firms divided by market value of smallest portfolio of firms	124 (in 1975)	182 (average over test period)	188 (average over test period)	113 (in 1970)
Average annualized return of the smallest portfolio of firms ²	21.24%	15.90%	14.04%	19.78%
Average annualized return of the largest portfolio of firms	11.16%	11.15%	7.80%	10.64%
The size premium (small–large)	10.08%	4.75%	6.24%	9.14%
Average beta of smallest portfolio:				
– standard beta ³	1.45	0.31	1.01	0.36
– adjusted beta ⁴	1.60	0.64	N.A.	0.50
Average beta of largest portfolio:				
– standard beta ³	0.96	1.01	0.98	1.00
– adjusted beta ⁴	0.93	1.02	N.A.	0.95

¹ The evidence for the U.S. is from BANZ (1982), for the U.K. from LEVIS (1985), for Belgium from HAWAWINI et al. (1986) and for Finland from BERGLUND and WAHLROOS (1983).

² For the U.S. there are 501 to 1530 NYSE stocks ranked by size into 5 portfolios with equal numbers of securities; for the U.K. there are 1500 to 2400 stocks ranked into 10 such portfolios; for Belgium there are 170 stocks ranked into 5 portfolios; and for Finland there are 50 stocks ranked into 10 portfolios. Annualized returns are monthly returns multiplied by 12.

³ Standard betas are estimated using ordinary least square regression (see footnote 6).

⁴ Adjusted betas are estimated using the DIMSON (1979) method. This method adjust for the thin trading that characterizes smaller firms. The adjusted betas of smaller firms tend to be larger than their standard betas.

ed States (over a 54-year test period); it is equal to 4.75 percent in the United Kingdom (over a 25-year test period), 6.24 percent in Belgium (over a 13-year test period), and 9.14 percent in Finland (over a 12-year period). The size effect exists in the four countries. But since the sample periods are different, we cannot be certain whether the magnitude of the effect is significantly different in these countries.

What about risk? Can differences in risk between the smallest and largest portfolios explain the size premium? The evidence is found in Table 1. In the three European countries, the market risk of the smallest firms is either lower than the market risk of the largest firms (United Kingdom and Finland) or the same (Belgium). It is not clear why small firms in the United Kingdom and Finland have, on average, lower betas than large firms but the implication is clear. If we adjust for differences in risk, the size premium would be higher than the one reported in Table 1 in the case of the United Kingdom and Finland (since small firms have lower betas) and about the same in Belgium (since small and large firms have, on average, roughly the same betas in that country). In the United States we have the opposite phenomenon: small firms have, on average, *higher* market risk than large firms. This is intuitively more appealing. Small firms are, on average, *more* risky than large firms in the United States. But the higher market risk of small firms in the United States does not account for the size premium in that country. The risk-adjusted size premium is smaller than the 10.08 percent reported in Table 1 for the United States but still significantly different from zero.

6. The size effect is not a stationary phenomenon

One obvious question at this point is whether the size effect is a stationary phenomenon. The evidence suggests that there is little stationarity. There are periods of time over which *large* firms, on average, outperform *small* firms. Also, the magnitude of the size premium is not constant. For example, in the case of the United States reported in Table 1, the size premium varied between a low of -10.20 percent over the subperiod 1926-1930 and a high of 43.80 percent over the subperiod 1931-1935. Also the magnitude and the sign of the size premium are

not the same, on average, in all months of the year. For example, in the United Kingdom, almost 50 percent of the total average size premium reported in Table 1 is earned during the month of *May* alone. In the case of Belgium we have reported in Table 1 a total average size premium of 6.24 percent. But the annualized average size premium is equal to 29.76 percent in *August* alone.

The size effect is clearly time dependent. It depends on the length of time over which the size premium is averaged out. It also varies throughout the year.

7. Other stock market anomalies

Size is not the only characteristic of firms which could be used to earn 'abnormal' returns, that is, returns exceeding those predicted by the CAPM. BASU (1977) postulated the existence of a relation between P/E-ratio and return. He examined the average annual returns, from April 1957 through March 1971, of over 750 NYSE stocks assigned to one of five portfolios on the basis of the magnitude of their year-end price-earnings ratio. He found that the lowest P/E portfolio earned, on average, 8 percent more than the highest P/E portfolio after adjusting returns for risk according to the CAPM. Is the P/E effect another anomaly or is it simply another manifestation of the size effect? Low P/E firms are mostly small firms and the P/E effect could be just a proxy for the size effect. This is the conclusion reached by several authors who examined both effects (REINGANUM, 1981a, and BANZ and BREEN, 1986). Once we control returns for differences in risk and size, and eliminate the bias introduced by the inadequate use of historical data, the P/E effect disappears.

Other stock market anomalies include a price-to-book value ratio effect (STATTMAN, 1980), a 'neglected'-firms effect (ARBEL and STREBEL, 1982), a period-of-listing effect (BARRY and BROWN, 1984) and a dividend-yield effect (KEIM, 1985). Firms with relatively low price-to-book value ratios seem to outperform, on average, firms with relatively high price-to-book value ratios. Firms which are not followed regularly by financial analysts and which are not widely held by institutional investors tend to outperform firms which are scrutinized by analysts and adopted by institutional investors.

Firms listed on the NYSE for the *least* number of months earn, on average, abnormal returns. So do firms with either *zero* or high dividend yields⁹. Some of these phenomena are related to the size effect (the price-to-book value effect and the 'neglected'-firms effect). Others seem to persist even after returns are adjusted for size (the period-of-listing effect and the dividend-yield effect). Some also exhibit seasonality. For example, the dividend-yield effect manifests itself mostly in the month of January. All these phenomena were uncovered using U.S. stock market data. At the time of this writing there were no similar studies based on European data but our suspicion is that anomalies similar to those found in U.S. stock market data will soon be uncovered in European stock market data.

8. A look at the CAPM in light of the size effect

We have reported the results of the FAMA and MACBETH test of the CAPM in a previous section and concluded that the evidence was consistent with equity pricing according to the CAPM. But if there exists a size premium in the equity market, the FAMA and MACBETH test would not capture it. The relationship between average returns and market risk must be re-examined in light of the size effect.

The first test of the CAPM with an explicit recognition of size was performed using U.S. stock market data (BANZ, 1981). Average returns were found to be positively related to market risk and negatively related to firm size. But the relationship between average returns, market risk and firm size is unstable and seasonal (TINIÇ and WEST, 1986). The evidence for two European markets, the United Kingdom and Belgium, is summarized in Table 2. When the entire sample is considered, portfolio returns are, on average, unrelated to market risk (note that the t-statistics for the risk premium are smaller than 1.70) and inversely related to size (the t-statistics for the size premium are greater than 1.70). The conclusion is clear: over the 27-year period from 1957 to 1983 in the United Kingdom and over the 13-year period from 1971 to 1983 in Belgium, a strategy that consists of holding portfolios of small firms would have outperformed a strategy that consists of holding portfolios of high-risk firms.

Table 2: Estimation of the average risk premium and the average size premium based on the relationship:

$$R_i = \gamma_0 + \gamma_1 \cdot \beta_i + \gamma_2 \cdot \text{SIZE}_i^*$$

Estimated coefficient average over	$\tilde{\gamma}_1$ (risk premium)	$\tilde{\gamma}_2$ (size premium)
<i>U.K. (1/57 to 12/83)</i>		
All months	-0.0032 (-1.44)	-0.0008 ** (-1.96)
April only	0.0157 ** (2.10)	-0.0004 ** (-2.13)
<i>Belgium (1/71 to 12/83)</i>		
All months	-0.0020 (-1.30)	-0.0008 ** (-2.30)
January only	0.0172 ** (3.05)	-0.0022 ** (-1.95)

* For the United Kingdom, size is measured by the logarithm of the market value of the firm. See CORHAY et al. (1987a). For Belgium, size is measured relative to the average market value of all firms, i.e., by the ratio: $(V_i - V_m)/V_m$ where V_i is the market value of firm i and V_m is the average market value. See HAWAWINI et al. (1986).

** Indicates coefficients that are significantly different from zero. The absence of the double asterisks indicates that the coefficient is not significantly different from zero.

But we said earlier that the relationship between return, market risk and size is seasonal in the United States. This is also the case in Europe. A look at Table 2 indicates that returns are significantly positively related to market risk and significantly negatively related to size only in *April* in the United Kingdom and only in *January* in Belgium. The conclusion is that a strategy that consists of holding portfolios containing firms that are *both* small and risky would have outperformed the market average in April in the United Kingdom and January in Belgium. As a matter of fact, a high-beta, small-firm strategy implemented during January in Belgium would have earned 10.31 percent (monthly return) over the 13-year period from January 1971 to December 1983. Over the same period, the average monthly market return was 0.84 percent. The January, small-firm, high-beta strategy earned a return over 12 times greater than a market-average strategy (exclusive of transaction costs)!

9. Interpreting the size effect

There are basically two possible interpretations of the small firm effect. The first is that the phenomenon does not exist; its appearance is simply the result of poor measurement proce-

dures. If stock returns and market risk were measured properly, the size effect would disappear. The second interpretation is that the size effect is the result of an incorrect specification of the pricing model (the CAPM) we are using to calculate the risk-adjusted size premium. If we had a 'correct' model of the risk-return relationship, which would incorporate all aspects of risk relevant for investors, then the risk-adjusted size-premium would disappear.

Suppose that the market risk of small firms is systematically underestimated and that the market risk of large firms is systematically overestimated. In other words, the 'true' market risk of small firms is higher than their estimated market risk and the 'true' market risk of large firms is lower than their estimated market risk. If that is the case, then the 'true' risk-adjusted return on small firms will be lower than the observed return (and that of large firms will be higher) with the end result that the risk-adjusted size premium may disappear. This outcome will be reinforced if the 'true' return on small firms were lower than those estimated from the data. This may be the case if we account, for example, for the higher costs of trading small firms. In this case the size-premium measured *net* of transaction costs will be smaller if small firms have higher transaction costs than their larger counterparts¹⁰.

But why should the estimated market risk of small firms be below its 'true' value? The estimated beta coefficient of infrequently traded stocks is lower than their 'true' beta coefficient, and since small firms tend to trade relatively infrequently, their beta coefficients are underestimated. Adjusted betas, which correct for thin trading smaller firms, have been developed by several authors (DIMSON, 1979, and COHEN et al., 1983). A look at Table 2 indicates that the adjusted betas of small firms are indeed higher than their standard betas. But even with adjusted betas, the size premium remains. Hence, adjusted betas reduce but do not eliminate the size premium.

The second possible explanation of the size effect is that the CAPM does not adequately adjust for risk and hence is not a reliable model to calculate risk-adjusted returns¹¹. This explanation is, of course, to some extent a truism. The CAPM is an incomplete model as the evidence has shown. But there are several kinds of risk which might be responsible for the size

anomaly. Suppose smaller firms are perceived to be riskier because investors have relatively less information about them than about larger firms. If that is the case, then the excess return earned on small firms is simply a compensation for holding riskier securities. This extra risk may not be captured by the beta coefficient and size may simply act as a proxy for a missing risk factor introduced by differential information across firms.

Note that the above interpretations of the small firm effect do not explain its seasonal behavior¹². Why is the risk-adjusted size-premium earned during particular months of the year? Specifically, January in the United States and Belgium and April in the United Kingdom? January is the first month of the fiscal year in the United States and Belgium. April is the first month of the fiscal year in the United Kingdom. It is possible that the small firm effect is partly tax induced. According to that hypothesis, investors can reduce their taxes by selling the stocks on which they lost money during the year. In doing so they realize capital losses that are deductible from their taxable income. The sale of securities at the end of the fiscal year depresses their prices which recover at the beginning of the next fiscal year as stocks move back to their equilibrium value. Because small-firm stocks are more volatile than large-firm stocks they are more likely to be candidates for year-end tax-selling and hence the observed January size-premium in the United States and Belgium and the April size-premium in the United Kingdom. The evidence is, however, far from conclusive with regard to this 'tax-loss selling effect' (CHAN, 1986)¹³.

At this time we can only offer interpretations and hypotheses. We do not yet have a definitive explanation of the small firm effect.

10. Implication for corporate treasurers and portfolio managers

We conclude with some practical implications of the small firm effect. Assuming that the phenomenon is not just the outcome of measurement errors then the size premium earned on small firms implies that investors demand (and obtain) higher return in order to hold smaller firms. This, in turn, implies that smaller firms have a higher cost of equity than their

larger counterparts. Another way to look at this phenomenon is that firms can reduce their cost of equity capital by increasing their market capitalization. This result offers a slim financial justification of conglomerate mergers.

From a portfolio manager's perspective, the small firm effect (as well as the other stock market anomalies) presents, in principle, an opportunity to increase portfolio performance. Are these opportunities real once we take transaction costs into account? The rewards suggested by the seasonal anomalies are substantial but academic research generally assumes no transaction costs¹⁴. Given the limited liquidity of small stocks and their high bid-ask spreads, most strategies which involve active trading will not lead to the desired success. While it may appear irresistible to invest in small Belgian firms every January, commissions and especially the market impact of the trades would most likely wipe out all or most of the gain. The higher costs associated with trading in and gathering information about small firms suggest that an approach which minimizes portfolio turnover, i.e., a buy-and-hold-strategy, is the most feasible one. But the fundamental question remains: is the extra return earned on small-firm stocks a 'free lunch' or is it simply a fair compensation on a riskier investment?

Footnotes

¹ Note the distinction between expected returns and average historical returns. The former is an *ex ante* measure and the latter is an *ex post* measure. If a market is characterized by 'rational expectations' then average historical returns are unbiased estimators of the unobservable returns.

² The expected return is calculated as follows: $\frac{1}{2}(5\%) + \frac{1}{2}(15\%) = 10\%$.

³ The standard deviation is calculated as follows: $\sqrt{\frac{1}{2}(5\% - 10\%)^2 + \frac{1}{2}(15\% - 10\%)^2} = 5\%$.

⁴ Note that stocks with identical betas could have very different firm-specific risks and vice-versa.

⁵ Note that empirical work indicates that the total risk of a security is mostly firm-specific. Market-related risk usually represents less than one-third of the total risk of a security (HAWAWINI, 1984).

⁶ The beta coefficient of a stock can be estimated using standard regression analysis. The estimated slope of the linear regression of stock returns on market index returns is the stock's beta coefficient.

⁷ The t-statistic is the ratio of the coefficient's mean value to its standard deviation. The higher the t-statistic the more 'precise' is the estimate.

⁸ Since the coefficient of beta squared is not significantly different from zero we can conclude that the relationship is linear.

⁹ FORBES (January 1987) reports the following annual returns (capital gains only) during 1986 for a large sample of U.S. common stocks (all stocks for which daily quotations are available): 23% for large firms and 41% for small firms; 27% for low-beta firms and 57% for high-beta firms, 16% for firms with a P/E ratio over 25 and 27% for firms with a P/E ratio under 5; 20% for firms with a dividend yield over 7.5% and 33% for firms with no dividend payments.

¹⁰ Transaction costs are measured by the broker's commission as well as the dealer's bid-ask spread. Smaller firms have relatively wider bid-ask spread and hence higher transaction costs (AMIHUD and MENDELSON, 1986).

¹¹ Recent attempts to examine the size effect in the United States using a different pricing model than the CAPM have not produced conclusive results. The Arbitrage Pricing Theory (APT) developed by Ross (1976) is a multifactor model of asset pricing which generalizes the CAPM. Employing the APT, REINGANUM (1981a) reports that even a five-factor model is unable to explain the size anomaly whereas CHAN (1983) reports that firms size does not have sufficient explanatory power to reject the APT model. GULTEKIN and GULTEKIN (1986) show that there is a strong seasonal pattern in the estimates of the risk premia from the APT model similar to those found for the CAPM (CORHAY, HAWAWINI and MICHEL, 1987b). They conclude that the APT model, like the CAPM, can explain the risk-return relationship in January only in the United States.

¹² Some authors have shown that the risk of small U.S. firms (both total risk and market risk) is higher in January than during the rest of the year (TINIÇ and ROGALSKI, 1986). Hence, the higher return earned by small firms in January may simply reward investors for holding securities that have higher risk in January. But the question of why we observe higher return and higher risk in January for small U.S. firms is still unanswered.

¹³ For example, a January effect has been observed in countries with no taxes on capital gains such as the Netherlands (VAN DEN BERGH and WESSELS, 1985) and Japan (KORAJCZYK and VIALLET, 1987).

¹⁴ See footnote 10.

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