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Options and Portfolio Insurance

1. Introduction

Portfolio insurance, like any other kind of insurance, provides a means of protection against losses. For example, a portfolio manager in charge of a ten million dollar equity portfolio might pay an insurance premium today for protection against losses on the portfolio at the end of the year. If the portfolio is worth ten million or more at the end of the year, he would not have made use of the insurance (but he probably would have slept well during the year); if the portfolio is worth less than ten million, the seller of the insurance will have to make up the shortfall. The situation is similar to the case of the purchase of house fire insurance. In this case, if the house does not burn down during the year the owner loses the premium that he paid, but if there is a fire, the insurance company will make up or pay any losses caused by the fire.

Although portfolio insurance has similarities with other kinds of insurance such as fire, theft, life, and automobile insurance, it also has some basic differences. In traditional insurance situations, the company selling the policy can largely diversify away the risk it is incurring because to a large degree, the risks are independent. However in the case of portfolio insurance it is not possible to diversify away the risks because when one portfolio does poorly, it is highly probable that most of the insured portfolios do poorly. In essence, the overall market risk of a portfolio cannot be diversified away since it must be borne eventually by someone in the economy. Therefore, if a portfolio manager buys insurance, there must be some agent in the economy who takes the risks on the other side. Fortunately, as a result of recent developments in capital markets there now exist new securities which are designed for such risk-sharing arrangements. These securities are options: call and put options.

The purpose of this paper is to show how it is possible to use options to insure a portfolio. For the most part, we will be talking about options on stock market indices like the Standard and Poor 100 or the Standard and Poor 500, and how these options can be used to insure portfolios which correlate highly with these respective indices. The basic ideas, however, apply also when the objective is to insure a position on an individual stock, a position in gold or silver, or a position in foreign currency. There exist options on stocks, options on gold and silver, and options on currencies that can be used to insure these positions. Moreover, it will be shown that even those positions which do not have traded options on them are possible to insure because dynamic strategies can be employed to reproduce options.

There has been some debate in the academic literature on the subject of the conditions under which an investor would want to buy portfolio insurance[1]. This paper will not concern itself with the conditions under which a particular investor would want to buy portfolio insurance or to what degree he may want to insure his portfolio. Rather, the issue addressed here will be how a portfolio manager or an investor can insure his portfolio if he has decided that he does want this insurance.

In Section 2 of the paper, calls and put options are described. The reader familiar with these securities can skip directly to Section 3 which shows how these options can be used to insure a portfolio. Section 4 introduces some practical considerations that the portfolio manager must take into account if he wants to in-

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sure a portfolio. Section 5 shows how it is possible to replicate an option with positions in equities and cash or treasury bills. This procedure allows the insurance of portfolios when no suitable options exist. Section 6 discusses some of the limitations of the analysis and Section 7 provides some concluding remarks.

2. Put and call options

A call option is a contract giving its owner the right to buy a security at a fixed exercise price on or before the maturity of the option. The general term "security" is used to denote all the underlying assets on which options can be written: individual stocks, stock indices, precious metals, foreign currency, bonds, treasury bill, futures contracts, and so on. If the price of the underlying security exceeds the exercise price at the maturity of the option, the holder of the call option will exercise the option and its value will be the difference between the security price and the exercise price. On the other side, if at the maturity of the option, the security price is below the exercise price, it will not be in the interest of the holder of the option to exercise it, and its value will be zero. The value of the call option at maturity can then be expressed mathematically as

$$C^* = \text{Max}(S^* - X, 0) \quad (1)$$

where C^* is the value of the call option at maturity, S^* is the value of the underlying security at the maturity of the option and X is the exercise price of the option. Figure 1 gives a graphical representation of this relationship. Note

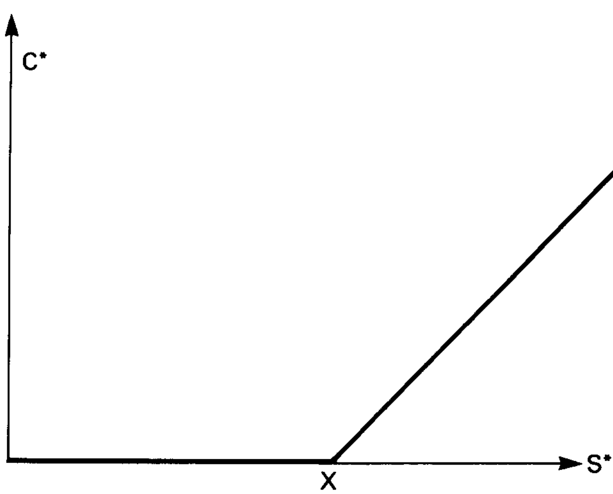

Figure 1: Value of a call option at maturity.

that the value of the call option shown in the figure does not take into account the initial payment for the call; for the holder of the call to break even it is necessary that the terminal value of the underlying security exceeds the exercise price by the amount initially paid for the option.

A *put option* is a contract giving its owner the right to sell the security at a fixed price on or before the maturity of the option. Given that a put is an option to sell the underlying security, the holder of the option will exercise this right only if the value of the underlying security at the maturity of the option is below the exercise price, and its value will be the difference between the exercise price and the security price at maturity. This can be stated as

$$P^* = \text{Max}(X - S^*, 0) \quad (2)$$

where P^* is the value of the put option at maturity. Figure 2 gives a graphical representation of this relationship. Here again the figure does not take into account the initial price paid for the put. The exercise price must exceed the terminal price of the underlying security by the price initially paid for the put for its holder to break even.

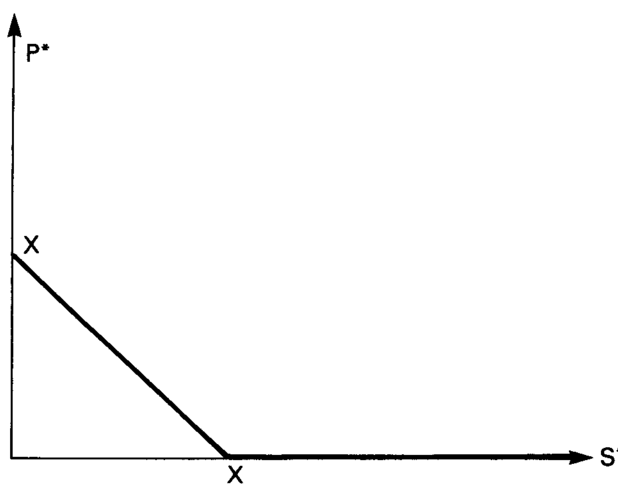

Figure 2: Value of a put option at maturity.

When the options can be exercised at any time before maturity, as described above, the options are said to be of the American type. If the options can only be exercised at maturity, they are called European options. Most traded options are of the American type. European options, however, will be shown to be more relevant in the process of portfolio insurance.

Parallel to the rapid development of the option markets in the last fifteen years has been a corresponding progress in academic research concerned with the evaluation of these new securities. In two seminal papers published in 1973, Black and Scholes (1973) and Merton (1973) provided the basic framework of option pricing theory. The Black-Scholes formula for valuing European options has become the most widely used formula by practitioners in the options industry. The Black-Scholes formula will be discussed in more detail in Section 5 when the procedures for replicating options with positions in equities and cash are discussed.

3. How to use options to insure a portfolio

Consider a portfolio manager who is managing a portfolio with a current value of S. (If you prefer to consider more concrete cases, assume that S is a portfolio of ten million dollars.) Figure 3 shows the value of the uninsured portfolio sometime in the future, say one year, for different values of the stocks that form the portfolio. The movements along the 45 degree line

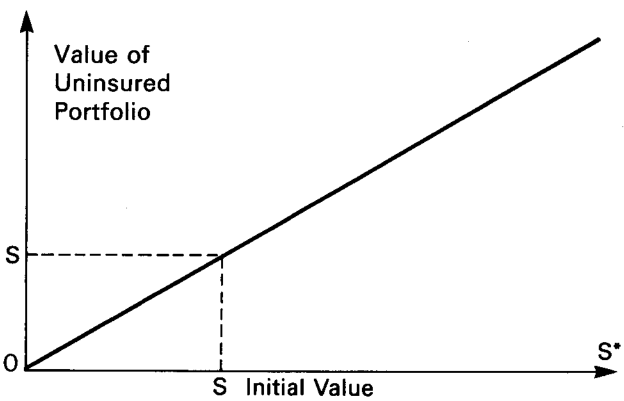

Figure 3: Terminal value of portfolio without insurance.

represent the possible terminal values of the underlying security: the essence of the problem is that there is uncertainty about this terminal value. This figure is not very interesting on its own because it simply says that the terminal value of the uninsured portfolio will be equal to the terminal value of the stocks forming the portfolio. It shows, however, the possibility of losses and serves as a base for combining the portfolio with options.

The idea behind portfolio insurance is to ensure that the terminal value of the portfolio will not be worth less than its current value (S). By combining Figures 2 and 3 into Figure 4 it can be seen that this objective can easily be achieved if the manager buys a put option on the portfolio with an exercise price equal to the current value of the portfolio.

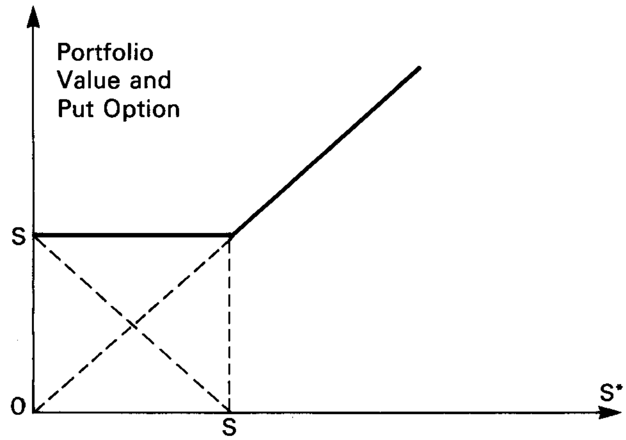

Figure 4: Terminal value of portfolio with insurance.

Notice from Figure 4 that if the terminal value of the portfolio falls below S (the initial value), the put option will be worth $S-S^*$ (since the exercise price of the option has been set equal to the initial value of the portfolio), and the total value of the pòrtion will be worth $S^*+(S-S^*)=S$. That is, the portfolio will have been perfectly insured. On the other side, if the terminal value of the portfolio is greater than its initial value the put is worthless and the total value of the portion is S^*. That is, the owner of the portfolio gets all the benefits of a positive market reaction. Since by buying a put the portfolio manager is able to perfectly insure his portfolio against losses, the cost of the insurance (i.e., the insurance premium) is the price of the put option. Figure 4 shows that the terminal value of the portfolio is perfectly insured against losses, but it does not show that the cost of the insurance is the initial price of the put option. To break even the value of the portfolio has to increase by the cost of the put option.

To simplify matters the example has assumed that the portfolio manager wants to buy insurance for exactly the initial value of the portfolio. He could also, however, insure the portfolio for only 95% of its initial value (an insurance with a 5% "deductible") or even 105% of its initial value. This can be easily accomplished by setting the exercise price of the put option at

0.95 S or 1.05 S, respectively. It should be clear that the second option will be more expensive than the first. To keep things easier to understand, this paper will deal mainly with full portfolio insurance (no deductible), when the premium is paid at the time of purchase of insurance, and when the insured portfolio gets all the benefits of an increase in the value of the portfolio[2].

So far, the concept of portfolio insurance has been introduced using put options. It will now be shown that the same result can be achieved by the use of call options. Consider again a portfolio manager who wants to insure a portfolio with an initial value of S, but this time instead of leaving his money in the portfolio and buying a put, he sells the portfolio (if you don't like the transaction costs involved in this procedure assume that our portfolio manager is starting with cash and is thinking about buying the portfolio) and buys a call option and treasury bills. More specifically, he buys one call option with an exercise price of S and the amount of treasury bills that will produce exactly S at the maturity of the option (i.e., the present value of S: $S/(1+r)^t$, where r is the interest rate and t is the insurance period assumed to be one year). It is easy to see from Figure 5 that this combination will produce the same outcome as the portfolio plus put combination. The treasury bills will be worth S at maturity whatever the

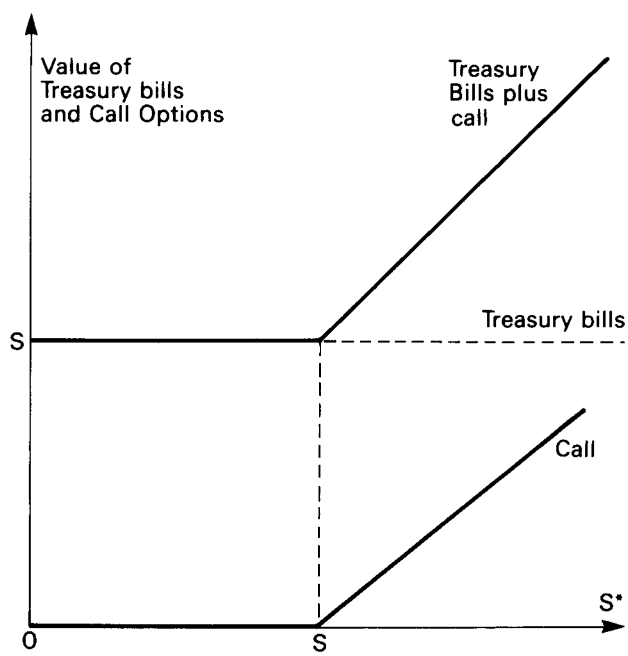

Figure 5: Terminal value using call option.

terminal value of the portfolio, and the call option will be worth something at maturity only if the terminal value of the portfolio is greater than its initial value. So again we have that if the terminal value of the portfolio is less than its initial value, the owner of the portfolio retains S, and otherwise he receives S*.

One of the most fundamental principles in finance is that two portfolios (or securities) that produce the same outcome in every possible future state of the world, should have the same value today. If they did not, riskless arbitrage profits would be possible by selling the more valuable and buying the cheaper portfolio. For the particular situation discussed above, this principle gives rise to the well-known put-call parity relationship for European options which relates the values of European puts and calls under very general conditions:

$$S + P = C + X/(1+r)^t \qquad (3)$$

In equation (3) both the put and the call have an exercise price of X (in the discussion above the exercise price was set equal to the inital value of the portfolio S). In words, equation (3) says that the current value of the portfolio plus a put is equal to a call plus the present value of the exercise price. The put-call parity relationship can also be easily modified to take account of dividend payments which we have so far ignored. Unfortunately the put-call parity relationship only holds for European options, since the early exercise possibility of American options produces different outcomes for some possible states of the world for the two combinations considered. Fortunately, however, portfolio insurance is conceptually more related to European options for which there is a nice, simple relationship between puts and calls.

4. Some practical considerations

The analysis of the previous section seems to indicate that the purchase of portfolio insurance is a simple exercise. From a conceptual point of view it is very straightforward. However, there are a number of factors that complicate the practical implementation of portfolio insurance. In this section some of these complications will be briefly considered.

A major difficulty is that in most cases traded options on the particular portfolio under consideration do not exist. This is particularly likely to be the case in Europe, but may also be a problem in the U.S. where the traded options on the major indices might not appropriately represent the portfolio to be insured[3].

Another difficulty is that even if there are traded options on an index that closely resembles the portfolio, traded options are of the American type. They are usually of short maturity, and sometimes the market for these options is very thin.

American options are worth at least as much as European options and usually are worth more. The reason for this is that it is possible to exercise an option before maturity, something holders of European options are unable to do. Portfolio insurance is more related to European options since the objective is that the value of the portfolio be guaranteed at the expiration of the option. If all the traded options are of the American type the purchaser of portfolio insurance would be paying a higher premium than would be necessary, given his objective. On the positive side the minimum value of the portfolio would be guaranteed during the whole life of the option and not only at maturity.

Most of the trading in index options is in contracts close to expiration (usually three months), though options up to nine months exist. Therefore, if the objective of a portfolio manager is to insure his portfolio for one or two years he would not be able to find an appropriate option. This problem could be partially overcome by rolling over short-term options, but this procedure could be more expensive and there is some uncertainty as to the total premium to be paid.

The market for options on some of the most popular indices, like the S and P 500, is too thin to allow for the insurance of a large portfolio. If the market is too thin, the purchase of options to insure a portfolio could affect the price of these options, increasing the cost of insurance.

The problems discussed above might lead the reader to believe that portfolio insurance is of little practical value. Fortunately, this is not the case. Not only have markets deepened, but recent developments in option pricing theory have provided the means of determining the equilibrium value of puts and calls and, also, critically important for the issue of portfolio insurance, the procedure by which the pay-offs of options can be replicated by positions in the underlying security (portfolio) and treasury bills. This is the topic of the following section.

5. Replicating options with positions in equities and treasury bills

To be able to understand how it is possible to replicate the pay-offs of options by dynamic investments in equities and treasury bills it is necessary to understand the logic behind the derivation of the BLACK-SCHOLES formula for valuing options. This section will start, then, with a brief, non-technical description of this logic.

Under certain assumptions about the random changes in security prices (basically a "random-walk"), it is possible to show that for a very short period of time the random movement of the price of a call option is perfectly correlated with the random movement of the price of the underlying security. This means that for short time periods when the security price moves up, the call price *always* moves up, and vice versa. This perfect instantaneous correlation between security and call prices implies that it is possible to find offsetting positions in the two assets (for example, buy one call option and take an offsetting short position in the security) such that the portfolio of the two assets is insensitive to movements in the security price. This position is called a perfect hedge because it is instantaneously riskless.

The key insight by BLACK and SCHOLES (1973) was that if it is possible to form a riskless portfolio by appropriate positions in the call option and the underlying security, to avoid the possibility of arbitrage profits, the return on this riskless portfolio must be the riskless interest rate. The absence of riskless arbitrage profits in the economy is a widely accepted principle in Financial Economics: even if they exist in certain markets on certain occasions, individuals would immediately come in to profit from them and by doing so eliminate them. In the above context, the absence of arbitrage opportunities implies that the return on the perfect hedge is equal to the return on a Treasury bill (both for a very short period of time), since both are riskless.

The above analysis and some complex mathematical manipulations produce a partial differential equation for the value of the call option as a function of the price of the underlying security and time to maturity (in addition to the characteristics of the option and the economic environment). The solution of this partial differential equation, subject to the terminal condition (1) for a European call option on a non-dividend paying security, is the famous BLACK-SCHOLES formula.

The formula is:

$$C = S \cdot N(d) - \frac{X}{(1 + r)^t} N(d - \sigma\sqrt{t}) \qquad (4)$$

where

C = the theoretical value of the European call option
S = the current value of the underlying security
X = the exercise price of the option
t = the time to expiration of the option
r = the riskless interest rate (Treasury bill rate)
σ = the standard deviation (volatility) of the rate of return on the underlying security

$d = [\ln(S/X) - (r - 1/2\sigma^2)\, t]/\sigma\sqrt{t}$

and

N(d) = the cumulative standard normal density function

The standard normal density function (with a mean of zero and a standard deviation of one) is shown in Figure 6. The total area under the bell-shaped curve is equal to one since it is the probability of all possible outcomes, Z, occurring. The cumulative function N(d) is the area under the curve up to the value of d (see Figure 6). Clearly, the value of N(d) lies between zero and one. The value of a call option given by equation (4) can therefore be intuitively interpreted as a weighted difference between the security price and the present value of the exercise price of the option, in which the weights can only take values between zero and one.

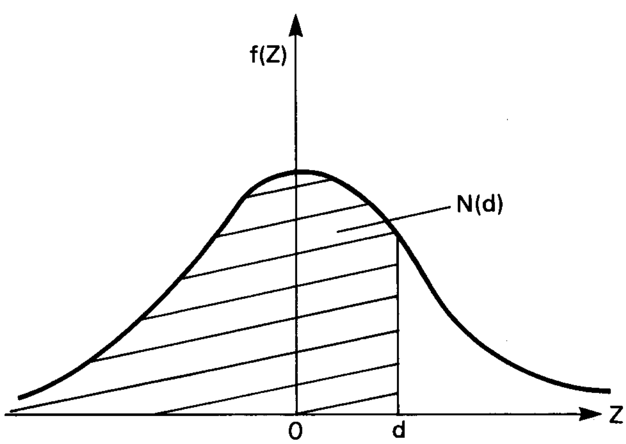

Figure 6: Normal density function.

When the option is very far "out of the money", which means that there is practically no possibility that it will be worth anything at maturity, both weights are equal to zero and the call option value is also equal to zero[4]. When the option is very far "in the money", which means there is no chance that the security price will fall below the exercise price, both weights have a value of one and the call option has a value equal to the difference between the security price and the present value of the exercise price. Loosely speaking the weights represent the probability that the option will expire in the money (where the security price is greater than the exercise price).

The variables that effect the value of the call option are seen very clearly in equation (4). They are, the security price, the exercise price, the time to maturity, the interest rate and the volatility of the security price. Of these only the last one is not directly observable although it can be estimated using a time series of past security prices. An important feature of equation (4) is that the call value does not depend on the expected rate of return of the underlying security. The reason for this is that the current price of the security already embodies the market expectations about future security prices.

The above analysis is of interest not only because it gives us a formula to value call options, but also because it tells us the *hedge ratio.* The hedge ratio is the number of shares of the underlying security that must be sold short to perfectly hedge a purchase of one call option to changes in the security price. Or equivalently, the hedge ratio is the number of shares that must be bought to perfectly hedge the writing of one call option[5].

The hedge ratio is given by:

$$\text{hedge ratio} = N(d) \qquad (5)$$

where d is the same as in equation (4).

In Figure 7 the curve gives the value of the call option for different values of the underlying security for a given time to maturity of the option. The hedge ratio is given by the slope of this curve. Note that for very high prices of the underlying security the hedge ratio is equal to one, so one share of the security must be sold to perfectly hedge one long position in the call. At the other extreme, for very low security prices the hedge ratio is close to zero and almost none of the security is needed to perfectly hedge a worthless call.

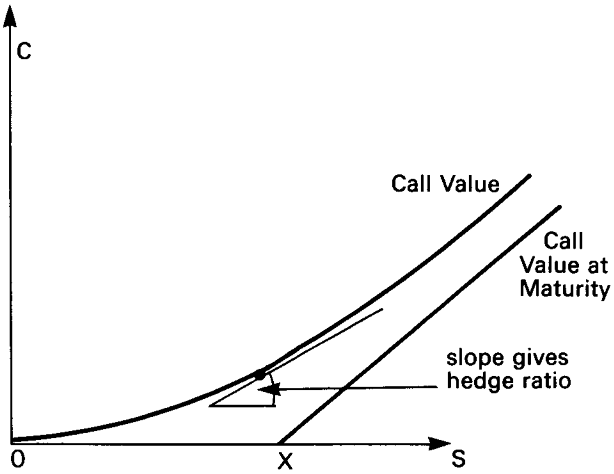

Figure 7: Call values and hedge ratios.

Let us recapitulate the above discussion to highlight the key insights that relate to portfolio insurance. We have learned that it is possible to buy a call option and sell an appropriate amount (determined by the hedge ratio) of the underlying security to create a riskless portfolio with a return equal to the riskless interest rate. Because the short position in the security is larger than the long position in the call, the portfolio is equivalent to borrowing at the riskless interest rate. But if a long position in a call and a short in the security is equivalent to borrowing, it is easy to see that a call can be *reproduced* by a long position in the security and borrowing, that is, a levered security purchase. It should be clear, however, that to reproduce the pay-offs on the call option it is necessary to periodically adjust the position in the underlying security to reflect the changes that the passage of time and changes in security prices have on the hedge ratio.

Just as it is possible to reproduce a call option by a dynamic (changing with time) investment in the underlying security and borrowing, so it is also possible to reproduce a put option by a short position in the security and lending (or buying the Treasury bills). Panel A of Figure 8 shows these relationships.

In section 3 it was shown that an insured portfolio can be obtained either by an investment in the portfolio and a put option or by a call option and lending. By reproducing the call and put options with appropriate investments in the portfolio and borrowing or lending it is easy to see that whichever way you look at the problem the insured portfolio can be reproduced by taking long positions in the portfolios and lending (buying Treasury bills). This relationship is shown in Panel B of Figure 8.

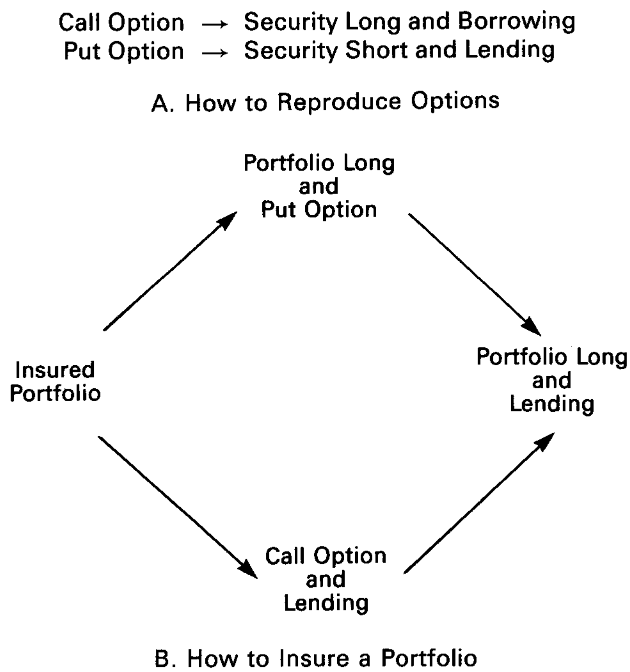

Figure 8: How to reproduce options and how to insure a portfolio.

By moving funds between the equities in the portfolio and Treasury bills in a way that exactly replicates the option the objective of the portfolio insurance can be obtained without the existence of the actual traded options[6]. The terminal value of the portfolio will be insured at the chosen exercise price and the cost of insurance will be the theoretical value of the put option.

The proportion of the funds invested in the portfolio is determined by the hedge ratio. When equity prices rise, the hedge ratio rises (see Figure 7) and Treasury bills are sold to invest in equities. When equity prices fall, the hedge ratio also falls and equities are sold to invest in Treasury bills. For very high equity prices (hedge ratio equals one) the funds are fully invested in equities, and for very low equity prices (hedge ratio equals zero) the funds are fully invested in Treasury bills. Following this procedure produces the desired outcome at the end of the insurance period: if the portfolio value goes up you end up in equities, and if the portfolio value goes down you end up in Treasury bills.

6. Limitations of the analysis

The previous section explaining the possibility of replicating options by dynamic allocation of funds to equities and to Treasury bills overlooked some important limitations. This section will consider these limitations, and assess their relevance for the practical application of dynamic portfolio insurance.

A number of simplifying assumptions have been made to obtain the relatively simple formulas for the value of a call option, equation (4), and the hedge ratio, equation (5). In particular, it has been assumed that continuous trading is allowed, that there are no transaction costs, that there are no jumps in security prices, that the interest rate is constant, that the volatility of security prices is constant, and that securities pay no dividends.

Since in practice there are transaction costs it is not possible to adjust the hedge ratio at every instant in time[7]. The solution to this problem is to change the allocation of funds between the portfolio and Treasury bills at fixed intervals of time (for example, after each week or each month) or even better, only when the underlying portfolio value changes by more than a predetermined amount (usually 1 or 2%). However, there are two negative consequences of not trading continuously. First, the cost of portfolio insurance will be higher than the value of the put option due to the transaction costs. Second, the insurance is not perfect since if there are very fast changes in the value of the portfolio (say 5% in a day) it will not be feasible to adjust the allocation of funds fast enough to maintain the required hedge ratio.

If the dividends to be received on the equities forming a portfolio during the insurance period are known, it is easy to modify the formulas for option values and hedge ratios to take them into account. If there is great uncertainty about dividends the situation is more problematic. However, in general dividends are slow to change. Therefore, the presence of dividends does not seem to be critical in the practical applications of the portfolio insurance procedure.

Major changes in interest rates during the insurance period may affect the actual cost of insurance since the true cost of the put option and the hedge ratios will depend on interest rates. This effect unlike that of transaction costs, can go either way, potentially increasing or decreasing the cost of insurance.

The most critical variable affecting option values and hedging ratios is the volatility of the underlying security prices. Although it is not difficult to estimate price volatilities using time series of past security prices, the problem is that volatilities are not constant. Different volatility estimates are obtained depending on the length of the previous period over which the estimates are calculated and there is no clear criteria to select the most appropriate one. Moreover, past security price volatility, whatever the length of the period over which it is estimated, is an imperfect measure of the expected *future* volatility during the life of the option. Option traders are well aware of the essential role that forward looking volatilities play in option prices; for those who hedge their positions options trading is really a "volatility game". Dynamic portfolio insurance strategies are also affected by changing volatilities. The actual cost of insurance will depend on the realized volatility during the insurance period. To deal with this problem some firms instead of selling insurance for a given time period of, say, one year, sell insurance for a given amount of cumulative volatility. Therefore, if realized volatility is less than expected, the insurance lasts longer, and if volatility is more than expected, the insurance period is shorter.

In spite of the complications, dynamic strategies are increasingly being used in practice to insure portfolios. Many innovations with respect to the simple approach described in this paper have been implemented to partially deal with the limitations described. For example, to decrease transaction costs incurred in purchasing and selling the large number of stocks included in any well-diversified portfolio, whenever it is possible the actual hedging is done with index futures where transaction costs are much lower. Index futures are now widely traded in the United States.

7. Summary and conclusions

The purpose of this paper has been to show the relationship between options and portfolio insurance. Having established this relationship, the issue of portfolio insurance has been shown to be one of finding the appropriate combination of options to insure a given portfolio.

Recent developments in the markets for traded options and in the theory of option pricing have facilitated the process of introducing the new concept of portfolio insurance. There is a growing interest among practitioners, particularly in the U.S., to consider the use of these tools in the management of portfolios.

Portfolio managers have been using various procedures for controlling the risk of their portfolios[8]. A larger proportion of fixed income securities, for example, may reduce the upside potential of an upswing in the equity markets, but it also reduces the possibility of losses in the case of a downswing. Appropriate stop-loss orders could also limit the exposure of an equity portfolio. Portfolio insurance offers a new and interesting tool that adds to those used in the past. As the procedures become generally known, the practice seems bound to increase. The manager who does not learn how to use options or dynamic strategies to insure portfolios does so at his and his clients' peril.

Footnotes

[1] See for example, BRENNAN and SOLANSKI (1981), LELAND (1980), BENNINGA and BLUME (1985).

[2] Using some simple properties of European options it is also possible to insure a portfolio for its full initial value, without any apparent payment for the insurance premium. The trick here is that the owner of the portfolio will not get the full benefit of a market upswing. Most of the commercial users of portfolio insurance in the U.S. employ this procedure.

[3] Even in those cases where options on the individual stocks of the portfolio exist, a portfolio of options is not equal to an option on a portfolio. A portfolio of put options would insure for example, each of the stocks in the portfolio individually; this is clearly superior insurance than a put option on the total portfolio, but it would also be correspondingly more expensive.

[4] An "out of money" option is one where the exercise price for a call option is higher than the market price, or where the exercise price for a put option is lower than the market price.

[5] The writer of a call option is the person who agrees to deliver the security if the buyer finds it worthwhile exercising the option. The writer of a put option is the person who agrees to buy the security if the put is exercised.

[6] These ideas were first presented by BRENNAN and SCHWARTZ (1976, 1979).

[7] For a recent and novel discussion of options with transaction costs see LELAND (1985).

[8] For a recent discussion of some of these ideas see RUBINSTEIN (1985.)

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